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Mathematical model of sexual orientations with advocacy

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Abstract

Sexual orientation is the attraction one feels to men, women, or both, while advocacy is any effort or activity geared towards supporting people of diverse sexual orientations to eradicate discrimination within a community. In this paper, a comprehensive mathematical model of sexual orientations with advocacy was developed, and the efficacy of advocacy is analysed. The model divided the population into ten compartments: Males, Females, gays, lesbians, heterosexual males, heterosexual females, bisexual males, bisexual females, recovery centres for males, and recovery centres for females. The positivity, boundedness, equilibria points, and control reproduction numbers were studied. The Routh-Hurwitz criteria were used to assess the local stability of the disease-free equilibrium, the Metzler Matrix method was used to determine global stability, while Lyapunov functions were used to study the global stability of the endemic equilibrium point. Bifurcation analysis was also conducted. Lastly, a sensitivity analysis and numerical simulations were conducted using Wolfram Mathematica and MATLAB software. Numerical results showed that the model with advocacy increases the gay, lesbian, bisexual male, and bisexual female populations by 5.7%, 15.6%, 2.6%, and 15.3%, respectively, compared with the model without advocacy. In conclusion, the study showed that while incorporating advocacy in the community significantly increased homosexual populations in the society, Lesbian and bisexual female populations were increased the most.

Keywords: Advocacy, homosexuality, stability, reproduction number, sexual orientation

1. Introduction

Sexual orientation has been defined as the attraction one feels to men, women, or both ^[1, 2]. It has been a challenge conceptualising the sexual orientation phenomenon since it has been characterised by shifting societal and scientific perceptions. In past years, sexual orientation has always been seen through a binary lens where heterosexuality was perceived as the standard. In contrast, other orientations were seen as a deviation from the norm. Magnus Hirschfeld and Alfred Kinsey began to change this perception by pushing for a deeper understanding of human sexuality ^[3]. Since then, there has been increased research work showing sexual orientations as a varied spectrum, leading to recognition of lesbians, gays and Bisexual Individuals (LGB) ^[4]. In 2015, the Supreme Court of the United States of America stated that a permit should be issued to same-sex couples ^[5]. Despite these efforts to recognize different spectrums in sexual orientations, a lot of LGB people still go for therapy to change their sexual attraction. In the United States, for instance, 60% of sexual minority people visit psychiatrists, psychologists, and religious leaders to change their sexual orientation ^[6]. Advocacy is the deliberate process of influencing those who make or have responsibility for implementing policy decisions. At the same time, an advocate publicly supports or recommends a particular cause or policy, or pleads on another's behalf ^[7, 8]. Advocacy is an action that aims to change laws, policies, practices, and attitudes. It can include public activities designed to gain media attention, such as flashmobs, demonstrations, or marches, non-public activities such as meetings with politicians, workshops, and trainings for key decision-makers and community leaders.

Lobbying in international forums; or mass campaigning through letter-writing, public meetings, or social media ^[9].

Mathematicians have also furthered knowledge in this study area by modelling phenomena revolving around this topic and other real-life situations ^[10, 11]. Such studies include, but are not limited to, HIV/AIDS models considering sexual preferences ^[12], analysing the impacts of bisexual behaviour in a global community ^[13], a model of sexual transmission of Hepatitis B considering sexual orientations ^[14] among LGBT populations.

This paper seeks to develop a comprehensive mathematical model of sexual orientations with advocacy, to further knowledge in the existing literature.

2. Model formulation

This study uses classical SIRS (susceptible, infected, recovered and susceptible) to model sexual orientations with advocacy

Model assumptions

- It was assumed that homosexuality (same-sex marriage) is not hereditary; instead, people are convinced to practice it.
- One can only be influenced by a gay, a lesbian, a bisexual male, or a bisexual female into homosexuality.
- Advocacy is only done by Gay, Lesbian, Bisexual males or Bisexual females.

2.1 Model development

The model is formulated into ten compartments, Males (M), Females (F), gays (G), lesbians (L), heterosexual males (H_m), heterosexual females (H_f), bisexual males (B_m), bisexual females (B_f), recovery centers for male (R_m) and recovery centers for females (R_f). Gays, lesbians, bisexual males and bisexual females will be referred to as homosexual populations. Below is the schematic representation of the model.

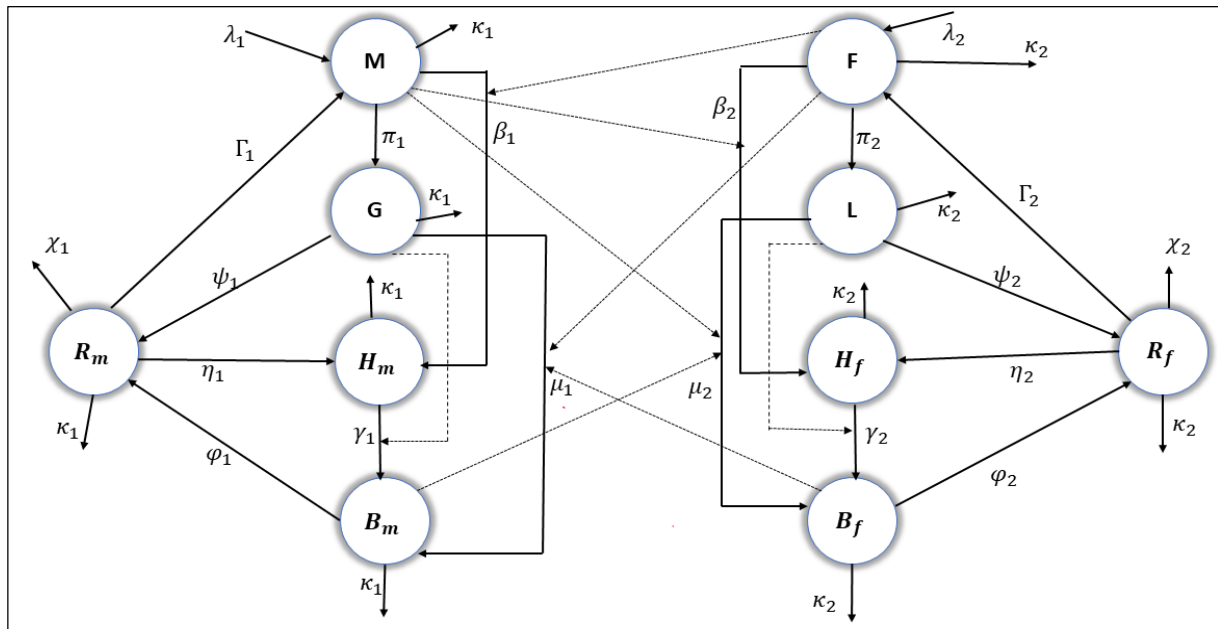


Fig 1: Model flow diagram

2.2 Model Equations

$$\left\{ \begin{array}{l} \frac{dM}{dt} = \lambda_1 + \Gamma_1 R_m - (\kappa_1 + \beta_1 F + \pi_1) M \\ \frac{dG}{dt} = \pi_1 M - (\kappa_1 + \psi_1) G - \mu_1 (F + \theta_1 B_f) G \\ \frac{dH_m}{dt} = \beta_1 M F + \eta_1 R_m - (\kappa_1 + \gamma_1) H_m \\ \frac{dB_m}{dt} = \gamma_1 H_m + \mu_1 (F + \theta_1 B_f) G - (\kappa_1 + \phi_1) B_m \\ \frac{dR_m}{dt} = \phi_1 B_m + \psi_1 G - (\kappa_1 + \eta_1 + \chi_1 + \Gamma_1) R_m \\ \frac{dF}{dt} = \lambda_2 + \Gamma_2 R_f - (\kappa_2 + \beta_2 M + \pi_2) F \\ \frac{dL}{dt} = \pi_2 F - (\kappa_2 + \psi_2) L - \mu_2 (M + \theta_2 B_m) L \\ \frac{dH_f}{dt} = \beta_2 M F + \eta_2 R_f - (\kappa_2 + \gamma_2) H_f \\ \frac{dB_f}{dt} = \gamma_2 H_f + \mu_2 (M + \theta_2 B_m) L - (\kappa_2 + \phi_2) B_f \\ \frac{dR_f}{dt} = \phi_2 B_f + \psi_2 L - (\kappa_2 + \eta_2 + \chi_2 + \Gamma_2) R_f \end{array} \right. \quad (1)$$

