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Air resistance and the earth's shadow caused by solar pressure in the system of two inextensible cable-connected satellites in orbit are represented by the equation of relative motion

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Abstract

The current study addresses the circular orbit's differential equations of motion. Concerning the centre of mass by putting $e=0$ and hence $\rho = \frac{1}{1+e \cos v} = 1$ and $\rho' = 0$ in the equation (1.5.12) for three dimensional case.

Keywords: Earth's shadow, solar radiation, perturbative, satellites

Introduction

The purpose of the proposed work is to analyze how the earth's shadow affects the motion of two satellites connected by a thin, flexible, and inextensible cable in the earth's gravitational field when it is perturbed by atmospheric resistance and solar radiation pressure. In this manner, the system has been permitted to move through the earth's shadow beam while in orbit around the planet due to the perturbative impact of solar radiation pressure. The shadow cast by the earth is thought to be cylindrical. The effect of the other Perturbative forces like oblateness of the earth and magnetic force of the earth etc. have been neglected during our analysis. The mathematical idealization of the physical problem is based on the following natural assumptions, which will form the foundation of the analytic approach to the further investigation of the problem:

Two satellites are considered as material particles.

The connecting cable of the two satellites is so chosen that it may be light, flexible and inextensible in character.

Earth's central force of attraction is taken as the main force governing the motion of the system.

Solar pressure, Air resistance, magnetic force and shadow of the earth oblateness of the earth etc. are taken as Perturbative forces.

But in our investigation for the motion of the system, only the influence of atmospheric resistance, shadow of the oblate earth and solar pressure.

Equation of motion in Nechvill's co-ordinate system

We shall use Nechvill's co-ordinate system for our further investigation of the motion of the system which are given by

$$\xi = \rho x$$

$$\eta = \rho y$$

$$\zeta = \rho z$$

$$\text{Where } \rho = \frac{R}{P} = \frac{1}{1+e \cos v}$$

..... (1)

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P= semi lotus rectum or focal parameter.

e= eccentricity of the orbit of centre of mass of the system.

Instead of time t, we shall use true anomaly v of the centre of mass as independent variable. The time t and true anomaly v are given by the differential relation.

$$\dot{v} = \frac{dv}{dt} = \frac{\sqrt{\mu P}}{P^2} \frac{1}{\rho^2} \dots\dots\dots (2)$$

Where $\frac{\sqrt{\mu P}}{P^2}$ remains constraint for a given orbit.

Let dashes denote differentiations with respect to v.

Now,

$$\begin{aligned} \dot{\xi} &= \frac{d}{dt}(\rho x) = \frac{d}{dv}(\rho x) \frac{dv}{dt} = (\rho' x + \rho x') \dot{v} \\ \dot{\eta} &= (\rho' y + \rho y') \dot{v} \text{ and} \\ \dot{\zeta} &= (\rho' z + \rho z') \dot{v} \dots\dots\dots (3) \end{aligned}$$

From (3), we obtain

$$\begin{aligned} \ddot{\xi} &= (\rho'' x + 2\rho' x' + \rho x'') \dot{v}^2 + (\rho' x + \rho x') \dot{v} \dot{v}' \\ \ddot{\eta} &= (\rho'' y + 2\rho' y' + \rho y'') \dot{v}^2 + (\rho' y + \rho y') \dot{v} \dot{v}' \text{ and} \\ \ddot{\zeta} &= \dot{v}^2 + (\rho' z + \rho z') \dot{v} \dot{v}' \dots\dots\dots (4) \end{aligned}$$

From (2), we obtain

$$\begin{aligned} \dot{v} &= \frac{\sqrt{\mu P}}{P^2} \frac{1}{\rho^2} \\ \Rightarrow \ddot{v} &= \frac{\sqrt{\mu P}}{P^2} \left(\frac{-2\rho\rho'}{\rho^4} \dot{v} \right) \\ &= \frac{\sqrt{\mu P}}{P^2} \frac{2\rho'}{\rho^3} \dot{v} \\ \therefore 2\dot{v}^2 \rho' + \ddot{v} \rho &= 0 \dots\dots\dots (5) \end{aligned}$$

$$\text{But, } \dot{v}' = -2 \frac{\sqrt{\mu P}}{P^2} \frac{\rho'}{\rho^3}$$

$$\therefore \dot{v}' \dot{v} = 2 \frac{\sqrt{\mu P}}{P^2} \frac{\rho' v}{\rho^3} = \ddot{v}$$

$$\therefore \ddot{v} = \dot{v}' \dot{v}$$

Putting $\ddot{v} = \dot{v}' \dot{v}$ in (5), we get

$$\dot{v}(2\dot{v}\rho' + \rho\dot{v}') = 0$$

$$\therefore 2\dot{v}\rho' + \rho\dot{v}' = 0 \quad \dots\dots\dots (6)$$

considered. The other disturbing forces on the motion of the system are neglected.

