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Study of the general nature of motion and stability in elliptical orbit of two connected satellites system under several influences

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Abstract

The derive a specific problem in stability theory when two artificial satellites connected by a light flexible non-conducting and inelastic cable and orbit around the earth under the influence of several perturbative forces like shadow of the earth, solar radiation pressure, oblateness of the earth air resistance and earth magnetic field. The central gravitational force plays a huge role in satellites. Technology without motion of satellites in space it is not possible us to use satellites for communication. The gravitational field earth is the main force to obtain the Jacobian integral of motion for this system. Further simulation and the equations of motion give rise to one equilibrium positions of motion of the system concerned under the above mentioned perturbative forces.

Keywords: Equation of motion, elliptical orbit, two satellites system, magnetic field, light flexible

Introduction

The linearised and normalized equations of relative motion of two cable connected satellites system, many space configurations of cable-connected that the systems have been proposed and analysed by different authors.

Beletsky and Novikova (1969) studied the motion of system of two cable connected satellites in central gravitational field of force relative to its centre of mass. this study assumed that the two satellites are moving in the plane of the centre of mass. Singh and Demin (1972) and Singh (1973) investigated the problem in two and three dimensional cases, Das et. al. (1976). These work conducted the analysis of relative motion of the system for the elliptical orbit of the centre of mass in two dimensional as well as three dimensional cases. Naryan and Singh (1987); Markaev et. al. (2003) studied the planar oscillation of a satellites in circular orbit. Ayub Khan (2011) investigated Chaotic motion in problem of dumbel satellites.

Kumar and Srivastava (2006) studied evolutionary and non-evolutional motion of a system of two cable connected artificial satellites under some perturbative forces.

Kumar and Singh (2010) studied motion and stability of two interconnected satellites system in circular orbit of the centre of mass. This work direct generalization of the works done by Das S.K. and Bhattacharya S.K., Singh A.K.P. Kumar R.K. and Singh N.K.

Equation of Motion in Case of Circular Orbit of the Centre of Mass of the System

The normalised and linearised differential equation of motion of one of the two satellites when the centre of mass moves along Keplerian Elliptical orbit in Nechvill's co-ordinate's system have been derived in the form:

$$x'' - 2y' - 3x\rho - \frac{4\beta x}{\rho} + A\rho^3\psi_1 \cos \epsilon \cos(\nu - \alpha) = -\bar{\lambda}_\alpha \rho^4 \left[1 - \frac{\ell_o}{\alpha \sqrt{x^2 + y^2 + z^2}} \right] x \quad .(1)$$

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$$y'' + 2x' + \frac{\beta y}{\rho} - A\rho^3 \psi_1 \cos \epsilon \sin(\nu - \alpha) = -\bar{\lambda}_\alpha \rho^3 \left[1 - \frac{\ell_o}{\sqrt{x^2 + y^2 + z^2}} \right] y$$

$$z'' + z + \frac{\beta z}{\rho} - A\rho^3 \psi_1 \sin \epsilon = -\bar{\lambda}_\alpha \rho^4 \left[1 - \frac{\ell_o}{\sqrt{x^2 + y^2 + z^2}} \right] z$$

where

$$A = \frac{\rho^3}{\mu} \left[\frac{B_1}{m_1} - \frac{B_2}{m_2} \right]$$

$$B = \frac{3k_2}{\rho^2}$$

$$\bar{\lambda}_\alpha = \frac{\rho^3}{\mu} \lambda \alpha$$

ψ_1 = shadow of earth parameter.

ℓ_o is the Length of cable connecting two satellites of masses m_1 and m_2 respectively. μ is the product of gravitational constant with mass of attracting centre (the earth) ρ is the semi latus ractum

The condition of constraint is given by:

$$x^2 + y^2 + z^2 \leq \frac{\ell_o}{\rho^2} \quad \text{..... (2)}$$

The system of satellites are under the shadow of the earth. When the shadow function $\psi_1 = 0$ and in that case the system will have no sunlight effect and hence the Left hand side equation (1) the portubative terms containing ψ_1 will vanished but as soon as the system will express the sun light effect the shadow function $\psi_1 = \rho$. Since general solution of (1) is beyond our reach, so, we restrict myself to the case of circular orbit we have

$$\rho = 0$$

So,

$$\rho = \frac{1}{1 + e \cos \nu} = 1 \text{ and } \rho' = 0 \quad \text{..... (3)}$$

putting $\rho = 1$ in (1) we get the equation of motion for two dimensional case.

$$\left. \begin{aligned} x'' - 2y' - (3 + 4B)x + A \cos \epsilon \cos(\tau - \alpha) &= -\bar{\lambda}_\alpha \left[1 - \frac{\ell_o}{r} \right] x \\ y'' + 2x' + By - A \cos \epsilon \sin(\tau - \alpha) &= -\bar{\lambda}_\alpha \left[1 - \frac{\ell_o}{r} \right] y \end{aligned} \right\} \quad \text{..... (4)}$$

where dash denotes the differentiation with respect to 't' and $r = \sqrt{x^2 + y^2}$.