Where,

$$\pi_1 = d_1(G + e_1 B_m + e_2 L + e_3 B_f)$$

$$\gamma_1 = d_2(G + e_1 B_m + e_2 L + e_3 B_f)$$

$$\gamma_2 = d_5(G + e_1 B_m + e_2 L + e_3 B_f)$$

$$\pi_2 = d_8(G + e_1 B_m + e_2 L + e_3 B_f)$$

Table 1: Parameter description

Parameter	Description	Parameter	Description
λ_1	Male Birth Rate	λ_2	Female birth rate
κ_1	The natural death rate for males	κ_2	Natural death rate for females
π_1	Conversion rate from Male to Gay	π_2	Conversion rate from females to lesbians
β_1	Conversion rate from male to heterosexual male	β_2	Conversion rate from female to heterosexual female
Γ_1	Conversion rate from the male recovery center to the male	Γ_2	Conversion rate from the female recovery center to the female
μ_1	Conversion rate from Gay to bisexual male	μ_2	Conversion rate from Lesbian to bisexual female
γ_1	Conversion rate from heterosexual male to bisexual male	γ_2	Conversion rate from heterosexual female to bisexual female
φ_1	The transfer rate from the Bisexual male to male Recovery center	φ_2	The transfer rate from bisexual females to the female Recovery center
η_1	Conversion rate from male recovery center to Heterosexual male	η_2	Conversion rate from the female recovery center to heterosexual females
ψ_1	The transfer rate from gay to a male recovery center	ψ_2	The transfer rate from lesbians to the recovery center
χ_1	The death rate due to the effects of the Male Recovery Center	χ_2	The death rate due to the effects of the Female Recovery Center
θ_1	The rate at which B_f converts G to B_m	θ_2	The rate at which B_m converts L to B_f
e_1	Infection rate among bisexual males (B_m)	e_3	Infection rate among bisexual females (B_f)
d_1	Gay recruitment rate	d_2	Bisexual male recruitment rate
d_5	Bisexual female recruitment rate	d_8	Lesbian recruitment rate
e_2	Infection rate among Lesbians		

3. Invariant regions

This section studies boundedness and positivity to establish if the model is in a feasible region for consideration.

3.1 Boundedness

Theorem 1

If the initial conditions of system 1 are in domain Q , then all solutions of system 1 enter and remain in the same domain Q .

Proof

It is first shown that all feasible solutions are uniformly bounded in a proper subset of Q . Splitting the system into the male component (N_m) and the female component (N_f) given as $N_m = M + G + H_m + B_m + R_m$ and $N_f = F + L + H_f + B_f + R_f$. Let $(N_m = M + G + H_m + B_m + R_m)$ be any system solution with non-negative initial conditions.

Since $\frac{dN_m}{dt} \leq \lambda_1 - N_m \kappa_1$ it follows that, $\lim_{t \rightarrow \infty} N_m(t) \leq \frac{\lambda_1}{\kappa_1}$. This implies that $0 \leq N_m \leq \frac{\lambda_1}{\kappa_1}$. Similarly, the female population simplifies to $0 \leq N_f \leq \frac{\lambda_2}{\kappa_2}$ [15]. Hence the proof.

3.2 Positivity

$$Q = \{M(t), G(t), L(t), H_m(t), H_f(t), B_m(t), B_f(t), R_m(t), R_f(t)\} \in R_+^{10} \text{ and } M(0) \geq$$

$$0, G(0) \geq 0, L(0) \geq 0, H_m(0) \geq 0, H_f(0) \geq 0, B_m(0) \geq 0, B_f(0) \geq 0, R_m(0) \geq$$

$$0, R_f(0) \geq 0 \text{ then, the region will be positively bounded.}$$

Proof

$$\frac{dM}{dt} = \lambda_1 + \Gamma_1 R_m - M(\kappa_1 + \beta_1 F + \pi_1)$$

$$\lambda_1 \geq 0, \Gamma_1 \geq 0$$

$$\frac{dM}{dt} \geq -M(\kappa_1 + \beta_1 F + \pi_1)$$

$$\frac{dM}{M} \geq \int_0^t -(\kappa_1 + \beta_1 F(s) + \pi_1) ds$$

$$M \geq e^{-\int_0^t (\kappa_1 + \beta_1 F(s) + \pi_1) ds} + C_1$$

Where, C_1 , it is a constant of integration. Putting initial conditions;

$$M(0) = C_1$$

$$M(t) \geq M(0)e^{-\int_0^t (\kappa_1 + \beta_1 F(s) + \pi_1) ds} \geq 0$$

Using a similar argument,

$$G(t) \geq G(0)e^{-\int_0^t (\kappa_1 + \psi_1 + \mu_1 F(s) + \mu_1 \theta_1 B_f(s)) ds} \geq 0$$

$$H_m(t) \geq H_m(0)e^{-(\kappa_1 + \gamma_1)t} \geq 0$$

$$B_m(t) \geq B_m(0)e^{-(\kappa_1 + \varphi_1)t} \geq 0$$

$$R_m(t) \geq R_m(0)e^{-(\kappa_1 + \eta_1 + \chi_1 + \Gamma_1)t} \geq 0$$

$$F(t) \geq F(0)e^{-\int_0^t (\kappa_2 + \beta_2 M(s) + \pi_2) ds} \geq 0$$

$$L(t) \geq L(0)e^{-\int_0^t (\kappa_2 + \psi_2 + \mu_2 M(s) + \mu_2 \theta_2 B_m(s)) ds} \geq 0$$

$$H_f(t) \geq H_f(0)e^{-(\kappa_2 + \gamma_2)t} \geq 0$$

$$B_f(t) \geq B_f(0)e^{-(\kappa_2 + \varphi_2)t} \geq 0$$

$$R_f(t) \geq R_f(0)e^{-(\kappa_2 + \eta_2 + \chi_2 + \Gamma_2)t} \geq 0$$

Hence, the region Q is positively invariant, and the model is well-posed mathematically. Since the model monitors human populations, all the variables and parameters are assumed to be non-negative for $t \geq 0$.

