

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

Maths 2025; 10(6): 18-28

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www.mathsjournal.com

Received: 09-04-2025

Accepted: 12-05-2025

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Mathematical model of sexual orientations in the presence of recovery centers

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DOI: <https://www.doi.org/10.22271/math.2025.v10.i6a.2053>

Abstract

Homosexuality has become a global trend both in developing and developed countries. While it has been legalized in some countries, it's still illegal in others. In this paper, a comprehensive mathematical model of sexual orientations with recovery centers was developed using a set of ordinary differential equations and solved using Wolfram Mathematica and the fourth-order Runge-Kutta method. The population was divided into ten compartments; Males (M), Females (F), gays (G), lesbians (L), heterosexual males (H_m), heterosexual females (H_f), bisexual males (B_m), bisexual females (B_f), recovery centers for males (R_m) and recovery centers for females (R_f). Positivity, boundedness, the homosexuality epidemic threshold, and equilibria for the model were determined, and stabilities were investigated. Moreover, control reproduction number and bifurcation analysis were also studied. The sensitivity of the parameters was investigated using the partial rank correlation coefficient (PRCC) method and Latin hypercube sampling (LHS). Numerical simulation was done using MATLAB 45-ODE Solver and showed that increasing homosexuality recruitment rates increased homosexuality in the population, and vice versa. It was also established numerically that increasing transfer rates from the bisexual classes to recovery centers increased the heterosexual populations. In conclusion, the effective transfer to recovery centers of the homosexual populations and low contact rates are most significant in reducing the spread of homosexuality in the community.

Keywords: Homosexuality, model, stability, reproduction number, sexual orientation

Introduction

Sexual orientation is the relative attraction one feels toward others (men, women, or both) ^[1]. Grasping sexual orientation has been a challenging and changing area of research, characterized by notable shifts in societal perceptions and scientific insights over time. Traditionally, sexual orientation was often seen through a binary framework, where heterosexuality was viewed as the standard and other orientations were marginalized or treated as disorders. This view began to shift in the late 1800s and early 1900s, as trailblazers like Magnus Hirschfeld and Alfred Kinsey ^[2] questioned the dominant beliefs and pushed for a deeper understanding of human sexuality. Since that time, research has increasingly acknowledged sexual orientation as a varied spectrum. Political rights of lesbian, gay, and bisexual (LGB) individuals have significantly improved in numerous Western nations over the past few decades ^[3]. In the United States, for instance, the Supreme Court determined in June 2015 that states are required to permit marriages between same-sex couples. Nearly 60% of sexual minority groups in the U.S. encountered Sexual Orientation and Gender Identity Change efforts (SOGIECE) from religious environments, counselors, psychologists, and psychiatrists ^[4].

Mathematical modeling is a method of simulating real-life situations with mathematical equations to predict future behavior by generating a simplified representation of a real system ^[5, 6]. Such models have been used in evaluating the effects of HIV/AIDS, considering sexual preferences under antiretroviral therapy ^[7]. Assessing the impacts of bisexual behavior in a global community ^[8].

Study the sexual transmission of the Hepatitis B virus (HBV) considering sexual orientation [9] study the spread of HIV/AIDS among LGBT populations [9]. This paper seeks to develop a mathematical model of sexual orientations in the presence of recovery centers.

2. Model formulation

Classical SIRS (susceptible, infected, recovered and susceptible) is used to model sexual orientations in the presence of recovery centers.

Model assumptions

- It was assumed that homosexuality (same-sex marriage) is not hereditary; rather, people are convinced to practice it.

- The male population can only be influenced by gay and bisexual males to practice homosexuality.
- The female population can only be influenced by lesbians and bisexual females to homosexuality.

2.1 Model development

The model is formulated into ten compartments; Males (M), Females (F), gays (G), lesbians (L), heterosexual males (H_m), heterosexual females (H_f), bisexual males (B_m), bisexual females (B_f), recovery centers for male (R_m) and recovery centers for females (R_f). Gays, lesbians, bisexual males and bisexual females will be referred to as homosexual populations. Below is the schematic representation of the model.