$$x'' - 2y' - 3\rho x - \frac{12k_2x}{\rho p^2} + \frac{a_1 p^3 \rho \rho'}{\sqrt{\mu p}} \\ = \frac{\bar{\lambda}_\alpha p^3}{\mu} \rho^4 x - \frac{p^3 \rho^3 m_1}{\mu l(m_1 + m_2)} \psi_1 \left(\frac{B_1}{M_1} - \frac{B_2}{M_2} \right) \cos \varepsilon \cos(v - \alpha) \\ x'' - 2y' - 3\rho x + \frac{4B}{\rho} x + f\rho\rho' + A\rho^3 \psi_1 \cos \varepsilon \cos(v - \alpha) - \lambda_\alpha \rho^4 x = 0 \quad \dots\dots\dots (7)$$

$$\text{Where } A = \frac{p^3 m_1}{\mu l(m_1 + m_2)} \left(\frac{B_1}{M_1} - \frac{B_2}{M_2} \right);$$

$$B = -\frac{3k_2}{p^2};$$

$$f = \frac{a_1 p^3}{\sqrt{\mu p}} \text{ and}$$

$$\lambda_\alpha = \frac{\bar{\lambda}_\alpha p^3}{\mu} = \frac{-\lambda p^3 (m_1 + m_2)}{\mu l m_1 m_2} \quad \dots\dots\dots (8)$$

Similarly, 2nd and 3rd equations may be written as

$$y'' + 2x' + f\rho^2 - \frac{By}{\rho} + A\rho^3 \psi_1 \cos \varepsilon \cos(v - \alpha) = \lambda_\alpha \rho^4 y \text{ and} \\ z'' + z + \frac{Bz}{\rho} - A\rho^3 \psi_1 \sin \varepsilon = \lambda_\alpha \rho^4 z \quad \dots\dots\dots (9)$$

Thus we get a new set of differential equation which given the motion of the particle of mass m_2 in the form

$$x'' - 2y' - 3\rho x + \frac{4B}{\rho} x + f\rho\rho' + A\rho^3 \psi_1 \cos \varepsilon \cos(v - \alpha) = \lambda_\alpha \rho^4 x; \\ y'' + 2x' + f\rho^2 - \frac{By}{\rho} - A\rho^3 \psi_1 \cos \varepsilon \sin(v - \alpha) = \lambda_\alpha \rho^4 y \\ \& z'' + z - \frac{Bz}{\rho} - A\rho^3 \psi_1 \sin \varepsilon = \lambda_\alpha \rho^4 z \quad \dots\dots\dots (10)$$

Where A, B, f and λ_α are given in (10), describes the motion of the particle of mass m_2 in the rotating frame of reference in Nechvill's coordinate system which is non-linear, non-homogeneous and non-autonomous whose general solution is beyond our reach Hence the equations of motion given by (10) take the form:

$$x'' - 2y' - 3x + 4Bx + A\psi_1 \cos \varepsilon \cos(v - \alpha) = \lambda_\alpha x \\ y'' + 2x' + f - By - A\psi_1 \cos \varepsilon \sin(v - \alpha) = \lambda_\alpha y \text{ and} \\ z'' + z - Bz - A\psi_1 \sin \varepsilon = \lambda_\alpha z \quad \dots\dots\dots (11)$$

Where A, B, f and λ_α have their usual meaning and are give in (10). The condition of constraint is given by (11) takes the form:

$$x^2 + y^2 + z^2 \leq 1 \quad \dots\dots\dots (12)$$

Multiplying 1st, 2nd and 3rd equations of (11) by $2x'$, $2y'$ and $2z'$ respectively and adding we get

$$\begin{aligned} & (2x'x'' + 2y'y'' + 2z'z'') - (3 - 4B)2xx' - 2Byy' - 2Bzz' + 2zz' \\ & + 2fy' - \frac{2A \cos \alpha \sin \theta x'}{\pi} - \frac{2A \sin \alpha \sin \theta y}{\pi} \\ & - \lambda_\alpha \cdot 2(xx' + yy' + zz') = 0 \quad \dots\dots\dots (13) \end{aligned}$$

Using (10) in (11) and integrating, we get

$$\begin{aligned} x'^2 + y'^2 + z'^2 &= (3 - 4B)x^2 + B(y^2 + z^2) - z^2 - 2fy \\ &+ \frac{2A \sin \theta}{\pi} (x \cos \alpha + y \sin \alpha) + h \quad \dots\dots\dots (14) \end{aligned}$$

Where h is the constant of integration.

(14) is known as Jacobian integral and can be interpreted as energy equation with modified potential v and is given by

$$\begin{aligned} V &= -\frac{1}{2} [(3 - 4B)x^2 - z^2 + B(y^2 + z^2)] + fy \\ &- \frac{A}{\pi} \sin \theta (x \cos \alpha + y \sin \alpha) \quad \dots\dots\dots (15) \end{aligned}$$

Thus we get a new set of differential equation which given the motion of the particle of mass m_2 in the form

$$\begin{aligned} x'' - 2y' - 3\rho x + \frac{4B}{\rho}x + f\rho\rho' + A\rho^3\psi_1 \cos \varepsilon \cos(v - \alpha) &= \lambda_\alpha \rho^4 x; \\ y'' + 2x' + f\rho^2 - \frac{By}{\rho} - A\rho^3\psi_1 \cos \varepsilon \sin(v - \alpha) &= \lambda_\alpha \rho^4 y \\ \& \ z'' + z - \frac{Bz}{\rho} - A\rho^3\psi_1 \sin \varepsilon = \lambda_\alpha \rho^4 z \quad \dots\dots\dots (16) \end{aligned}$$

Conclusion

Where A, B, f and λ_α are given in (16), describes the motion of the particle of mass m_1 and m_2 in the rotating frame of reference in Nechvill's coordinate system which is non-linear, non-homogeneous and non-autonomous whose general solution is beyond our reach.

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