4. Equilibrium states, reproductive number, and stability**4.1 Homosexuality-free equilibrium point (HFE).****Theorem 3**

System 1 has an HFE obtained by setting all the homosexual and recovered classes to zero. ^[16]

Proof

$$E^0 = (M^0, G^0, H_m^0, B_m^0, R_m^0, F^0, L^0, H_f^0, B_f^0, R_f^0) = (\zeta_1, 0, \zeta_2, 0, 0, \zeta_3, 0, \zeta_4, 0, 0)$$

$$\text{Where } \zeta_1 = M^0, \zeta_2 = H_m^0, \zeta_3 = F^0, \zeta_4 = H_f^0$$

$$\begin{aligned} \xi_1 &= \frac{2\kappa_2\lambda_1}{\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2 + \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (\kappa_1\kappa_2 + \beta_2\lambda_1 - \beta_1\lambda_2)^2}} \\ \xi_2 &= \frac{\kappa_1\kappa_2 + \beta_2\lambda_1 + \beta_1\lambda_2 - \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (\kappa_1\kappa_2 + \beta_2\lambda_1 - \beta_1\lambda_2)^2}}{2\kappa_1\beta_2} \\ \xi_3 &= \frac{-\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2 + \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (-\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2)^2}}{2\kappa_1\beta_2} \\ \xi_4 &= \frac{\beta_2\lambda_1(-\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2 + \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (-\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2)^2})}{\beta_1\kappa_2(\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2 + \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (\kappa_1\kappa_2 + \beta_2\lambda_1 - \beta_1\lambda_2)^2})} \end{aligned}$$

Hence the proof.

4.2 The Basic Reproduction Number

The next-generation method ^[17, 18] was used to obtain reproduction numbers. Using the notation f for a matrix of new infection terms and v for the matrix of the remaining transfer terms in system 1, yields

$$f = \begin{pmatrix} \pi_1 M \\ \gamma_1 H_m \\ \pi_2 F \\ \gamma_2 H_f \end{pmatrix}$$

$$v = \begin{pmatrix} (\kappa_1 + \psi_1)G + \mu_1(F + \theta_1 B_f)G \\ -\mu_1(F + \theta_1 B_f)G + (\kappa_1 + \varphi_1)B_m \\ (\kappa_2 + \psi_2)L + \mu_2(M + \theta_2 B_m)L \\ -\mu_2(M + \theta_2 B_m)L + (\kappa_2 + \varphi_2)B_f \end{pmatrix}$$

The Jacobi of f and v results to matrices F and V Respectively at HFE

$$F = \begin{pmatrix} Md_1 & Md_1e_1 & Md_1e_2 & Md_1e_3 \\ d_2H_m & d_2e_1H_m & d_2e_2H_m & d_2e_3H_m \\ Fd_8 & Fd_8e_1 & Fd_8e_2 & Fd_8e_3 \\ d_5H_f & d_5e_1H_f & d_5e_2H_f & d_5e_3H_f \end{pmatrix}$$

$$V = \begin{pmatrix} \kappa_1 + F^0\mu_1 + \psi_1 & 0 & 0 & 0 \\ -F^0\mu_1 & \kappa_1 + \varphi_1 & 0 & 0 \\ 0 & 0 & \kappa_2 + M^0\mu_2 + \psi_2 & 0 \\ 0 & 0 & -M^0\mu_2 & \kappa_2 + \varphi_2 \end{pmatrix}$$

It follows that basic reproduction numbers for model 1, denoted by R_0^1 , R_0^2 , R_0^3 and R_0^4 are given by evaluating the eigenvalues of $F.V^{-1}$ as follows

$$F.V^{-1} = \begin{pmatrix} \frac{Md_1}{\kappa_1 + F\mu_1 + \psi_1} & 0 & 0 & 0 \\ \frac{Fd_2H_m\mu_1}{(\kappa_1 + \varphi_1)(\kappa_1 + F\mu_1 + \psi_1)} & \frac{d_2e_1H_m}{\kappa_1 + \varphi_1} & 0 & 0 \\ 0 & 0 & \frac{Fd_8e_2}{\kappa_2 + M\mu_2 + \psi_2} & 0 \\ 0 & 0 & \frac{Md_5e_2H_f\mu_2}{(\kappa_2 + \varphi_2)(\kappa_2 + M\mu_2 + \psi_2)} & \frac{d_5e_3H_f}{\kappa_2 + \varphi_2} \end{pmatrix} \quad (2)$$

Thus, the basic reproduction numbers are:

$$\left\{ R_0^1 = \frac{Fd_8e_2}{\kappa_2 + M\mu_2 + \psi_2}, R_0^2 = \frac{Md_1}{\kappa_1 + F\mu_1 + \psi_1}, R_0^3 = \frac{d_5e_3H_f}{\kappa_2 + \varphi_2}, R_0^4 = \frac{d_2e_1H_m}{\kappa_1 + \varphi_1} \right\}$$

4.3 Local Stability Analysis of Homosexuality-Free Equilibrium Point (HFE)

Theorem 4

The HFE of model system 1 is locally asymptotically stable (LAS) if $R_0^1 < 1$, $R_0^2 < 1$, $R_0^3 < 1$, and $R_0^4 < 1$ and unstable otherwise^[17].

Proof

To establish the local stability, the Jacobian of the model was evaluated at E^0 . The stability of this steady state is then determined based on the signs of the eigenvalues of the corresponding Jacobian, which are functions of the model parameters.

$$\begin{pmatrix} -F^0\beta_1 - \kappa_1 & -M^0d_1 & 0 & -M^0d_1e_1 & \Gamma_1 & -M^0\beta_1 & -M^0d_1e_2 & 0 & -M^0d_1e_3 & 0 \\ 0 & M^0d_1 - \kappa_1 - F^0\mu_1 - \psi_1 & 0 & M^0d_1e_1 & 0 & 0 & M^0d_1e_2 & 0 & M^0d_1e_3 & 0 \\ \beta_1F^0 & -d_2H_m^0 & -\kappa_1 & -d_2e_1H_m^0 & \eta_1 & M^0\beta_1 & -d_2e_2H_m^0 & 0 & -d_2e_3H_m^0 & 0 \\ 0 & d_2H_m^0 + F^0\mu_1 & 0 & d_2e_1H_m^0 - \kappa_1 - \varphi_1 & 0 & 0 & d_2e_2H_m^0 & 0 & d_2e_3H_m^0 & 0 \\ 0 & \psi_1 & 0 & \varphi_1 & -(\Gamma_1 + \eta_1 + \kappa_1 + \chi_1) & 0 & 0 & 0 & 0 & 0 \\ -F^0\beta_2 & -F^0d_8 & 0 & -F^0d_8e_1 & 0 & -M^0\beta_2 - \kappa_2 & -F^0d_8 & 0 & -F^0d_8e_3 & \Gamma_2 \\ 0 & F^0d_8 & 0 & F^0d_8e_1 & 0 & 0 & F^0d_8e_2 - \kappa_2 - M^0\mu_2 - \psi_2 & 0 & F^0d_8e_3 & 0 \\ F^0\beta_2 & -d_5H_f^0 & 0 & -d_5e_1H_f^0 & 0 & M^0\beta_2 & -d_5e_2H_f^0 & -\kappa_2 & -d_5e_3H_f^0 & \eta_2 \\ 0 & d_5H_f^0 & 0 & d_5e_1H_f^0 & 0 & 0 & d_5e_2H_f^0 + M^0\mu_2 & 0 & d_5e_3H_f^0 - \kappa_2 - \varphi_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_2 & 0 & \varphi_2 & -(\Gamma_2 + \eta_2 + \kappa_2 + \chi_2) \end{pmatrix} \quad (3)$$

All eigenvalues of (3) were negative under the condition,

$\frac{(F\beta_1 + M\beta_2 + \kappa_1 + \kappa_2)^2}{(M\beta_2\kappa_1 + F\beta_1\kappa_2 + \kappa_1\kappa_2)} < 1$. Thus, the homosexuality Free Equilibrium (HFE) of the system is locally asymptotically stable, i.e. $R_0^1 < 1$, $R_0^2 < 1$, $R_0^3 < 1$, and $R_0^4 < 1$ and unstable otherwise.