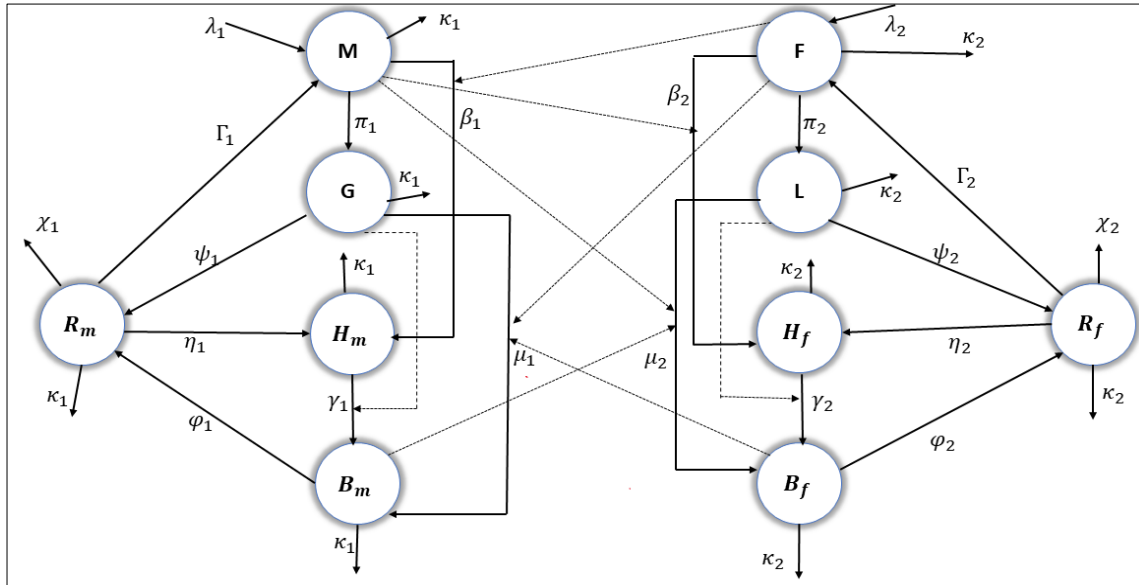


Fig 1: Model flow diagram

2.2 Model Equations

$$\left\{ \begin{array}{l} \frac{dM}{dt} = \lambda_1 + \Gamma_1 R_m - (\kappa_1 + \beta_1 F + \pi_1) M \\ \frac{dG}{dt} = \pi_1 M - (\kappa_1 + \psi_1) G - \mu_1 (F + \theta_1 B_f) G \\ \frac{dH_m}{dt} = \beta_1 M F + \eta_1 R_m - (\kappa_1 + \gamma_1) H_m \\ \frac{dB_m}{dt} = \gamma_1 H_m + \mu_1 (F + \theta_1 B_f) G - (\kappa_1 + \phi_1) B_m \\ \frac{dR_m}{dt} = \phi_1 B_m + \psi_1 G - (\kappa_1 + \eta_1 + \chi_1 + \Gamma_1) R_m \\ \frac{dF}{dt} = \lambda_2 + \Gamma_2 R_f - (\kappa_2 + \beta_2 M + \pi_2) F \\ \frac{dL}{dt} = \pi_2 F - (\kappa_2 + \psi_2) L - \mu_2 (M + \theta_2 B_m) L \\ \frac{dH_f}{dt} = \beta_2 M F + \eta_2 R_f - (\kappa_2 + \gamma_2) H_f \\ \frac{dB_f}{dt} = \gamma_2 H_f + \mu_2 (M + \theta_2 B_m) L - (\kappa_2 + \phi_2) B_f \\ \frac{dR_f}{dt} = \phi_2 B_f + \psi_2 L - (\kappa_2 + \eta_2 + \chi_2 + \Gamma_2) R_f \end{array} \right. \quad (1)$$

Where,

$$\begin{aligned} \pi_1 &= d_1 (G + e_1 B_m) \\ \pi_2 &= d_8 (L + e_3 B_f) \\ \gamma_1 &= d_2 (G + e_1 B_m) \\ \gamma_2 &= d_5 (L + e_3 B_f) \end{aligned}$$

Table 1: Parameter description

Parameter	Description	Parameter	Description
λ_1	Male Birth rate	λ_2	Female birth rate
κ_1	The natural death rate for males	κ_2	Natural death rate for females
π_1	Conversion rate from Male to Gay	π_2	Conversion rate from female to lesbians
β_1	Conversion rate from male to heterosexual male	β_2	Conversion rate from female to heterosexual female
Γ_1	Conversion rate from male recovery center to male	Γ_2	Conversion rate from female recovery center to female
μ_1	Conversion rate from Gay to bisexual male	μ_2	Conversion rate from Lesbian to bisexual female
γ_1	Conversion rate from heterosexual male to bisexual male	γ_2	Conversion rate from heterosexual female to bisexual female
ϕ_1	The transfer rate from Bisexual male to male Recovery center	ϕ_2	The transfer rate from bisexual females to the female Recovery center
η_1	Conversion rate from male recovery center to Heterosexual male	η_2	Conversion rate from female recovery center to heterosexual female
ψ_1	The transfer rate from gay to male recovery center	ψ_2	The transfer rate from lesbians to the recovery center
χ_1	The death rate due to the effects of the Male Recovery Center	χ_2	The death rate due to the effects of the Female Recovery Center
θ_1	The rate at which B_f converts G to B_m	θ_2	The rate at which B_m converts L to B_f
e_1	Infection rate by bisexual male (B_m)	e_3	Infection rate by bisexual female (B_f)
d_1	Gay recruitment rate	d_2	Bisexual male recruitment rate
d_5	Bisexual female recruitment rate	d_8	Lesbian recruitment rate

3. Invariant regions

The model system 1 will be analyzed in a suitable feasible region as follows.

3.1 Boundedness

Theorem 1

If the initial conditions of system 1 are in domain Q , then all solutions of system 1 enter and remain in the same domain Q .

Proof: It is first shown that all feasible solutions are uniformly bounded in a proper subset of Q . Splitting the

system into the male component (N_m) and the female component (N_f) given as $N_m = M + G + H_m + B_m + R_m$ and $N_f = F + L + H_f + B_f + R_f$. Let $(N_m = M + G + H_m + B_m + R_m)$ be any solution of the system with non-negative initial conditions.

Since $\frac{dN_m}{dt} \leq \lambda_1 - N_m \kappa_1$, it follows that, $\lim_{t \rightarrow \infty} N_m(t) \leq \frac{\lambda_1}{\kappa_1}$. This implies that $0 \leq N_m \leq \frac{\lambda_1}{\kappa_1}$. Similarly, the female population simplifies to $0 \leq N_f \leq \frac{\lambda_2}{\kappa_2}$ [10]. Hence the proof.

3.2 Positivity

$Q = \{M(t), G(t), L(t), H_m(t), H_f(t), B_m(t), B_f(t), R_m(t), R_f(t)\} \in R_+^{10}$ and $M(0) \geq 0, G(0) \geq 0, L(0) \geq 0, H_m(0) \geq 0, H_f(0) \geq 0, B_m(0) \geq 0, B_f(0) \geq 0, R_m(0) \geq 0, R_f(0) \geq 0$ then, the region will be positively bounded.

Proof

$$\frac{dM}{dt} = \lambda_1 + \Gamma_1 R_m - M(\kappa_1 + \beta_1 F + \pi_1)$$

$$\lambda_1 \geq 0, \Gamma_1 \geq 0$$

$$\frac{dM}{dt} \geq -M(\kappa_1 + \beta_1 F + \pi_1)$$

$$\frac{dM}{M} \geq \int_0^t -(\kappa_1 + \beta_1 F(s) + \pi_1) ds$$

$$M \geq e^{-\int_0^t (\kappa_1 + \beta_1 F(s) + \pi_1) ds} + C_1$$

Where C_1 Is a constant of integration. Putting initial conditions

$$M(0) = C_1$$

$$M(t) \geq M(0)e^{-\int_0^t (\kappa_1 + \beta_1 F(s) + \pi_1) ds} \geq 0$$

Using a similar argument

$$G(t) \geq G(0)e^{-\int_0^t (\kappa_1 + \psi_1 + \mu_1 F(s) + \mu_1 \theta_1 B_f(s)) ds} \geq 0$$

$$H_m(t) \geq H_m(0)e^{-(\kappa_1 + \gamma_1)t} \geq 0$$

$$B_m(t) \geq B_m(0)e^{-(\kappa_1 + \phi_1)t} \geq 0$$

$$R_m(t) \geq R_m(0)e^{-(\kappa_1 + \eta_1 + \chi_1 + \Gamma_1)t} \geq 0$$

$$F(t) \geq F(0)e^{-\int_0^t (\kappa_2 + \beta_2 M(s) + \pi_2) ds} \geq 0$$

$$\begin{aligned}
L(t) &\geq L(0)e^{-\int_0^t (\kappa_2 + \psi_2 + \mu_2 M(s) + \mu_2 \theta_2 B_m(s)) ds} \geq 0 \\
H_f(t) &\geq H_f(0)e^{-(\kappa_2 + \gamma_2)t} \geq 0 \\
B_f(t) &\geq B_f(0)e^{-(\kappa_2 + \varphi_2)t} \geq 0 \\
R_f(t) &\geq R_f(0)e^{-(\kappa_2 + \eta_2 + \chi_2 + \Gamma_2)t} \geq 0
\end{aligned}$$

Hence, the region Q is positively invariant, and the model is well-posed mathematically. Since the model monitors human populations, all the variables and parameters are assumed to be non-negative for $t \geq 0$.

