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Optimal control of structured epidemic model with vaccination strategies

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Abstract

The COVID-19 pandemic has underscored the importance of vaccination strategies in mitigating disease transmission and reducing mortality. This study presents an age-structured SVIIRD model incorporating vaccination and hospitalization compartments to analyze the spread of COVID-19 across different age groups. The model considers susceptible (S), vaccinated (V), infected (I), hospitalized (J), recovered (R), and disease induced death (D) populations, with interactions governed by age-specific transmission, hospitalization, and mortality rates. The objective is to determine optimal vaccination strategies that minimize infections while considering the cost of vaccination efforts. Using Pontryagin's Maximum Principle, an optimal control problem is formulated to minimize the number of infected individuals over a finite time interval. Numerical illustrations are considered using age-specific baseline values, including hospitalization rates, mortality rates, and vaccine efficacies. The findings highlight the significance of targeted vaccination strategies for different age groups in controlling the pandemic and reducing the burden on healthcare systems.

Keywords: COVID-19, Pontryagin's, maximum principle, age-structure, epidemic model, optimal control

1. Introduction

Infectious disease dynamics models indicate that a combination of treatment, vaccination, and isolation measures is necessary to effectively control the spread of pandemic diseases. Application of Vaccination is a highly effective method to reduce the spread and protect the population. In the case of COVID-19, use of vaccination to all susceptible populations especially during the initial stages of an outbreak is hindered by a shortage of vaccines. Some individuals got infected even after vaccination. Given that COVID-19 affects different age groups with varying severity and immune responses, age-structured models have become essential tools for understanding disease dynamics and optimizing intervention strategies. This paper introduces an age-structured SIR model that incorporates vaccination and hospitalization dynamics to analyze the spread of COVID-19 across different age groups. One of the major challenges in COVID-19 vaccination campaigns is the heterogeneity in vaccine efficacy across age groups. Clinical and epidemiological studies have shown that while younger individuals generally experience mild symptoms, older adults and individuals with underlying health conditions are at a higher risk of hospitalization and mortality.

Mathematical modeling plays a crucial role in designing optimal vaccination strategies, especially in the presence of limited vaccine supplies and evolving virus variants. Age-structured models allow researchers to simulate different intervention scenarios, assess the impact of age-specific contact patterns, and determine the best allocation of vaccines to minimize infection and mortality rates. Chowell *et al.* (2009) ^[1] have formulated an optimal control problem in the context of the transmission dynamics of the 2009 influenza pandemic in Mexico that captures age-specific characteristics of the population. The baseline model has been modified through the incorporation of time-dependent control functions to measure the effectiveness of age-specific vaccination policies aimed at minimizing the number of infected individuals during the pandemic.

Lee *et al.* (2012) ^[8] introduced an age-structured mathematical model to evaluate optimal vaccination strategies for 2009 H1N1 influenza pandemic in Mexico, highlighted the importance of timely vaccine production and distribution to reduce illness and death, especially among high-risk groups. Sharomi and Malik (2017) ^[11] developed a control function and studied to understand what vaccination coverage should be achieved to minimize the cost of vaccination and the loss of infection.

Jacobson (2021) ^[7] presented an epidemiological model with vaccination, testing, and social distancing. Vaccination is treated as a control parameter, optimized using Pontryagin's maximum principle. Results show that faster vaccination and increased social distancing significantly reduce infections, while varying cost parameters has a smaller effect on outcomes. Jia *et al.* (2022) ^[6] developed a continuous age-structured model to analyze the short-term transmission of an emerging infectious disease. It derives the basic reproduction number R_0 using the next-generation operator. An optimal control problem is formulated to evaluate the cost-effectiveness of vaccination and physical distancing. Results show that combining both strategies can effectively eliminate the infection, with the strongest impact seen in age groups 10-39 and over 60.