Numerical simulations support analytical results that in the absence of homosexuality, the number of individuals in homosexual classes will tend to zero, as shown below.

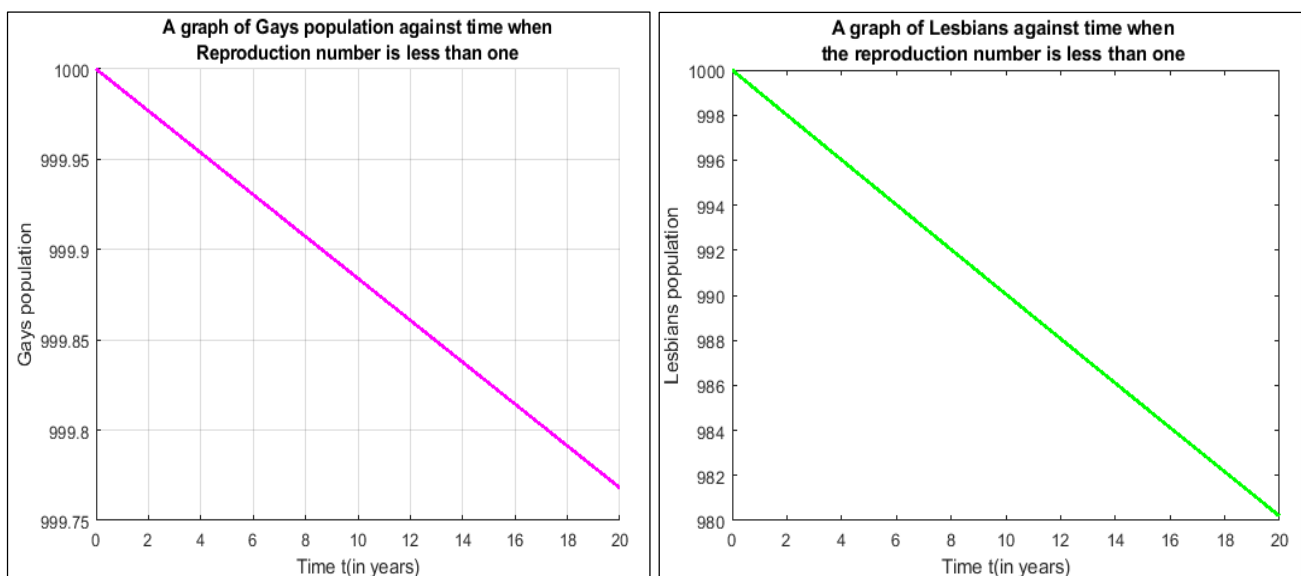


Fig 2: Graphs of gays and lesbians against time when $R_0^1 < 1$, $R_0^2 < 1$, $R_0^3 < 1$ and $R_0^4 < 1$.

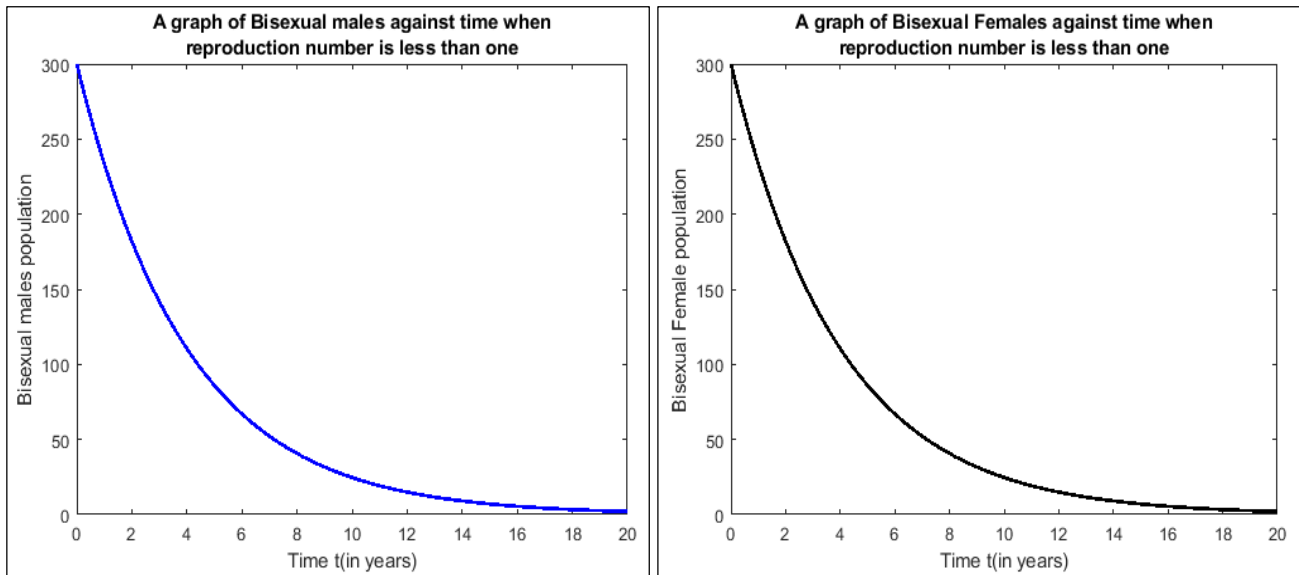


Fig 3: Graphs of bisexual males and bisexual females against time when $R_0^1 < 1$, $R_0^2 < 1$, $R_0^3 < 1$ and $R_0^4 < 1$

4.4 Global Stability of the homosexuality-free equilibrium (H.F.E)

Theorem 5

If system 1 can be written in the form $\frac{dX}{dt} = F(X, Z)$, $\frac{dZ}{dt} = G(X, Z)$, $G(X, 0) = 0$

Where $X = (M, H_m, R_m, F, H_f, R_f) \in R_+^6$ denotes non-infectious compartments and $Z = (G, B_m, L, B_f) \in R_+^4$ denotes the infectious compartments. $E_0 = (X^*, 0)$ represents the homosexuality-free equilibrium of the system if this point satisfies the following:

(i.) $\frac{dX}{dt} = F(X, 0)$ where X^* is globally asymptotically stable.