4. Equilibrium states, reproductive number, and stability

4.1 Homosexuality-free equilibrium point (HFE)

Theorem 3: The system 1 has an HFE that is obtained by setting all the homosexual and recovered classes to zero.^[11]

Proof

$$E^0 = (M^0, G^0, H_m^0, B_m^0, R_m^0, F^0, L^0, H_f^0, B_f^0, R_f^0) = (\zeta_1, 0, \zeta_2, 0, 0, \zeta_3, 0, \zeta_4, 0, 0)$$

$$\text{Where } \zeta_1 = M^0, \zeta_2 = H_m^0, \zeta_3 = F^0, \zeta_4 = H_f^0$$

$$\begin{aligned}
\zeta_1 &= \frac{2\kappa_2\lambda_1}{\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2 + \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (\kappa_1\kappa_2 + \beta_2\lambda_1 - \beta_1\lambda_2)^2}} \\
\zeta_2 &= \frac{\kappa_1\kappa_2 + \beta_2\lambda_1 + \beta_1\lambda_2 - \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (\kappa_1\kappa_2 + \beta_2\lambda_1 - \beta_1\lambda_2)^2}}{2\kappa_1\beta_2} \\
\zeta_3 &= \frac{-\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2 + \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (-\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2)^2}}{2\kappa_1\beta_2} \\
\zeta_4 &= \frac{\beta_2\lambda_1(-\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2 + \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (-\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2)^2})}{\beta_1\kappa_2(\kappa_1\kappa_2 - \beta_2\lambda_1 + \beta_1\lambda_2 + \sqrt{4\beta_1\kappa_1\kappa_2\lambda_2 + (\kappa_1\kappa_2 + \beta_2\lambda_1 - \beta_1\lambda_2)^2})}
\end{aligned}$$

Hence the proof.

4.2 The Basic Reproduction Number

The next-generation method^{[12, [13]]} was used to obtain reproduction numbers. Using the notation f for a matrix of new infection terms and v for the matrix of the remaining transfer terms in system 1, yields;

$$\begin{aligned}
f &= \begin{pmatrix} \pi_1 M \\ \gamma_1 H_m \\ \pi_2 F \\ \gamma_2 H_f \end{pmatrix} \\
v &= \begin{pmatrix} (\kappa_1 + \psi_1)G + \mu_1(F + \theta_1 B_f)G \\ -\mu_1(F + \theta_1 B_f)G + (\kappa_1 + \varphi_1)B_m \\ (\kappa_2 + \psi_2)L + \mu_2(M + \theta_2 B_m)L \\ -\mu_2(M + \theta_2 B_m)L + (\kappa_2 + \varphi_2)B_f \end{pmatrix}
\end{aligned}$$

The Jacobi of f and v results to matrices F and V Respectively at HFE

$$\begin{aligned}
F &= \begin{pmatrix} M^0 d_1 & M^0 d_1 e_1 & 0 & 0 \\ H_m^0 d_2 & H_m^0 d_2 e_1 & 0 & 0 \\ 0 & 0 & F^0 d_8 & F^0 d_8 e_3 \\ 0 & 0 & H_f^0 d_5 & H_f^0 d_5 e_3 \end{pmatrix} \\
V &= \begin{pmatrix} \kappa_1 + F^0 \mu_1 + \psi_1 & 0 & 0 & 0 \\ -F^0 \mu_1 & \kappa_1 + \varphi_1 & 0 & 0 \\ 0 & 0 & \kappa_2 + M^0 \mu_2 + \psi_2 & 0 \\ 0 & 0 & -M^0 \mu_2 & \kappa_2 + \varphi_2 \end{pmatrix}
\end{aligned}$$

It follows that basic reproduction numbers for model 1, denoted by R_0^m and R_0^f are given by the eigenvalues of $F.V^{-1}$ as

$$\begin{aligned}
R_0^m &= \frac{d_2 e_1 H_m^0}{\kappa_1 + \varphi_1} + \frac{M^0 d_1 \kappa_1}{(\kappa_1 + \varphi_1)(\kappa_1 + F^0 \mu_1 + \psi_1)} + \frac{F^0 M^0 d_1 e_1 \mu_1}{(\kappa_1 + \varphi_1)(\kappa_1 + F^0 \mu_1 + \psi_1)} + \frac{M^0 d_1 \varphi_1}{(\kappa_1 + \varphi_1)(\kappa_1 + F^0 \mu_1 + \psi_1)} \\
R_0^f &= \frac{d_5 e_3 H_f^0}{\kappa_2 + \varphi_2} + \frac{F^0 d_8 \kappa_2}{(\kappa_2 + \varphi_2)(\kappa_2 + M^0 \mu_2 + \psi_2)} + \frac{F^0 M^0 d_8 e_3 \mu_2}{(\kappa_2 + \varphi_2)(\kappa_2 + M^0 \mu_2 + \psi_2)} + \frac{F^0 d_8 \varphi_2}{(\kappa_2 + \varphi_2)(\kappa_2 + M^0 \mu_2 + \psi_2)}
\end{aligned}$$

4.3 Local Stability Analysis of Homosexuality Free Equilibrium Point (HFE)

Theorem 4: The HFE of model system 1 is locally asymptotically stable (LAS) if $R_0^m < 1$ and $R_0^f < 1$ and unstable otherwise ^[12].