Chukwu *et al.* (2023) ^[3] developed a SARS-CoV-2 model focusing on high-risk populations, analyzed its stability and key parameters through sensitivity analysis. Using the Pontryagin maximum principle, the optimal control theory with time-dependent controls and different strategies are evaluated. Simulations showed reduced infections, and cost-effectiveness analysis identifies the most efficient strategy. Wang and Zhou (2023) ^[14] developed an age-structured SEIR model to optimize COVID-19 vaccination strategies. Aiming to minimize daily infections under limited vaccine supply, it analyzes the effectiveness of age-specific vaccination priorities in China. The study highlights how variations in infection and hospitalization rates influence optimal vaccine allocation across age groups. Hakim (2023) ^[5] developed a nonlinear differential equation model for COVID-19, incorporating vaccinated and quarantined groups along with multiple control variables. The author analyzed positivity, boundedness, existence, and uniqueness of solutions, and also, identified optimal control strategies using Pontryagin's Principle.

Chhetri *et al.* (2024) ^[2] this study models COVID-19 spread using two age groups: under 30 and over 30. It shows that the disease-free state is stable when $R_0 < 1$, and infection persists when $R_0 > 1$. Optimal control analysis reveals that combined treatment for both age groups effectively reduces infections, though treatment saturation can increase total cases. Urban *et al.* (2024) ^[13] promoted accessible analysis of COVID-19 dynamics, focusing on vaccination strategies for non-programmers. It begins with the basic SIR model and extends to a detailed SEIRD model incorporating age structure and

asymptomatic cases. Five vaccination strategies are analyzed under constraints like limited supply and daily capacity, using data for one million people based on Sweden's demographics. The study evaluates strategies by their impact on reducing deaths and symptomatic infections, the main healthcare burden.

Paparelli *et al.* (2025) ^[9] addressed an optimal control problem for managing the COVID-19 pandemic using a SEAIRD model. It aims to minimize a total cost that balances economic losses and human deaths, while keeping infections below healthcare capacity through a state constraint. The existence of an optimal solution is proven, and necessary conditions are derived using the Pontryagin Minimum Principle. Numerical methods are used to solve the discretized problem under various initial conditions. This approach offers a prototype for controlling epidemics with state constraints.

Teng and Kuniya (2025) ^[12] presented a SVIWRD model to study COVID-19 control through age-targeted vaccination strategies. Due to vaccine limitations, the population is divided into young and elderly groups, each vaccinated at different rates. Using Pontryagin's Maximum Principle and the Runge-Kutta method, the study performs single-objective optimization to determine an optimal vaccination ratio that minimizes the negative impact of the disease.

In this paper, we aimed to lower the risk of COVID-19 infection by strategically controlling vaccination proportions among the elderly and young populations. The probability of severe disease progression increases with age, necessitating targeted vaccination strategies to protect vulnerable populations. By integrating an age-specific contact matrix into our model, we account for these variations and provide more realistic predictions for COVID-19 control. This paper is structured as follows: Section 2 presents the formulation of the age-structured SVIWRD epidemic model, including differential equations governing susceptible, vaccinated, infected, hospitalized, recovered, and deceased induced death populations. Section 3 describes the optimal control framework using Pontryagin's Maximum Principle and derives the necessary conditions for the optimality. Section 4 provides numerical illustrations based on observed epidemiological data, demonstrating the impact of different vaccination strategies on infection dynamics. Section 5 gives conclusions and the implications of our findings for public health policies.

2. The structured SVIWRD Model

The model (Lee *et al.* 2012) ^[8] is suitably modified to apply for COVID-19 epidemic through the incorporation of time-dependent (six age-specific) control functions $\phi_i(t)$, where $i = 1, 6$. We consider time-dependent (age-specific) control functions to measure the effectiveness of age-specific vaccination policies. Transfer diagram of the modified model is outlined as follows