(ii.) $\frac{dZ}{dt} = D_Z G(X, 0)Z - G(X, Z) \geq 0$ for all

$X, Z \in \Omega$, where $A = D_Z G(X^*, 0)$ is an M-matrix (the off-diagonal elements of A are non-negative) and Ω is the region where the model makes biological sense [16, [17][19]. Then we can conclude that E_0 is a globally asymptotically stable (GAS) equilibrium of model system 1, provided that $R_0^1 < 1$, $R_0^2 < 1$, $R_0^3 < 1$, and $R_0^4 < 1$

Proof

Applying Theorem 5 to Model System 1 gives

$$\hat{G}(X, Z) = \begin{pmatrix} \left(1 - \frac{G}{N}\right)(G + e_1 B_m + e_2 L + e_3 B_f) d_1 \\ \left(1 - \frac{B_m}{N}\right)(G + e_1 B_m + e_2 L + e_3 B_f) d_2 \\ \left(1 - \frac{L}{N}\right)(G + e_1 B_m + e_2 L + e_3 B_f) d_8 \\ \left(1 - \frac{B_f}{N}\right)(G + e_1 B_m + e_2 L + e_3 B_f) d_5 \end{pmatrix} \geq 0$$

Since $\hat{G}(X, Z) \geq 0$ by theorem 5, E_0 Globally asymptotically stable.

4.5 Existence of Homosexuality Endemic Equilibrium point (HEEP) Conditions for the existence of equilibrium are sought for which the model is endemic in the population^[17]. For HEEP to exist, the infectious classes of the system 1 must be greater than zero, that is, $G > 0$, $B_m > 0$, $L > 0$, $B_f > 0$. Setting infectious classes to zero and substituting them with each other, and letting them be greater than zero, results in

$$\frac{F d_8 e_2}{\kappa_2 + M \mu_2 + \psi_2} > 1,$$

$$\frac{M d_1}{\kappa_1 + F \mu_1 + \psi_1} > 1,$$

$$\frac{d_5 e_3 H_f}{\kappa_2 + \varphi_2} > 1,$$

$$\frac{d_2 e_1 H_m}{\kappa_1 + \varphi_1} > 1.$$

Thus, HEEP has existed since $R_0^1 > 1$, $R_0^2 > 1$, $R_0^3 > 1$, and $R_0^4 > 1$

4.6 Global stability of the Homosexuality Endemic Equilibrium point (HEEP)

Theorem 6

The HEEP is globally asymptotically stable whenever $R_0^1 < 1$, $R_0^2 < 1$, $R_0^3 < 1$, and $R_0^4 < 1$. [20]

Proof

The following Lyapunov Function was proposed

$$\begin{aligned} k(M, G, H_m, B_m, R_m, F, L, H_f, B_f, R_f) = & M - M^* - M^* \ln \frac{M}{M^*} + y_1(G - G^* - \\ & G^* \ln \frac{G}{G^*}) + y_2 \left(H_m - H_m^* - H_m^* \ln \frac{H_m}{H_m^*} \right) + y_3 \left(B_m - B_m^* - B_m^* \ln \frac{B_m}{B_m^*} \right) + y_4(R_m - \\ & R_m^* - R_m^* \ln \frac{R_m}{R_m^*}) + \left(F - F^* - F^* \ln \frac{F}{F^*} \right) + y_5 \left(L - L^* - L^* \ln \frac{L}{L^*} \right) + y_6(H_f - H_f^* - \\ & H_f^* \ln \frac{H_f}{H_f^*}) + y_7 \left(B_f - B_f^* - B_f^* \ln \frac{B_f}{B_f^*} \right) + y_8 \left(R_f - R_f^* - R_f^* \ln \frac{R_f}{R_f^*} \right) \end{aligned}$$

If the Lyapunov function $k(M, G, H_m, B_m, R_m, F, L, H_f, B_f, R_f)$ satisfies the condition

$k(M^*, G^*, H_m^*, B_m^*, R_m^*, F^*, L^*, H_f^*, B_f^*, R_f^*) = 0$ and $k(M^0, G^0, H_m^0, B_m^0, R_m^0, F^0, L^0, H_f^0, B_f^0, R_f^0) > 0$ is said to be positive definite.

Now the positive constants $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8$ are chosen such that the coefficients of $(M, G, H_m, B_m, R_m, F, L, H_f, B_f, R_f)$ are equal to zero. Solving gives

$$\begin{aligned} y_1 = 1, y_2 = \frac{\nabla_5 \nabla_2 - \Gamma_1}{\eta_1}, y_3 = \frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1}, y_4 = \frac{\nabla_2}{\psi_1}, y_5 = 1, y_6 = \frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \\ y_7 = \frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2}, y_8 = \frac{\nabla_7}{\psi_2} \end{aligned}$$

Where,

$$\begin{aligned} \nabla_1 = (\kappa_1 + \beta_1 F + \pi_1), \nabla_2 = (\kappa_1 + \psi_1 + \mu_1 F + \mu_1 \theta_1 B_f), \nabla_3 = (\kappa_1 + \gamma_1 G), \nabla_4 = (\kappa_1 + \varphi_1), \nabla_5 = (\kappa_1 + \eta_1 + \chi_1 + \Gamma_1), \nabla_6 = (\kappa_2 + \beta_2 M + \pi_2), \nabla_7 = (\kappa_2 + \\ \psi_2 + \mu_2 M + \mu_2 \theta_2 B_m), \nabla_8 = (\kappa_2 + \gamma_2), \nabla_9 = (\kappa_2 + \varphi_2), \nabla_{10} = (\kappa_2 + \eta_2 + \chi_2 + \Gamma_2). \end{aligned}$$

Letting P^* denote all the positive values and Q^* denote all the negative values of the solution after substituting evaluated constants, results in

$$\begin{aligned} P^* = & M^* \nabla_1 + \Gamma_1 R_m + M \left(\frac{d_1 G}{N_m} + \frac{d_1 e_1 B_m}{N_m} \right) + \frac{M^*}{M} M \nabla_1 + \frac{M^*}{M} M \nabla_1 + \pi_1 M + \\ & \frac{G^*}{G} G \nabla_2 + \beta_1 M F \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) + \eta_1 R_m \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) + \frac{H_m^*}{H_m} H_m \nabla_3 \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) + \\ & \gamma_1 H_m \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) + \mu_1 (F + \theta_1 B_f) G \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) + \frac{B_m^*}{B_m} B_m \nabla_4 \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) + \varphi_1 B_m \left(\frac{\nabla_2}{\psi_1} \right) + \psi_1 G \left(\frac{\nabla_2}{\psi_1} \right) + \\ & \frac{R_m^*}{R_m} R_m \nabla_5 \left(\frac{\nabla_2}{\psi_1} \right) + \lambda_2 + \Gamma_2 R_f + F \left(\frac{d_8 L}{N_f} + \frac{d_s e_3 B_f}{N_f} \right) \frac{F^*}{F} \nabla_6 F + \pi_2 F + \frac{L^*}{L} \nabla_7 L + \beta_2 M F + \\ & \eta_2 R_f \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) + \frac{H_f^*}{H_f} \nabla_8 H_f \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) + \gamma_2 H_f \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) + \mu_2 (M + \theta_2 B_m) L \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) + \\ & \frac{B_f^*}{B_f} \nabla_9 B_f \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) + \varphi_2 B_f \left(\frac{\nabla_7}{\psi_2} \right) + \psi_2 L y_8 + \frac{R_f^*}{R_f} \nabla_{10} R_f \left(\frac{\nabla_7}{\psi_2} \right) \\ Q^* = & -\Gamma_1 R_m^* - M \left(\frac{d_1 G}{N_m} + \frac{d_1 e_1 B_m}{N_m} \right) - \frac{M^*}{M} \lambda_1 - \frac{M^*}{M} \Gamma_1 R_m - G \nabla_2 - \frac{G^*}{G} \pi_1 M - \end{aligned}$$