Proof

To establish the local stability, the Jacobian of the model was evaluated at E^0 . The stability of this steady state is then determined based on the signs of the eigenvalues of the corresponding Jacobian, which are functions of the model parameters.

$$\begin{pmatrix} -F^0\beta_1 - \kappa_1 & -M^0d_1 & 0 & -M^0d_1e_1 & \Gamma_1 & -M^0\beta_1 & 0 & 0 & 0 & 0 \\ 0 & M^0d_1 - \kappa_1 - \psi_1 & 0 & M^0d_1e_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1F^0 & 0 & -\kappa_1 & 0 & \eta_1 & M^0\beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\kappa_1 - \varphi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_1 & 0 & \varphi_1 & -(\Gamma_1 + \eta_1 + \kappa_1 + \chi_1) & 0 & 0 & 0 & 0 & 0 \\ -F^0\beta_2 & 0 & 0 & 0 & 0 & -M^0\beta_2 - \kappa_2 & -F^0d_8 & 0 & -F^0d_8e_3 & \Gamma_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & F^0d_8 - \kappa_2 - \psi_2 & 0 & F^0d_8e_3 & 0 \\ F^0\beta_2 & 0 & 0 & 0 & 0 & M^0\beta_2 & 0 & -\kappa_2 & 0 & \eta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\kappa_2 - \varphi_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_2 & 0 & \varphi_2 & -(\Gamma_2 + \eta_2 + \kappa_2 + \chi_2) \end{pmatrix} \quad (2)$$

All eigenvalues of (2) were negative under the conditions, $\frac{M^0d_1}{\kappa_1 + \psi_1} < 1$ and $\frac{F^0d_8}{\kappa_2 + \psi_2} < 1$. Thus, the homosexuality Free Equilibrium (HFE) of the system is locally asymptotically stable, i.e. $R_0^m < 1, R_0^f < 1$ and unstable otherwise.

4.4 Global Stability of the homosexuality-free equilibrium point (H.F.E)

Theorem 5

If system 1 can be written in the form $\frac{dX}{dt} = F(X, Z)$, $\frac{dZ}{dt} = G(X, Z)$, $G(X, 0) = 0$

Where $X = (M, H_m, R_m, F, H_f, R_f) \in R_+^6$ denotes non-infectious compartments and $Z = (G, B_m, L, B_f) \in R_+^4$ denotes the infectious compartments. $E_0 = (X^*, 0)$ represents the homosexuality-free equilibrium of the system if this point satisfies the following:

(i) $\frac{dX}{dt} = F(X, 0)$ where X^* is globally asymptotically stable

(ii) $\frac{dZ}{dt} = D_Z G(X, 0)Z - G(X, Z) \geq 0$ for all $X, Z \in \Omega$

Where $A = D_Z G(X^*, 0)$ is an M-matrix (the off-diagonal elements of A are non-negative) and Ω is the region where the model makes biological sense ^[16, 17, 14]. Then we can conclude that E_0 is a globally asymptotically stable (GAS) equilibrium of model system 1 provided that $R_0^m < 1$ and $R_0^f < 1$.

Proof

Applying theorem 5 to model system 1 gives

$$\hat{G}(X, Z) = \begin{pmatrix} \left(1 - \frac{N_m}{N}\right)(G + e_1 B_m) d_1 \\ 0 \\ \left(1 - \frac{N_f}{N}\right)(L + e_3 B_f) d_8 \\ 0 \end{pmatrix} \geq 0$$

Since $\hat{G}(X, Z) \geq 0$ by theorem 5, E_0 globally asymptotically stable

4.5 Existence of Homosexuality Endemic Equilibrium point (HEEP)

Conditions for the existence of equilibrium are sought for which the model is endemic in the population ^[12]. For HEEP to exist, the infectious classes of the system 1 must be greater than zero, that is, $G > 0, B_m > 0, L > 0, B_f > 0$. Setting G, B_m classes to zero and substituting them with each other, and letting them be greater than zero results to

$$\frac{d_2 e_1 H_m^0}{\kappa_1 + \varphi_1} + \frac{M^0 d_1 \kappa_1}{(\kappa_1 + \varphi_1)(\kappa_1 + F^0 \mu_1 + \psi_1)} + \frac{F^0 M^0 d_1 e_1 \mu_1}{(\kappa_1 + \varphi_1)(\kappa_1 + F^0 \mu_1 + \psi_1)} + \frac{M^0 d_1 \varphi_1}{(\kappa_1 + \varphi_1)(\kappa_1 + F^0 \mu_1 + \psi_1)} > 1$$

and similar argument for L and B_f gives

$$\frac{d_5 e_3 H_f^0}{\kappa_2 + \varphi_2} + \frac{F^0 d_8 \kappa_2}{(\kappa_2 + \varphi_2)(\kappa_2 + M^0 \mu_2 + \psi_2)} + \frac{F^0 M^0 d_8 e_3 \mu_2}{(\kappa_2 + \varphi_2)(\kappa_2 + M^0 \mu_2 + \psi_2)} + \frac{F^0 d_8 \varphi_2}{(\kappa_2 + \varphi_2)(\kappa_2 + M^0 \mu_2 + \psi_2)} > 1$$

Thus, HEEP has existed since $R_0^m > 1, R_0^f > 1$.

The numerical results show that as the time approaches infinity, $\frac{dG}{dt}, \frac{dB_m}{dt}, \frac{dL}{dt}, \frac{dB_f}{dt}$ approaches zero, which supports the analytical results

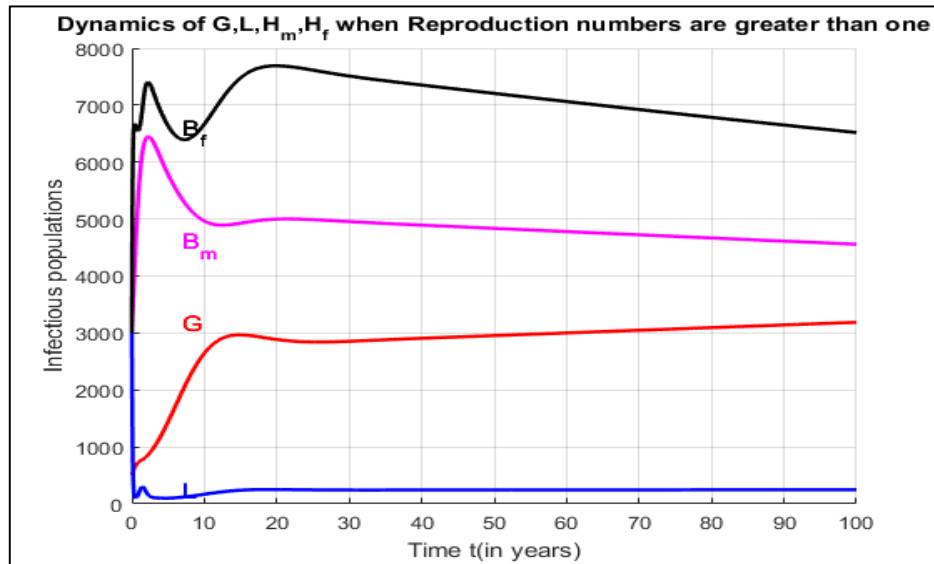


Fig 2: Graph showing the existence of endemic equilibrium points over time

4.6 Global stability of the Homosexuality Endemic Equilibrium point (HEEP)

Theorem 6

The HEEP is globally asymptotically stable whenever $R_0^m < 1$ and $R_0^f < 1$. [15]