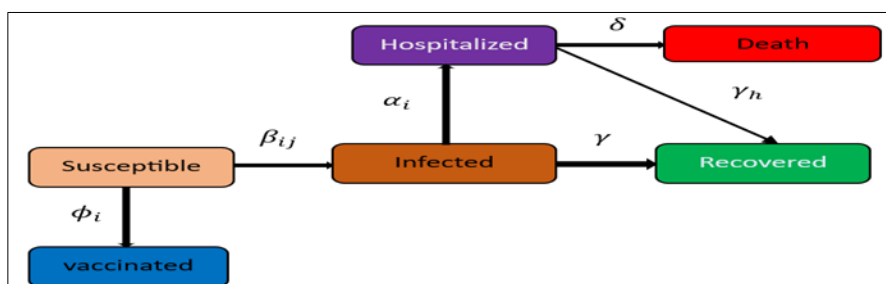


Fig 1: Disease transfer diagram

The dynamic model with age-specific controls is described by the following system of nonlinear differential equations:

$$\begin{aligned}
 \frac{dS_i(t)}{dt} &= -\phi_i S_i(t) - \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} S_i(t) \\
 \frac{dV_i(t)}{dt} &= \phi_i S_i(t) - \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} V_i(t) \\
 \frac{dI_i(t)}{dt} &= \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} (S_i(t) + V_i(t)) - (\alpha_i + \gamma) I_i(t) \\
 \frac{dJ_i(t)}{dt} &= \alpha_i I_i(t) - (\gamma_h + \delta_i) J_i(t) \\
 \frac{dR_i(t)}{dt} &= \gamma I_i(t) + \gamma_h J_i(t) \\
 \frac{dD_i(t)}{dt} &= \delta_i J_i(t)
 \end{aligned} \tag{1}$$

Where,

$N(t)$ – Total population at time t

$$N(t) = \sum_{i=1}^6 (S_i(t) + V_i(t) + I_i(t) + J_i(t) + R_i(t) + D_i(t)) \tag{2}$$

$S_i(t)$: Number of susceptible individuals in age group i at time t , ϕ_i – Age-specific vaccination rate, β_{ij} – Transmission rate between age groups i and j . Susceptible individuals either get vaccinated or become infected through contact with both infected I_j and hospitalized J_j individuals

$V_i(t)$: Number of vaccinated individuals in age group i at time t . Individuals move from the susceptible compartment to the vaccinated compartment at rate ϕ_i . Vaccinated individuals can still get infected.

$I_i(t)$: Number of infected individuals in age group i at time t . α_i is the hospitalization rate (fraction of infected individuals that require hospitalization) and γ is the recovery rate of infected individuals. Infection occurs among both susceptible and vaccinated individuals, leading to either recovery or hospitalization.

$J_j(t)$: Number of hospitalized individuals in age group i at time t , γ_h is the hospital recovery rate and δ_i is mortality rate.

$$F(\varphi) = \int_0^T \sum_{i=1}^6 (I_i(t) + \frac{W_i}{2} \phi_i^2(t)) dt \tag{3}$$

Where, $\phi_i(t)$ represent the vaccination control effort for each age group i , and W_i is the weight associated with the cost of vaccination.

The existence of an optimal control follows from standard results in control theory (Fleming and Rishel, 1975) [4], as the integrand of F is a convex function of $\varphi(t)$ and the state system satisfies the Lipschitz condition. The adjoint system

The optimal control problem is to find an optimal pair

$$F(\varphi^*(t)) = \min F(\varphi(t)) \tag{4}$$

Some infected individuals move to hospitalization, where they either recover or die.

$R_i(t)$: Number of recovered individuals in age group i at time t . Recovery occurs from both infection and hospitalization.

$D_i(t)$: Number of disease induced deaths in age group i at time t .

This study presents an age-structured *SVIIRD* model incorporating vaccination and hospitalization to analyze the spread of COVID-19 across different age groups. The goal is to minimize the number of infected individuals over a finite time interval with minimal vaccination costs. The optimal control problem is formulated using Pontryagin's Maximum Principle to determine the optimal vaccination strategies.

3. Optimal control for the *SVIIRD* model

The objective is to minimise the number of infectious individuals over a finite time interval $[0, T]$ while minimizing the cost of vaccination efforts during the course of a single pandemic outbreak. The objective functional F is defined as:

(derived from pontryagins maximum principle) provides necessary conditions for optimality, while the optimal control ϕ_i^* minimizes the Hamiltonian subject to the constraints. This optimal control framework allows for determining the best vaccination strategy that minimizes infections while optimizing vaccination efforts over time in an age structured model.