$$\begin{aligned}
& H_m \nabla_3 \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) - \frac{H_m^*}{H_m} \beta_1 M F \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) - \frac{H_m^*}{H_m} \eta_1 R_m \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) - B_m \nabla_4 \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) - \\
& \frac{B_m^*}{B_m} \gamma_1 H_m \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) - \frac{B_m^*}{B_m} (\mu_1 (F + \theta_1 B_f) G) \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) - R_m \nabla_5 \left(\frac{\nabla_2}{\psi_1} \right) - \frac{R_m^*}{R_m} \varphi_1 B_m \left(\frac{\nabla_2}{\psi_1} \right) - \\
& \frac{R_m^*}{R_m} \psi_1 G \left(\frac{\nabla_2}{\psi_1} \right) - F \left(\frac{d_8 L}{N_f} + \frac{d_8 e_3 B_f}{N_f} \right) - \frac{F^*}{F} \lambda_2 - \frac{F^*}{F} \Gamma_2 R_f - \nabla_7 L - \frac{L^*}{L} \pi_2 F - \nabla_8 H_f \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) - \\
& \frac{H_f^*}{H_f} \beta_2 M F \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) - \frac{H_f^*}{H_f} \eta_2 R_f \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) - \nabla_9 B_f \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) - \frac{B_f^*}{B_f} \gamma_2 H_f \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) - \\
& \frac{B_f^*}{B_f} \mu_2 (M + \theta_2 B_m) L \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) - \nabla_{10} R_f \left(\frac{\nabla_7}{\psi_2} \right) - \frac{R_f^*}{R_f} \varphi_2 B_f \left(\frac{\nabla_7}{\psi_2} \right) - \frac{R_f^*}{R_f} \psi_2 L \left(\frac{\nabla_7}{\psi_2} \right)
\end{aligned}$$

Then $\frac{dk}{dt} = 0$ holds only when $M = M^*, G = G^*, H_m = H_m^*, B_m = B_m^*, R_m = R_m^*, F = F^*, L = L^*, H_f = H_f^*, B_f = B_f^*, R_f = R_f^*$ so the maximal compact invariant set in $S; E; T \in \Pi: \frac{dV}{dt} = 0$ is the singleton E^* using Lassalle's invariance principle^[18, 19], $\frac{dk(M, G, H_m, B_m, R_m, F, L, H_f, B_f, R_f)}{dt} < 0$ if $R_0^1 > 1, R_0^2 > 1, R_0^3 > 1$ and $R_0^4 > 1$, that is $P^* > Q^*$.

4.7 Bifurcation Analysis

The Centre Manifold Theorem^[21] was used to investigate the nature of the bifurcation of Model 1. Letting P_k be the k^{th} component of p and

$$\begin{aligned}
a &= \sum_{k,l,j=0}^n V_k u_i u_j \frac{\partial^2 p_k}{\partial y_i \partial y_j} (0,0) \\
b &= \sum_{k,l,j=0}^n V_k v_i \frac{\partial^2 p_k}{\partial y_i \partial R_0} (0,0)
\end{aligned}$$

Considering a case where $R_0^{*1} = 1, R_0^{*2} = 1, R_0^{*3} = 1$ and $R_0^{*4} = 1, d_1 = d_1^*, d_2 = d_2^*, d_5 = d_5^*$ and $d_8 = d_8^*$ were chosen as bifurcation parameters as follows:

$$\begin{aligned}
d_1^* &= \frac{\kappa_1 + F\mu_1 + \psi_1}{M} \\
d_2^* &= \frac{\kappa_1 + \varphi_1}{e_1 H_m} \\
d_8^* &= \frac{\kappa_2 + M\mu_2 + \psi_2}{F e_2} \\
d_5^* &= \frac{\kappa_2 + \varphi_2}{e_3 H_f}
\end{aligned}$$

Jacobian J^* was obtained:

$$\begin{pmatrix}
-F^0 \beta_1 - \kappa_1 & -M^0 d_1^* & 0 & -M^0 d_1^* e_1 & \Gamma_1 & -M^0 \beta_1 & -M^0 d_1^* e_2 & 0 & -M^0 d_1^* e_3 & 0 \\
0 & M^0 d_1^* - \kappa_1 - F^0 \mu_1 - \psi_1 & 0 & M^0 d_1^* e_1 & 0 & 0 & M^0 d_1^* e_2 & 0 & M^0 d_1^* e_3 & 0 \\
\beta_1 F^0 & -d_2^* H_m^0 & -\kappa_1 & -d_2^* e_1 H_m^0 & \eta_1 & M^0 \beta_1 & -d_2^* e_2 H_m^0 & 0 & -d_2^* e_3 H_m^0 & 0 \\
0 & d_2^* H_m^0 + F^0 \mu_1 & 0 & d_2^* e_1 H_m^0 - \kappa_1 - \varphi_1 & 0 & 0 & d_2^* e_2 H_m^0 & 0 & d_2^* e_3 H_m^0 & 0 \\
0 & \psi_1 & 0 & \varphi_1 & -(\Gamma_1 + \eta_1 + \kappa_1 + \chi_1) & 0 & 0 & 0 & 0 & 0 \\
-F^0 \beta_2 & -F^0 d_8^* & 0 & -F^0 d_8^* e_1 & 0 & -M^0 \beta_2 - \kappa_2 & -F^0 d_8^* & 0 & -F^0 d_8^* e_3 & \Gamma_2 \\
0 & F^0 d_8^* & 0 & F^0 d_8^* e_1 & 0 & 0 & F^0 d_8^* e_2 - \kappa_2 - M^0 \mu_2 - \psi_2 & 0 & F^0 d_8^* e_3 & 0 \\
F^0 \beta_2 & -d_5^* H_f^0 & 0 & -d_5^* e_1 H_f^0 & 0 & M^0 \beta_2 & -d_5^* e_2 H_f^0 & -\kappa_2 & -d_5^* e_3 H_f^0 & \eta_2 \\
0 & d_5^* H_f^0 & 0 & d_5^* e_1 H_f^0 & 0 & 0 & d_5^* e_2 H_f^0 + M^0 \mu_2 & 0 & d_5^* e_3 H_f^0 - \kappa_2 - \varphi_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \psi_2 & 0 & \varphi_2 & -(\Gamma_2 + \eta_2 + \kappa_2 + \chi_2)
\end{pmatrix}$$

Solving for the right and left eigenvector at HFE of the Jacobian J^* resulted to $u = [u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}]^T = 0$ and left eigenvectors as $v = [v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}] = 0$.

Since the right and left eigenvectors resulted in zero, i.e., ($a = 0$ and $b = 0$), then the Centre Manifold theorem cannot be used to study the bifurcation of this system, or this system doesn't have bifurcation.