Proof

The following Lyapunov Function was proposed

$$k(M, G, H_m, B_m, R_m, F, L, H_f, B_f, R_f) = M - M^* - M^* \ln \frac{M}{M^*} + y_1(G - G^* - G^* \ln \frac{G}{G^*}) + y_2(H_m - H_m^* - H_m^* \ln \frac{H_m}{H_m^*}) + y_3(B_m - B_m^* - B_m^* \ln \frac{B_m}{B_m^*}) + y_4(R_m - R_m^* - R_m^* \ln \frac{R_m}{R_m^*}) + (F - F^* - F^* \ln \frac{F}{F^*}) + y_5(L - L^* - L^* \ln \frac{L}{L^*}) + y_6(H_f - H_f^* - H_f^* \ln \frac{H_f}{H_f^*}) + y_7(B_f - B_f^* - B_f^* \ln \frac{B_f}{B_f^*}) + y_8(R_f - R_f^* - R_f^* \ln \frac{R_f}{R_f^*})$$

If the Lyapunov function $k(M, G, H_m, B_m, R_m, F, L, H_f, B_f, R_f)$ satisfies the condition

$k(M^*, G^*, H_m^*, B_m^*, R_m^*, F^*, L^*, H_f^*, B_f^*, R_f^*) = 0$ and $k(M^0, G^0, H_m^0, B_m^0, R_m^0, F^0, L^0, H_f^0, B_f^0, R_f^0) > 0$ is said to be positive definite.

Now the positive constants $y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8$ are chosen such that the coefficients of $(M, G, H_m, B_m, R_m, F, L, H_f, B_f, R_f)$ are equal to zero. Solving gives

$$y_1 = 1, y_2 = \frac{\nabla_5 \nabla_2 - \Gamma_1}{\eta_1 \psi_1}, y_3 = \frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1}, y_4 = \frac{\nabla_2}{\psi_1}, y_5 = 1, y_6 = \frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2}$$

$$y_7 = \frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2}, y_8 = \frac{\nabla_7}{\psi_2}$$

Where,

$$\nabla_1 = (\kappa_1 + \beta_1 F + \pi_1), \nabla_2 = (\kappa_1 + \psi_1 + \mu_1 F + \mu_1 \theta_1 B_f), \nabla_3 = (\kappa_1 + \gamma_1 G), \nabla_4 = (\kappa_1 + \varphi_1), \nabla_5 = (\kappa_1 + \eta_1 + \chi_1 + \Gamma_1), \nabla_6 = (\kappa_2 + \beta_2 M + \pi_2), \nabla_7 = (\kappa_2 + \psi_2 + \mu_2 M + \mu_2 \theta_2 B_m), \nabla_8 = (\kappa_2 + \gamma_2), \nabla_9 = (\kappa_2 + \varphi_2), \nabla_{10} = (\kappa_2 + \eta_2 + \chi_2 + \Gamma_2).$$

Letting P^* denote all the positive values and Q^* denote all the negative values of the solution after substituting evaluated constants, results to

$$P^* = M^* \nabla_1 + \Gamma_1 R_m + M \left(\frac{d_1 G}{N_m} + \frac{d_1 e_1 B_m}{N_m} \right) + \frac{M^*}{M} M \nabla_1 + \frac{M^*}{M} M \nabla_1 + \pi_1 M +$$

$$\begin{aligned} & \frac{G^*}{G} G \nabla_2 + \beta_1 M F \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) + \eta_1 R_m \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) + \frac{H_m^*}{H_m} H_m \nabla_3 \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) + \\ & \gamma_1 H_m \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) + \mu_1 (F + \theta_1 B_f) G \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) + \frac{B_m^*}{B_m} B_m \nabla_4 \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) + \varphi_1 B_m \left(\frac{\nabla_2}{\psi_1} \right) + \psi_1 G \left(\frac{\nabla_2}{\psi_1} \right) + \\ & \frac{R_m^*}{R_m} R_m \nabla_5 \left(\frac{\nabla_2}{\psi_1} \right) + \lambda_2 + \Gamma_2 R_f + F \left(\frac{d_8 L}{N_f} + \frac{d_5 e_3 B_f}{N_f} \right) \frac{F^*}{F} \nabla_6 F + \pi_2 F + \frac{L^*}{L} \nabla_7 L + \beta_2 M F + \\ & \eta_2 R_f \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) + \frac{H_f^*}{H_f} \nabla_8 H_f \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) + \gamma_2 H_f \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) + \mu_2 (M + \theta_2 B_m) L \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) + \\ & \frac{B_f^*}{B_f} \nabla_9 B_f \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) + \varphi_2 B_f \left(\frac{\nabla_7}{\psi_2} \right) + \psi_2 L y_8 + \frac{R_f^*}{R_f} \nabla_{10} R_f \left(\frac{\nabla_7}{\psi_2} \right) \\ Q^* = & -\Gamma_1 R_m^* - M \left(\frac{d_1 G}{N_m} + \frac{d_1 e_1 B_m}{N_m} \right) - \frac{M^*}{M} \lambda_1 - \frac{M^*}{M} \Gamma_1 R_m - G \nabla_2 - \frac{G^*}{G} \pi_1 M - \\ & H_m \nabla_3 \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) - \frac{H_m^*}{H_m} \beta_1 M F \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) - \frac{H_m^*}{H_m} \eta_1 R_m \left(\frac{\nabla_5 \nabla_2 - \psi_1 \Gamma_1}{\psi_1 \eta_1} \right) - B_m \nabla_4 \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) - \\ & \frac{B_m^*}{B_m} \gamma_1 H_m \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) - \frac{B_m^*}{B_m} (\mu_1 (F + \theta_1 B_f) G) \left(\frac{\varphi_1 \nabla_2}{\nabla_4 \psi_1} \right) - R_m \nabla_5 \left(\frac{\nabla_2}{\psi_1} \right) - \frac{R_m^*}{R_m} \varphi_1 B_m \left(\frac{\nabla_2}{\psi_1} \right) - \\ & \frac{R_m^*}{R_m} \psi_1 G \left(\frac{\nabla_2}{\psi_1} \right) - F \left(\frac{d_8 L}{N_f} + \frac{d_5 e_3 B_f}{N_f} \right) - \frac{F^*}{F} \lambda_2 - \frac{F^*}{F} \Gamma_2 R_f - \nabla_7 L - \frac{L^*}{L} \pi_2 F - \nabla_8 H_f \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) - \\ & \frac{H_f^*}{H_f} \beta_2 M F \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) - \frac{H_f^*}{H_f} \eta_2 R_f \left(\frac{\nabla_7 \nabla_{10} - \psi_2 \Gamma_2}{\psi_2 \eta_2} \right) - \nabla_9 B_f \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) - \frac{B_f^*}{B_f} \gamma_2 H_f \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) - \\ & \frac{B_f^*}{B_f} \mu_2 (M + \theta_2 B_m) L \left(\frac{\varphi_2 \nabla_7}{\nabla_9 \psi_2} \right) - \nabla_{10} R_f \left(\frac{\nabla_7}{\psi_2} \right) - \frac{R_f^*}{R_f} \varphi_2 B_f \left(\frac{\nabla_7}{\psi_2} \right) - \frac{R_f^*}{R_f} \psi_2 L \left(\frac{\nabla_7}{\psi_2} \right) \end{aligned}$$