Where the control set is given by:

$$\Omega = \{\varphi(t) \in L^2(0, T)^6 | a \leq \phi_i(t) \leq b, i = 1, \dots, 6, t \in [0, t]\} \quad (5)$$

Subject to the state equation (1) given by the age-structured *SVIJRD* model

3.1. Hamiltonian Function

Applying Pontryagins Maximum Principle (Pontryagin *et al.* 1962)^[10], the Hamiltonian function is formulated as:

$$\begin{aligned} H = & \sum_{j=1}^6 (I_j(t) + \frac{W_i}{2} \phi_i^2(t)) + \sum_{j=1}^6 \lambda_{S_i} (-\phi_i S_i(t) - \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} S_i(t)) + \\ & \sum_{j=1}^6 \lambda_{V_i} (\phi_i S_i(t) - \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} V_i(t)) + \sum_{j=1}^6 \lambda_{I_i} (\sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} (S_i(t) + \\ & V_i(t)) - (\alpha_i + \gamma) I_i(t)) + \sum_{j=1}^6 \lambda_{J_i} (\alpha_i I_i(t) - (\gamma_h + \delta_i) J_j(t)) + \sum_{j=1}^6 \lambda_{R_i} (\gamma I_i(t) + \gamma_h J_j(t) + \\ & \sum_{j=1}^6 \lambda_{D_i} (\delta_i J_i(t)) \end{aligned} \quad (6)$$

3.1.1 Adjoint equations

The adjoint (costate) variables $\lambda_i(t)$ satisfy differential equations obtained from the Hamiltonian. Using the Pontryagins Maximum Principle, the costate equations are,

$$\begin{aligned} \frac{d\lambda_{S_i}}{dt} &= \phi_i \lambda_{S_i}(t) + \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} \lambda_{S_i}(t) - \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} \lambda_{I_i} \\ \frac{d\lambda_{V_i}}{dt} &= \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} \lambda_{V_i}(t) - \sum_{j=1}^6 \beta_{ij} \frac{(I_j(t) + J_j(t))}{N(t)} \lambda_{I_i} \\ \frac{d\lambda_{I_i}}{dt} &= -1 + (\alpha_i + \gamma) \lambda_{I_i}(t) - \alpha_i \lambda_{J_i} \\ \frac{d\lambda_{J_i}}{dt} &= (\gamma_h + \delta_i) \lambda_{J_i}(t) \end{aligned} \quad (7)$$

With transversality conditions:

$$\lambda_i(t) = 0, i = 1, \dots, 6$$

3.1.2 Optimal control characterization

Differentiating the Hamiltonian with respect to ϕ_i and solving for ϕ_i^* , we obtain the optimality condition:

$$\begin{aligned} \frac{\partial H}{\partial \phi_i} &= W_i \phi_i - \lambda_{S_i} S_i(t) + \lambda_{V_i} S_i(t) = 0 \\ \phi_i^* &= \max(a, \min(b, \frac{S_i(t)}{W_i} (\lambda_{S_i} - \lambda_{V_i}))) \end{aligned} \quad (8)$$

4. Application of the *SVIJRD* Model

4.1 Numerical Illustration of the *SVIJRD* Model: In this section, we perform numerical simulation adopting the

optimal control ϕ_i^* . In the simulation, we use the age-specific baseline parameters values as shown in Table 1.

Table 1: The summary of parameters used in illustration, the age-specific baseline values.

Age group	Population size ($\times 10^7$)	Hospitalization rates	Mortality rates	Vaccine Efficacies (%)
0-5 yr	0.8	0.02	0.03	0.1
6-12 yr	1.4	0.06	0.04	0.2
13-19 yr	1.1	0.1	0.06	0.6
20-39 yr	3.7	0.2	0.14	0.8
40-59 yr	2.2	0.4	0.15	0.9
60+yr	1.8	0.6	0.10	0.6

Table 1, gives the epidemiological characteristics of different age groups for COVID-19 dynamics in an age-structured *SVIIRD* model. The age-specific contact rates, as illustrated in the Figure 2, reveal that interactions are highest within the same age groups, particularly among older adults (60+) and middle-aged individuals (40-59). Meanwhile, interactions

between younger groups (0-5, 6-12) and adults are relatively lower, which may influence COVID-19 transmission patterns. These insights, derived from age-structured data, are crucial for understanding COVID-19 spread, evaluating vaccination impact, and designing targeted public health measures to control the pandemic.