4.8 Normalized Sensitivity analysis of the parameters

The technique employed in the sensitivity analysis was based on the reproduction numbers. This helps determine each parameter's contribution level to the reproduction number. This technique uses partial derivatives of the parameters^[22]. These indices are given by

$$\Lambda_{\rho}^{R_c^*} = \frac{\partial R_c^*}{\partial \rho} \times \frac{\rho}{R_c^*}$$

Table 2: Sensitivity indices of parameters of the reproduction numbers

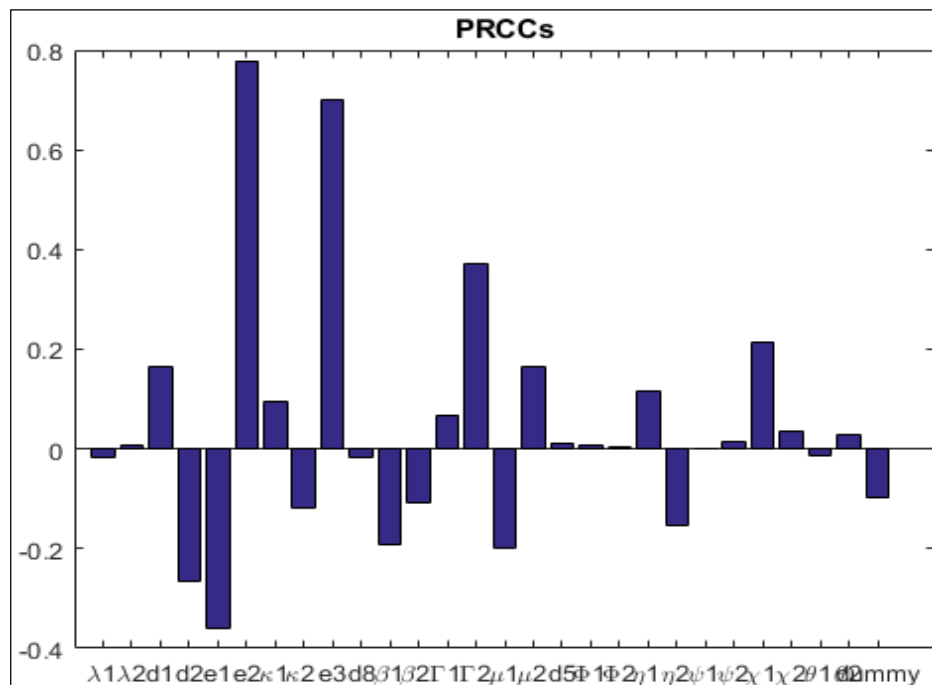
Parameter	Value	Parameter	Value
κ_1	-0.0000355464	κ_2	-6.4×10^{-8}
μ_1	-0.999556	μ_2	-0.999933
φ_1	-0.999994	φ_2	-1
ψ_1	-0.000222165	ψ_2	-0.0000666622
e_1	1	e_3	1
d_1	1	d_2	1
d_5	1	d_8	1
e_2	1		

4.9 Partial rank correlation coefficient (PRCC)

To identify parameters that were more influential to the model, PRCC analysis was carried out, where the strength of the outputs and inputs of the model was determined using the Latin hypercube sampling (LHS) method. ^[23]

Table 3: PRCC and P-Values Sensitivity indices

Parameter	PRCC	p-value	Parameter	PRCC	P-Value
λ_1	-0.016461	0.87087	λ_2	0.0079463	0.93746
κ_1	0.095777	0.34317	κ_2	-0.11973	0.23544
β_1	-0.19045	0.057703	β_2	-0.10652	0.29152
Γ_1	0.067819	0.50258	Γ_2	0.37077	0.00014623
μ_1	-0.19785	0.048478	μ_2	0.16563	0.099585
φ_1	0.0075675	0.94043	φ_2	0.002745	0.97838
η_1	0.11553	0.25237	η_2	-0.15164	0.13205
ψ_1	$2.0094e^{-05}$	0.99984	ψ_2	0.015286	0.88001
χ_1	0.21465	0.031989	χ_2	0.034371	0.73424
θ_1	-0.012038	0.90538	θ_2	0.028371	0.77932
e_1	-0.36005	0.00023377	e_3	0.70153	$4.3242e^{-16}$
d_1	0.16654	0.097708	d_2	-0.26636	0.0073912
d_5	0.012792	0.89948	d_8	-0.87365	0.87365
e_2	0.77802	$1.6907e^{-21}$			

**Fig 4:** Summary Graphs of the PRCC analysis

5. Numerical Simulations

Numerical simulations were done using MATLAB software to observe the dynamics of the sexual orientation model over time. The following parameter values were used in the simulation.

Table 4: Parameter estimations

Parameter	Value	Source	Parameter	Value	Source
λ_1	0.027	[24]	λ_2	0.027	[24]
κ_1	0.016	[24]	κ_2	0.016	[24]
β_1	0.0018	Estimate	β_2	0.0018	Estimate
Γ_1	0.2	[24]	Γ_2	0.92	Estimate
μ_1	0.000045	Estimate	μ_2	0.015	Estimate
ϕ_1	0.25	Estimate	ϕ_2	0.25	Estimate
η_1	0.005	Estimate	η_2	0.25	Estimate
ψ_1	0.00001	Estimate	ψ_2	0.001	[24]
χ_1	0.0015	Estimate	χ_2	0.015	Estimate
θ_1	0.75	Estimate	θ_2	0.0015	Estimate
e_1	0.001	Estimate	e_3	0.1015	Estimate
d_1	0.0021	Estimate	d_2	0.00145	Estimate
d_5	0.001	Estimate	d_8	0.00841	Estimate
e_2	0.5	Estimate			

For the model with advocacy, the following forces of infection were used:

$$\pi_1 = d_1(G + e_1B_m + e_2L + e_3B_f)$$

$$\gamma_1 = d_2(G + e_1B_m + e_2L + e_3B_f)$$

$$\gamma_2 = d_5(G + e_1B_m + e_2L + e_3B_f)$$

$$\pi_2 = d_8(G + e_1B_m + e_2L + e_3B_f)$$

For the model without advocacy, these forces of infection were used:

$$\pi_1 = d_1(G + e_1B_m),$$

$$\gamma_1 = d_2(G + e_1B_m),$$

$$\gamma_2 = d_5(L + e_3B_f),$$

$$\pi_2 = d_8(L + e_3B_f)$$

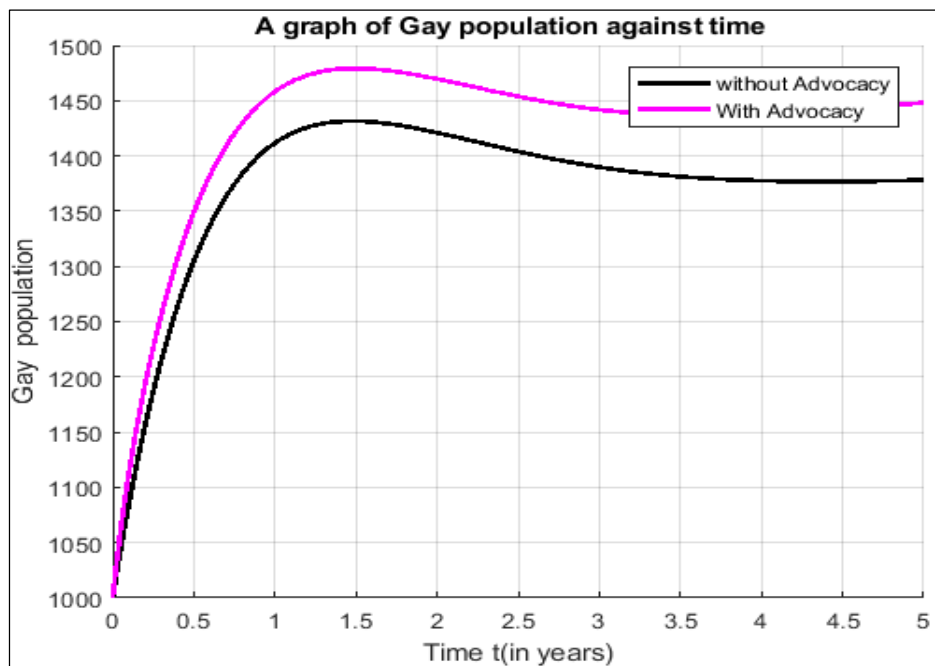


Fig 5: Graph showing the gay population with and without advocacy.