Then $\frac{dk}{dt} = 0$ holds only when $M = M^*, G = G^*, H_m = H_m^*, B_m = B_m^*, R_m = R_m^*, F = F^*, L = L^*, H_f = H_f^*, B_f = B_f^*, R_f = R_f^*$ so the maximal compact invariant set in $S; E; T \in \Pi: \frac{dV}{dt} = 0$ is the singleton E^* using Lassalle's invariance principle [18, ^[19], $\frac{dk(M, G, H_m, B_m, R_m, F, L, H_f, B_f, R_f)}{dt} < 0$ if $R_0^m > 1$ and $R_0^f > 1$, that is $P^* > Q^*$.

4.7 Bifurcation Analysis

The Centre Manifold Theorem ^[16] was used to investigate the nature of the bifurcation of the model 1. Letting P_k be the k^{th} component of p and

$$\begin{aligned} a &= \sum_{k, i, j=0}^n V_k u_i u_j \frac{\partial^2 p_k}{\partial y_i \partial y_j} (0, 0) \\ b &= \sum_{k, i, j=0}^n V_k v_i \frac{\partial^2 p_k}{\partial y_i \partial R_0} (0, 0) \end{aligned}$$

Considering a case where $R_0^{*m} = 1$ and $R_0^{*f} = 1, d_1 = d_1^*$ and $d_8 = d_8^*$ were chosen as bifurcation parameters as follows:

$$\begin{aligned} d_1^* &= - \frac{(d_2 e_1 H_m - \kappa_1 - \varphi_1)(\kappa_1 + F \mu_1 + \psi_1)}{M(\kappa_1 + F e_1 \mu_1 + \varphi_1)} \\ d_8^* &= - \frac{(d_5 e_3 H_f - \kappa_2 - \varphi_2)(\kappa_2 + M \mu_2 + \psi_2)}{F(\kappa_2 + M e_3 \mu_2 + \varphi_2)} \end{aligned}$$

Jacobian J^* was obtained

$$\begin{pmatrix} -F^0 \beta_1 - \kappa_1 & -M^0 d_1^* & 0 & -M^0 d_1^* e_1 & \Gamma_1 & -M^0 \beta_1 & 0 & 0 & 0 & 0 \\ 0 & M^0 d_1^* - \kappa_1 - \psi_1 & 0 & M^0 d_1^* e_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 F^0 & 0 & -\kappa_1 & 0 & \eta_1 & M^0 \beta_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\kappa_1 - \varphi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_1 & 0 & \varphi_1 & -(\Gamma_1 + \eta_1 + \kappa_1 + \chi_1) & 0 & 0 & 0 & 0 & 0 \\ -F^0 \beta_2 & 0 & 0 & 0 & 0 & -M^0 \beta_2 - \kappa_2 & -F^0 d_8^* & 0 & -F^0 d_8^* e_3 & \Gamma_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & F^0 d_8^* - \kappa_2 - \psi_2 & 0 & F^0 d_8^* e_3 & 0 \\ F^0 \beta_2 & 0 & 0 & 0 & 0 & M^0 \beta_2 & 0 & -\kappa_2 & 0 & \eta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\kappa_2 - \varphi_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_2 & 0 & \varphi_2 & -(\Gamma_2 + \eta_2 + \kappa_2 + \chi_2) \end{pmatrix}$$

At HFE the Jacobian J^* was found to have the right eigenvector given by $u = [u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}]^T$ and left eigenvectors as $v = [v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}]$. Solving these vectors resulted to $u_1 = 0, u_2 = 0, u_3 = 0, u_4 = 0, u_5 = 0, u_6 = 0, u_7 = 0, u_8 = 0, u_9 = 0, u_{10} = 0$ and $v_1 = 0, v_2 = 0, v_3 = 0, v_4 = 0, v_5 = 0, v_6 = 0, v_7 = 0, v_8 = 0, v_9 = 0, v_{10} = 0$.

Since the values of right and left eigenvectors obtained would give us $a = 0$ and $b = 0$, this either means that the system doesn't have a bifurcation, or we can't use the Center Manifold theorem to investigate the bifurcation of this system.

4.8 Normalized Sensitivity analysis of the parameters

The technique employed in the sensitivity analysis was based on the reproduction numbers. This helps in determining the level of contribution of each parameter to the reproduction number. This technique uses partial derivatives with respect to the parameters

^[17]. These indices are given by $\Lambda_{\rho}^{R_C^*} = \frac{\partial R_C^*}{\partial \rho} \times \frac{\rho}{R_C^*}$

Table 2: Sensitivity indices of parameters of the reproduction numbers

Parameter	Value	Parameter	Value
κ_1	-0.0000355474	κ_2	-4.80141×10^{-9}
μ_1	-0.999556	μ_2	-0.0120087
ϕ_1	-0.000186179	ϕ_2	-0.0736024
ψ_1	-0.000222164	ψ_2	-5.6784×10^{-6}
e_1	0.00018618	e_3	0.0736024
d_1	0.999994	d_2	6.21469×10^{-6}
d_5	0.000435031	d_8	0.0851817

4.9 Partial rank correlation coefficient (PRCC)

To identify parameters that were more influential to the model, PRCC analysis was carried out, where the strength of the outputs and inputs of the model was determined using the LHS method ^[18]. The closer a value was to +1 or -1, the stronger the linear association between x and y variables, and the farther a value was from +1 or -1, the weaker the linear association between x and y variables.