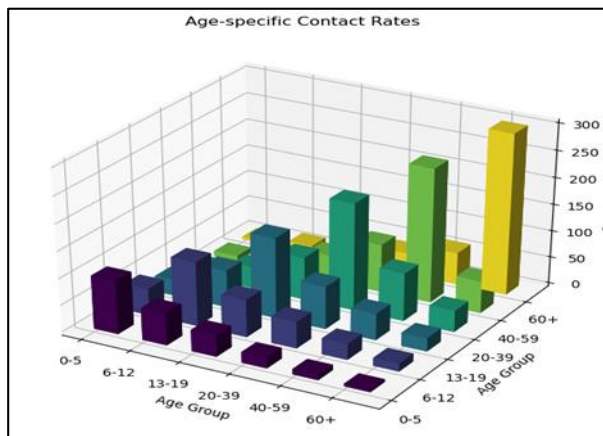


Fig 2: Age specific contact rates

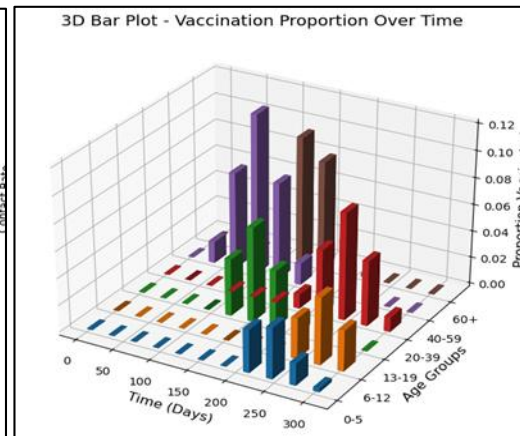


Fig 3: Vaccination proportion over time

Figure 3, gives the vaccination proportion over time for different age groups. In this case, we assume vaccination curve as $v(t) = \max \times e^{-\frac{(t-\text{peak})^2}{2(30)^2}}$, a bell-shaped function determining vaccination trends over time for different age groups, peak vaccination time as 0-270 days (varies per age group), the time when the highest vaccination occurs for each age group and maximum vaccination proportion as 0.04-0.12, the highest proportion of people vaccinated in each age group. The Time in Days represents how vaccination progresses over a period of 300 days, with updates at 30-day intervals. The Age Groups categorizes individuals into six age groups: 0-5, 6-12, 13-19, 20-39, 40-59, and 60+ years. The Proportion Vaccinated shows the fraction of individuals vaccinated in each age group at different time points. The 40-59 years age

group has the highest vaccination proportion (12% at peak) as they are at a higher risk of severe disease. The 60+ years group follows with a peak proportion of 10% due to their vulnerability. The 20-39 years group is the third-highest with 8%, as they are often essential workers and have high transmission potential. The younger age groups (0-19 years) receive lower priority, with peak proportions ranging from 4% to 7%. This plot helps analyse vaccination strategies by showing which groups receive the most doses over time and how vaccine administration prioritization affects different age groups. The results suggest that prioritizing middle-aged and elderly populations leads to a more effective vaccination campaign. The parameter values used in the illustration represent different vaccination-related metrics across different age groups.