The condition of constraint given by (2)

$$x^2 + y^2 \leq \ell_o^2 \quad \text{..... (5)}$$

Thus after the averaging the periodic terms of (4) we have –

$$\frac{1}{2\pi} \left[\int_{-\theta}^{\theta} A \cos \epsilon \cos(\tau - \alpha) d\tau + \int_{\theta}^{2\pi - \theta} A \cos \epsilon \cos(\tau - \alpha) d\tau \right] = \frac{-A \cos \epsilon \cos \alpha \sin \theta}{\pi}$$

$$\frac{1}{2\pi} \left[\int_{-\theta}^{\theta} A \cos \epsilon \sin(\tau - \alpha) d\tau + \int_{\theta}^{2\pi - \theta} A \cos \epsilon \sin(\tau - \alpha) d\tau \right] = \frac{A \cos \epsilon \sin \alpha \sin \theta}{\pi}$$

..... (6)

Equation of motion of the system are being described by using averaged values (6) in the form –

$$\left. \begin{aligned} x'' - 2y' - (3 + 4B) - \frac{-A \cos \epsilon \cos \alpha \sin \theta}{\pi} &= -\bar{\lambda}_{\alpha} \left[1 - \frac{\ell_o}{r} \right] x \\ y'' + 2x' + By - \frac{A \cos \epsilon \sin \alpha \sin \theta}{\pi} &= -\bar{\lambda}_{\alpha} \left[1 - \frac{\ell_o}{r} \right] y \end{aligned} \right\} \quad \text{..... (7)}$$

Multiplying the two equation (7) by $2x'$ and $2y'$ and adding, we get after integrating, we get the Jacobian integral -

$$x'^2 + y'^2 - (3 + 4B)x^2 - \frac{2Ax \cos \epsilon \cos \alpha \sin \theta}{\pi} - \frac{2Ay}{\pi} \cos \epsilon \sin \alpha \sin \theta +$$

$$+\bar{\lambda}_{\alpha} \left[(x^2 + y^2) - 2\ell_o \sqrt{x^2 + y^2} \right] = h \quad \text{..... (8)}$$

Hence we conclude that the satellites of mass m_1 will move inside the boundary of different curve of zero velocity represental (8) for different values of Jacobian constant.

Equilibrium-solution of the Problem

We have obtained a set of equations given by (4) for motion of the system in the rotating frame of reference. It has been assumed that the system is moving with effective constraints and the connecting cable of two satellites of masses m_1 and m_2 respectively will always remains tight. The equilibrium position of the system is given by the constant values of the coordinates in the rotating frame of reference.

Now let

$x = x_0$ and $y = y_0$ give the equilibrium position where x_0 and y_0 are constant

$$\left. \begin{aligned} x^1 = x_0^1 = 0 = x_0'^1 \\ y' = y_0' = 0 = y_0'' \end{aligned} \right\} \quad \text{..... (9)}$$

using (9) in (8), we get

$$\left. \begin{aligned} -(3 + 4B)x_0 - \frac{A}{\pi} \cos \epsilon \cos \alpha \sin \theta &= -\bar{\lambda}_{\alpha} \left[1 - \frac{\ell_o}{r_o} \right] x_0 \\ By_0 - \frac{A}{\pi} \cos \epsilon \sin \alpha \sin \theta &= -\bar{\lambda}_{\alpha} \left[1 - \frac{\ell_o}{r} \right] y_0 \end{aligned} \right\} \quad \text{..... (10)}$$

Where

$$r = \sqrt{x_0^2 + y_0^2}$$

Now we shall discuss particular solution of the system of equation (10) as putting $\alpha = 0$ as follows.

$$\left. \begin{aligned} -(3+4B)x_0 - \frac{A}{\pi} \cos \epsilon \sin \theta &= -\bar{\lambda}_\alpha \left[1 - \frac{\ell_o}{r_o} \right] x_0 \\ By_0 &= -\bar{\lambda}_\alpha \left[1 - \frac{\ell_o}{r_o} \right] y_0 \end{aligned} \right\} \dots\dots\dots (11)$$

From (10) we get the equilibrium point.

$$[a, 0] = \left[\frac{\bar{\lambda}_\alpha \ell_o \pi + A \cos \epsilon \sin \theta}{\pi (\lambda_\alpha - 3 - 4B)} 0 \right]$$

and second equilibrium points $(b, 0)$, putting $x_o = b$; $y_o = 0$ in (10) we get

$$\left. \begin{aligned} -(3+4B)b - \frac{A}{\pi} \cos \epsilon \sin \theta &= -\bar{\lambda}_\alpha \left[1 - \frac{\ell_o}{r_o} \right] b \\ Bc &= -\bar{\lambda}_\alpha \left[1 - \frac{\ell_o}{r_o} \right] c \end{aligned} \right\} \dots\dots\dots (12)$$