Table 3: PRCC and P-Values Sensitivity indices

Parameter	PRCC	P-Value	Parameter	PRCC	P-Value
λ_1	-0.096408	0.33999	λ_2	-0.085649	0.39684
κ_1	-0.074297	0.46255	κ_2	0.097847	0.3328
β_1	-0.071783	0.47788	β_2	-0.2308	0.020875
Γ_1	-0.12313	0.2223	Γ_2	0.22952	0.021613
μ_1	0.11161	0.26893	μ_2	0.084446	0.40353
γ_1	-0.71351	$8.0601e^{-17}$	γ_2	-0.087415	0.38714
ϕ_1	-0.025908	0.79805	ϕ_2	-0.15405	0.12593
η_1	-0.046069	0.64901	η_2	0.055011	0.58672
ψ_1	-0.0090479	0.92881	ψ_2	-0.1635	0.10408
χ_1	0.2125	0.03379	χ_2	-0.28452	0.0041184
θ_1	0.070599	0.48519	θ_2	-0.10136	0.31563
e_1	-0.91166	$1.2431e^{-39}$	e_3	-0.31612	0.0013553
d_1	-0.16511	0.10066	d_2	-0.074916	0.45882
d_5	0.021105	0.8349	d_8	0.13173	0.19141

5. Numerical Simulations

To observe the dynamics of the sexual orientation model over time, numerical simulations were done using MATLAB software. The following parameter values were used in the simulation.

Table 4: Parameter estimations

Parameter	value	Source	Parameter	Value	Source
λ_1	0.027	^[19]	λ_2	0.027	^[19]
κ_1	0.016	^[19]	κ_2	0.016	^[19]
β_1	0.0018	Estimated	β_2	0.0018	Estimated
Γ_1	0.2	^[19]	Γ_2	0.92	Estimated
μ_1	0.000045	Estimated	μ_2	0.015	Estimated
γ_1	0.015	Estimated	γ_2	0.05	Estimated
ϕ_1	0.25	Estimated	ϕ_2	0.25	Estimated
η_1	0.005	Estimated	η_2	0.25	Estimated
ψ_1	0.00001	Estimated	ψ_2	0.001	^[19]
χ_1	0.0015	Estimated	χ_2	0.015	Estimated
θ_1	0.75	Estimated	θ_2	0.0015	Estimated
e_1	0.001	Estimated	e_3	0.1015	Estimated
d_1	0.0021	Estimated	d_2	0.00145	Estimated
d_5	0.001	Estimated	d_8	0.00841	Estimated

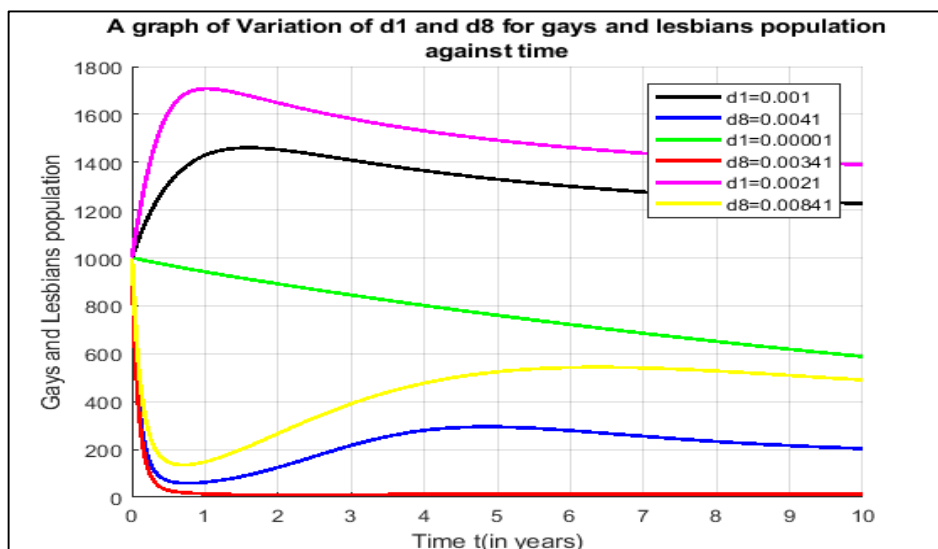


Fig 3 (a): Graph showing the effect of varying homosexuality recruitment rates on gay and lesbian populations.

Figure 3 (a) shows that increasing the gay recruitment rate (d_1) by 110% increases the gay population by 16%, while increasing the lesbian recruitment rate (d_8) by 105% increases the lesbian population by 362% at maximum values.

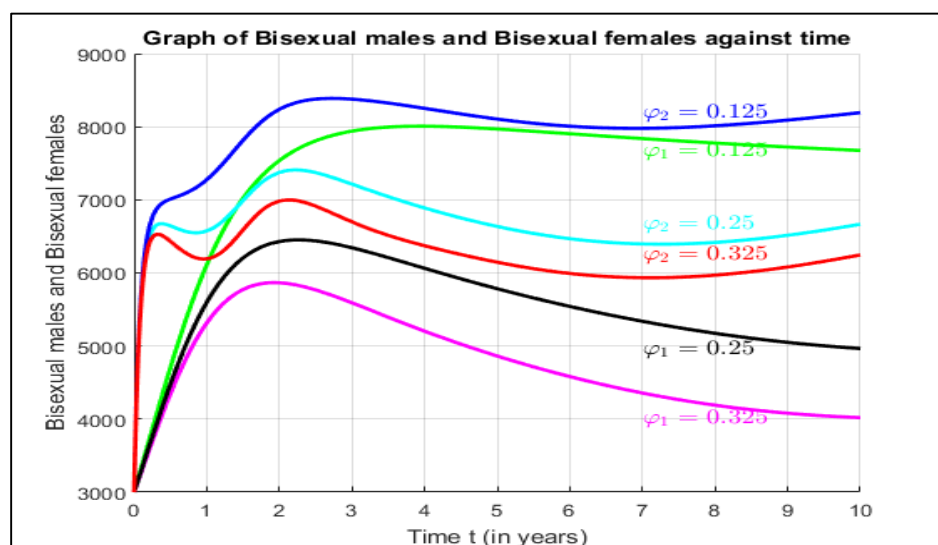


Fig 3 (b): Graphs showing the effects of varying transfer rates from the bisexual population to recovery centers.