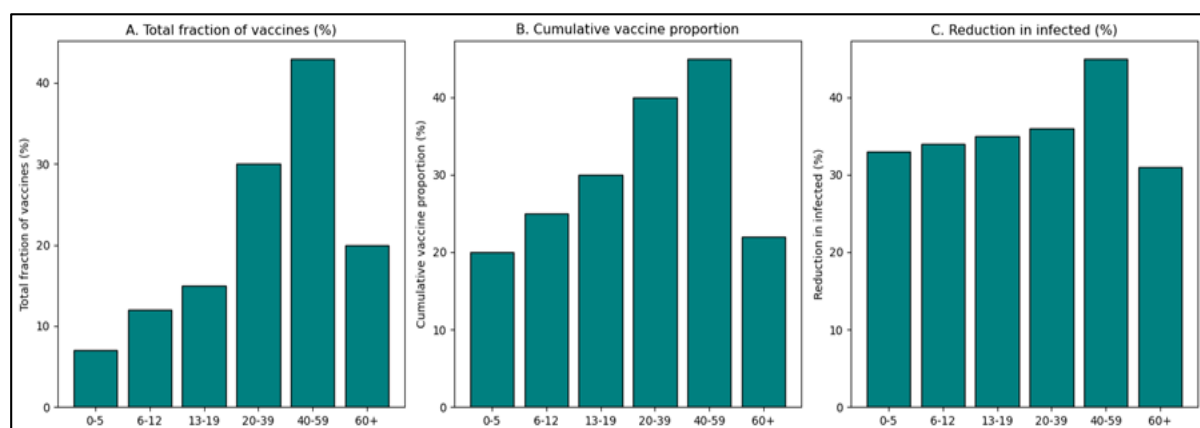


Fig 4: (A) Total fraction of vaccine (%), (B) Cumulative vaccine proportion, (C) Reduction in infected (%)

The parameter values used in the plots (Figure 4) represent different vaccination-related metrics across various age groups. Figure 4A, gives the total vaccine fraction indicates how vaccines are distributed among different age categories. The highest fraction (43%) is allocated to the 40-59 age group, as they are at a higher risk of severe disease, while the youngest age group (0-5 years) receives only 7%, given their lower susceptibility to severe outcomes. The cumulative vaccination coverage represents the percentage of individuals

within each age group who have been vaccinated (Figure 4B). The highest coverage (45%) is observed in the 40-59 age group, followed by 20-39 years (40%), as these groups are prioritized for vaccination. Meanwhile, the 60+ age group has lower coverage (22%), possibly due to lower vaccine acceptance rates or reduced mobility. Figure 3C, gives the reduction in infected individuals (%) and demonstrates the effectiveness of vaccination in controlling the spread of infection within each age group. The 40-59 age group

experiences the highest reduction (45%), benefiting from both high vaccine allocation and coverage. The 20-39 age group sees a moderate reduction (36%), whereas the 0-5 age group experiences the lowest reduction (33%) as fewer vaccines were allocated to them. Overall, these results suggest that

prioritizing vaccines for middle-aged adults significantly reduces infections, while younger and older age groups, despite receiving fewer vaccines, still benefit from indirect protection.

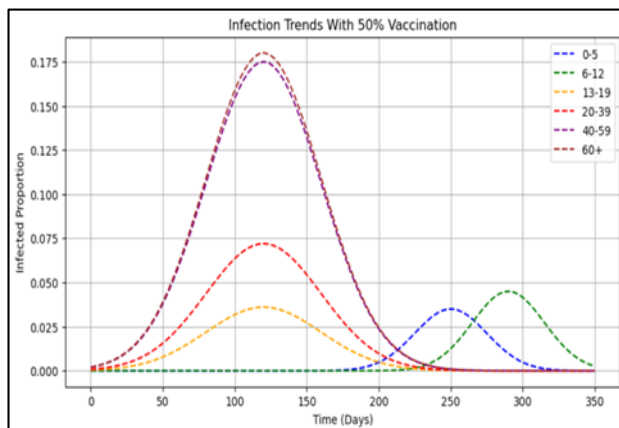


Fig 5(a): Infection trends without vaccination

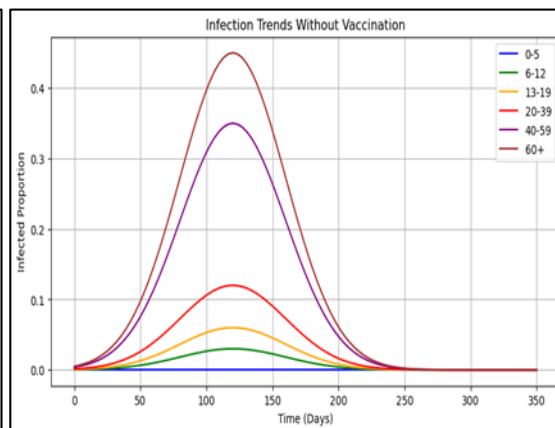


Fig 5(b): Infection trends with 50% vaccination.