The comparison of the model with and without advocacy in Figure 5 showed that advocacy increases the gay population by 5.7% at the maximum value compared to the model without advocacy.

In Figure 6, it was shown that advocacy increases the bisexual male population by 2.6% compared to the model without advocacy at the maximum value.

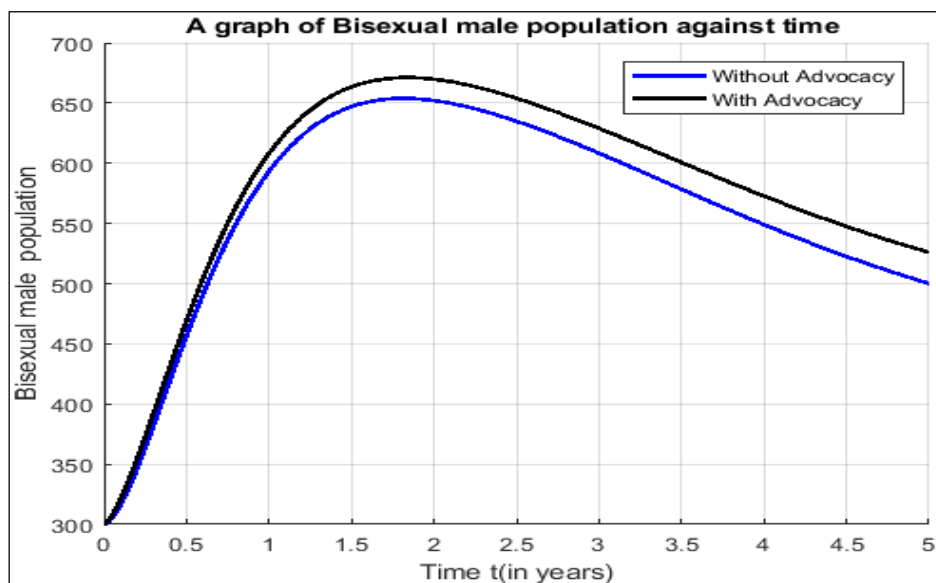


Fig 6: Graph showing the Bisexual male population with and without advocacy.

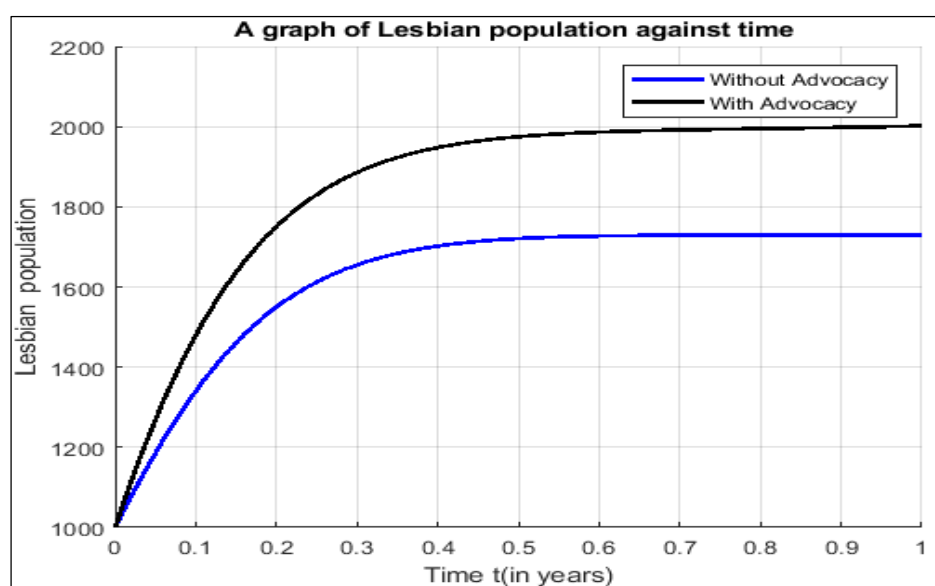


Fig 7: Graph showing the lesbian population with and without advocacy.

Figure 7 shows that in the presence of advocacy, the lesbian population is increased by 15.6% compared to the model without advocacy.



Fig 8: Graph of bisexual female populations against time.

Figure 8 shows that in the presence of advocacy, the bisexual female population is increased by 15.3% in comparison to the model without advocacy.

The numerical results above showed that, while advocacy increases the spread of homosexuality in the population, its effects are more pronounced in bisexual female and lesbian populations.

6. Results and Discussion

This research developed a deterministic mathematical model to analyze sexual orientations with advocacy. The positivity, boundedness, equilibria points, and control reproduction numbers were studied. The Routh-Hurwitz criteria were used to assess the local stability of the disease-free equilibrium, the Metzler Matrix method was used to determine global stability, while Lyapunov functions were used to study the global stability of the endemic equilibrium point. Bifurcation analysis was also conducted. Lastly, a sensitivity analysis and numerical simulations were performed using Wolfram Mathematica and MATLAB software.

The model was found to be bounded, lay in the positive region, and had Homosexuality-free equilibrium points given as $E^0 = (\xi_1, 0, \xi_2, 0, 0, \xi_3, 0, \xi_4, 0, 0)$. Control reproduction numbers were obtained as $R_0^1 = 0.280315$, $R_0^2 = 4.66546$, $R_0^3 = 0.0203$, and $R_0^4 = 0.144999$. The stability of HFE was found to be locally asymptotically stable (LAS) and globally asymptotically stable (GAS) whenever $R_0^1 < 1$, $R_0^2 < 1$, $R_0^3 < 1$, and $R_0^4 < 1$. The global stability of HEEP was found to be asymptotically stable (GAS) whenever $R_0^1 > 1$, $R_0^2 > 1$, $R_0^3 > 1$, and $R_0^4 > 1$.

Upon carrying out bifurcation analysis, it was observed that the Center Manifold theorem could not be used to get the bifurcation of this system since the right and left eigenvectors were zero. The sensitivity analysis based on the control reproduction number showed that d_1 , d_2 , d_5 , d_8 , e_1 , e_2 , e_3 , and ϕ_2 were the most influential parameters, and they would significantly influence the spread of homosexuality within the community.

The sensitivity analysis was performed based on the PRCC method and LHS to identify the highly sensitive parameters in the model. It showed e_2 and e_3 as the most influential parameters, in that order, and ψ_1 as the least influential.

Numerical results showed that the model with advocacy increases the gay population, the lesbian population, the bisexual male population, and the bisexual female population by 5.7%, 15.6%, 2.6%, and 15.3%, respectively, in comparison with the model without advocacy.

Thus, this study shows that while incorporating advocacy that promotes homosexuality in the community significantly increases homosexual populations in the society, Lesbian and bisexual female populations will be improved the most.

Conflict of interest

The authors declare that there is no conflict of interest.

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