Fig 3 (b) shows that increasing ϕ_1 (the rate at which the bisexual male population joins recovery centers) by 30% decreases the bisexual male population by 7.9% while increasing ϕ_2 (the rate at which the bisexual female population joins recovery centers) by 30% decreases the bisexual female population by 2.3% at the maximum value.

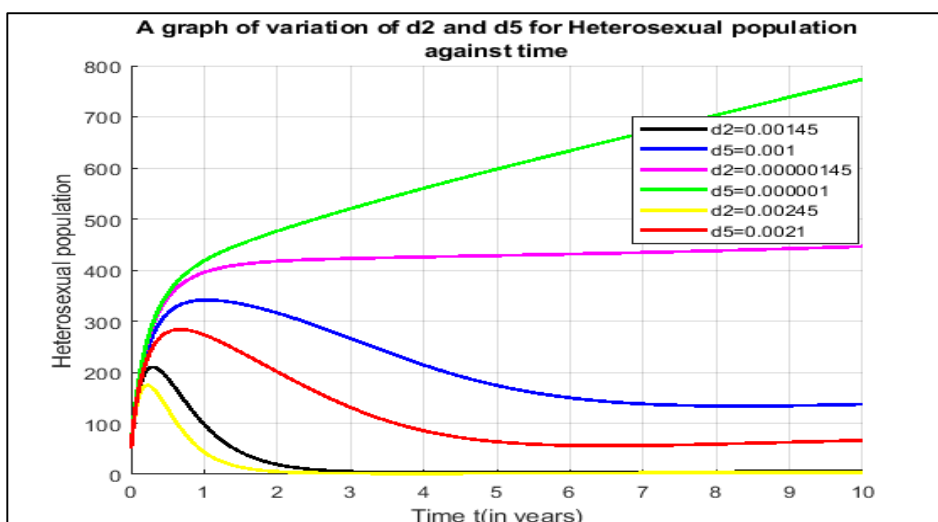


Fig 4 (a): Graphs showing the effect of varying bisexual recruitment rates

Figure 4 (a) shows that increasing the Bisexual males' recruitment rate (d_2) by 69% decreases the heterosexual male population by 17.8%, while increasing the bisexual females' recruitment rate (d_5) by 110% decreases the heterosexual female population by 12.3% at maximum value.

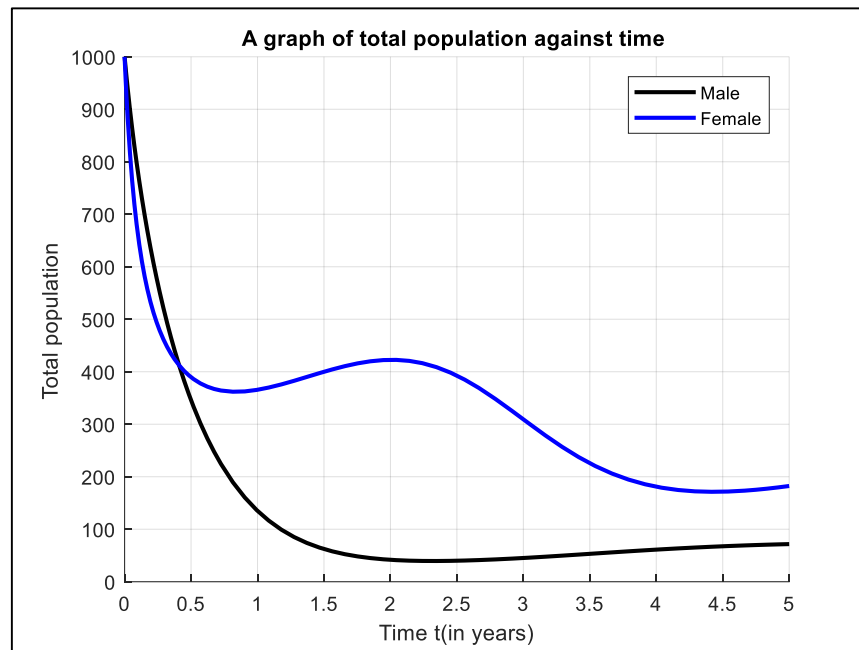


Fig 4 (b): Graph of male and female populations against time

Figure 4 (b) shows that the population of susceptible (male and female) populations declines with time and that the male population declines faster than the female population.

Results and Discussion

In this study, a deterministic mathematical model of sexual orientation with recovery centers was developed. The qualitative behavior of model 1 was analyzed by obtaining the positivity and boundedness of the solution, equilibrium points, and control reproduction numbers R_0^m and R_0^f were obtained using the next-generation matrix method. Local stability of HFE was obtained by Routh-Hurwitz criteria, and global stability by the Metzler Matrix method. Lyapunov function was used to determine the global stability of HEEP, and bifurcation analysis was also carried out.

From the analysis carried out, the model was bounded and lay in the positive region. The local stability of HFE was LAS since $R_0^m < 1$ and $R_0^f < 1$, and the global stability of HEEP was asymptotically stable when $R_0^m > 1$ and $R_0^f > 1$. It was also observed that the global stability of HFE was asymptotically stable, given that $R_0^m < 1$ and $R_0^f < 1$. Control reproduction numbers were obtained as $R_0^m = 3.9$ and $R_0^f = 3.2$.

Moreover, a sensitivity analysis of the system was also performed based on the control reproduction number, and it was observed that d_1, d_8 were the most influential parameters hence the conclusion that varying d_1, d_8 would significantly influence the spread of homosexuality within the community. To identify the highly sensitive parameters in the model, the sensitivity analysis was performed based on the PRCC method and LHS, and showed $(e_1, e_3, \Gamma_2, \chi_1, \chi_2)$ as the most influential parameters.

Finally, numerical simulations showed that increasing recruitment rates (d_1, d_2, d_5, d_8) increased homosexuality in the population and increasing ϕ_1 and ϕ_2 decreased bisexual male and bisexual female population, respectively. Therefore, this paper shows that the best way of reducing the spread is by

reducing the contact rate and increasing transfer rates from homosexual classes to recovery centers.

Acknowledgment

The authors appreciate the support accorded by the Kirinyaga University staff fraternity.

Conflict of interest

The authors declare that there is no conflict of interest.

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