Now,

$$[b, c] = \left[\frac{A \cos \epsilon \sin \theta}{\pi (3+5B)}, \sqrt{\left(\frac{\lambda_\alpha \ell_o}{\lambda_\alpha + B} \right)^2 - \frac{A^2 \cos^2 \epsilon \sin^2 \theta}{\pi^2 (3+5B)^2}} \right]$$

Let ξ and η denote variations in the co-ordinates for this position of equilibrium then.

$$\left. \begin{aligned} x &= a + \xi \quad \text{and} \quad y = \eta \\ x' &= \xi' \quad \text{and} \quad y' = \eta' \\ x'' &= \xi'' \quad \text{and} \quad y'' = \eta'' \end{aligned} \right\} \dots\dots\dots (13)$$

Using (12) in (4) we get,

$$\begin{aligned} \xi'' - 2\eta' - (3+4B)(a + \xi) - \frac{A \cos \epsilon \sin \theta}{\pi} &= -\bar{\lambda}_\alpha \left[1 - \frac{\ell_o}{r} \right] (a + \xi) \\ \eta'' + 2\xi' + B\eta &= -\bar{\lambda}_\alpha \left[1 - \frac{\ell_o}{r} \right] \eta \end{aligned} \dots\dots\dots (13)$$

$$\text{where } r^2 - (a + \xi)^2 + \eta^2 \dots\dots\dots (14)$$

as the original system of equations admits Jacob's integral so there must exist a Jacobi's integral for the system of Equation (13) moreover It is not different to deduce the Jacobi's integral for (13) in the form:

$$\begin{aligned} \xi'^2 + \eta'^2 - (1+4B)(a + \xi)^2 + B\eta^2 - \frac{2A\xi \cos \epsilon \sin \theta}{\pi} + \bar{\lambda}_\alpha \left[(a + \xi)^2 + \eta^2 \right] \\ - 2\bar{\lambda}_\alpha \ell_o \left[(a + \xi)^2 + \eta^2 \right] &= h \end{aligned} \dots\dots\dots (15)$$

where h is the constant of integration.

Now (15) can be put in the form:

$$\begin{aligned}
V(\xi^2, \eta^2, \xi, \eta) &= \xi^2 + \eta^2 + \xi^2 \left[\bar{\lambda}_\alpha - 3 - 4B - \frac{\bar{\lambda}_\alpha \ell_o}{a} \right] + \eta^2 \left[\bar{\lambda}_\alpha + B - \frac{\bar{\lambda}_\alpha \ell_o}{a} \right] \\
&+ \xi^2 \left[\bar{\lambda}_\alpha + B - \frac{\bar{\lambda}_\alpha \ell_o}{a} \right] + \xi \left[-2(3+4B)a - \frac{2A \cos \epsilon \sin \theta}{\pi} + 2\bar{\lambda}_\alpha a - 2\bar{\lambda}_\alpha \ell_o \right] \\
&+ a \left[-(3+4B)a + \bar{\lambda}_\alpha a - 2\bar{\lambda}_\alpha \ell_o \right] + 0(3)
\end{aligned}
\tag{16}$$

Conclusion

Where $0[3]$ stands for 3rd and higher order terms in the small equations ξ and η . To test the stability in the sense of Liapunov we take Jacobian integral given by (16) as a Liapunov function $V(\xi^2, \eta^2, \xi, \eta)$ and applying Liapunov's theorem on stability. It follows that the only interior for given Equilibrium position $[a, b]$ to be stable is that V must be positive definite and for this the following condition must be satisfied.

$$(i) \quad -2(3+4B)a - \frac{2A \cos \epsilon \sin \theta}{\pi} + 2\bar{\lambda}_\alpha a - 2\bar{\lambda}_\alpha \ell_o = 0$$

$$(ii) \quad \bar{\lambda}_\alpha - 3 - 4B - \frac{\bar{\lambda}_\alpha \ell_o}{a} > 0$$

putting the value of a & b for

$$(iii) \quad \bar{\lambda}_\alpha + B - \frac{\bar{\lambda}_\alpha \ell_o}{a} > 0$$

Now third condition

$$B + \bar{\lambda}_\alpha \left(1 - \frac{\ell_o}{a} \right) = B + \left[1 - \frac{\ell_o (\pi \bar{\lambda}_\alpha - 3\pi - 4B\pi)}{\bar{\lambda}_\alpha \ell_o \pi + A \cos \epsilon \sin \theta} \right] \bar{\lambda}_\alpha = B + \frac{\bar{\lambda}_\alpha \pi \ell_o (3+4B)}{\bar{\lambda}_\alpha \pi \ell_o + A \cos \epsilon \sin \theta} > 0$$

as

$$0 < \frac{\pi \theta}{2}, B > 0, \pi > 0$$

$\bar{\lambda}_\alpha > 0, \ell_o > 0$ and $A \cos \epsilon > 0$ are identically satisfied.

Thus we conclude that the equilibrium position $(a, 0)$ is stable in the sense of Liapunov.

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