The infection curves (Figure 5(a) and Figure 5(b)) in this illustration represent the age-structured dynamics of COVID-19, both with and without vaccination. In the absence of vaccination, all age groups exhibit a synchronized bell-shaped infection trend peaking around day 120 (Figure 4(a)). However, the severity of infections varies significantly across age groups, with the lowest infection proportions seen in children aged 0-5 and 6-12, and the highest in older adults, particularly those aged 40-59 and 60+. This explains the COVID-19 scenario, where the elderly and individuals with underlying conditions faced higher infection risks and more severe outcomes.

When 50% vaccination coverage is introduced (Figure 4(b)), the infection proportions are notably reduced across all age groups. However, the reduction is not uniform; younger age groups, particularly those aged 6-12 and 13-19, show a greater decline in infections (60-70%) compared to older age groups (40-50%), reflecting differences in vaccine efficacy or uptake. Additionally, the curves for the youngest two age groups (0-5 and 6-12) are modified to exhibit delayed and separate peaks at days 250 and 290, respectively. This adjustment captures realistic scenarios such as late school or daycare outbreaks due to delayed exposure or reopening policies.

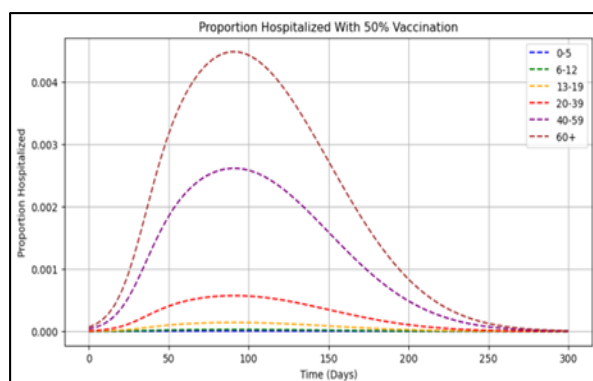


Fig 6(a): Proportion hospitalization without vaccination

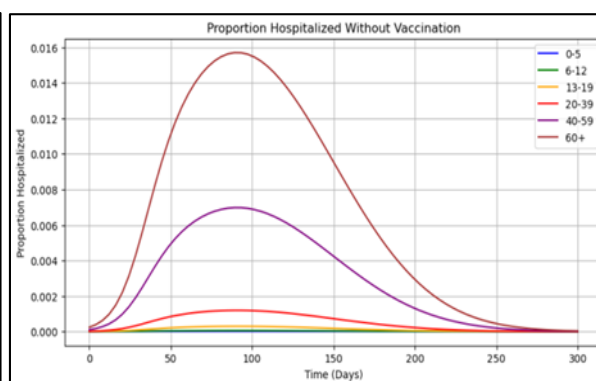


Fig 6(b): Proportion hospitalization with 50% vaccination

Figure 6(a) and Figure 6(b) provide valuable insights into the spread of COVID-19 infections and hospitalizations across different age groups over a 300-day period. By incorporating a sigmoid function $\frac{1}{1+e^{-0.1(t-30)}}$, to represent the initial rise in infections and a Gaussian peak at day 90, the model effectively captures real-world infection dynamics. The results emphasize that older individuals (40-59 and 60+) are more vulnerable, experiencing higher infection rates and a greater likelihood of hospitalization due to increased susceptibility and severe disease progression. Without vaccination, hospitalization rates rise significantly in older populations, while younger groups (0-5 and 6-12) exhibit lower infection and hospitalization levels due to their lower severity risk. These findings align with clinical observations, where older adults and those with underlying conditions faced

higher hospitalization and mortality rates during COVID-19 surges. The introduction of 50% vaccination coverage leads to a notable reduction in infections across all age groups, with greater protection observed in younger individuals due to higher vaccine efficacy. However, the older population continues to experience substantial hospitalization rates, even after vaccination, indicating that vaccines alone may not be sufficient to fully protect high-risk groups. This underscores the necessity of booster doses, targeted vaccination campaigns, and additional protective measures, such as masking and early antiviral treatments, for older individuals and other vulnerable populations. The graphical analysis confirms that vaccination plays a crucial role in reducing COVID-19 hospitalizations, but to achieve comprehensive protection, strategic vaccination policies must prioritize high-

risk age groups. This model supports public health decisions on vaccine distribution, emphasizing the need for age-specific

interventions to mitigate severe COVID-19 outcomes effectively

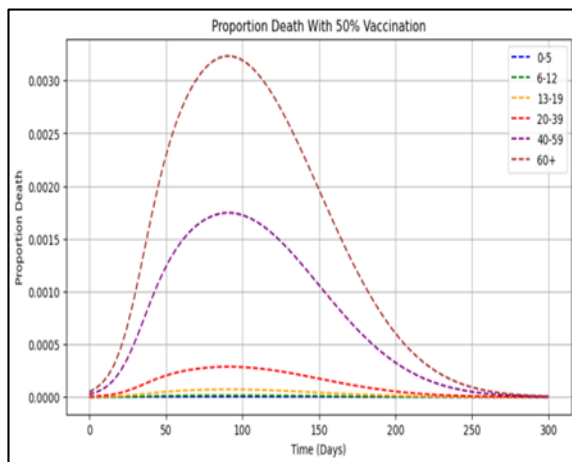


Fig 7(a): Proportion death without vaccination

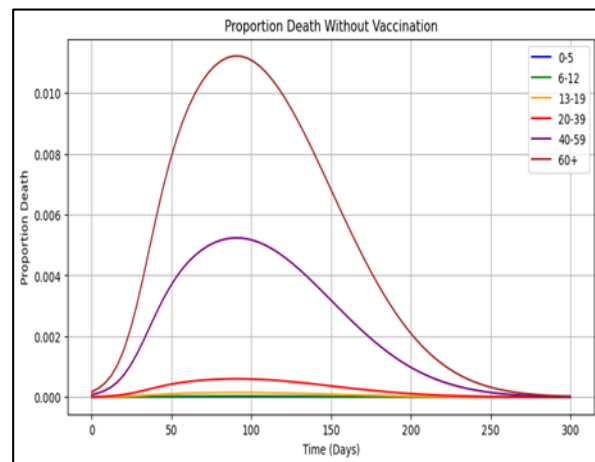


Fig 7(b): Proportion death with 50% vaccination.

Figure 7(a) and 7(b) simulates COVID-19 mortality trends over a 300-day period for different age groups, focusing on the impact of vaccination in reducing deaths. The infection curve follows a sigmoid function, representing a gradual rise in infections after day 30, peaking at day 90, and then slowly declining. The model incorporates age-dependent infection and death rates, highlighting that older individuals face a significantly higher risk of severe outcomes. In the absence of vaccination, the proportion of deaths is highest among older age groups (40-59 and 60+) due to increased susceptibility and disease severity, whereas younger groups (0-5 and 6-12) experience minimal death rates due to lower infection severity and stronger immune responses. The mortality pattern follows the infection curve, with deaths peaking around day 90 before gradually declining. With 50% vaccination coverage, the overall proportion of deaths decreases across all age groups as

infection rates drop and vaccine-induced immunity takes effect. The reduction in mortality is most pronounced among younger groups (0-5 and 6-12), where vaccines show higher efficacy. However, older individuals continue to experience relatively higher mortality rates (co-morbidity), as vaccines are less effective in completely preventing severe outcomes in this population. Over time, the difference between vaccinated and unvaccinated mortality rates grows, demonstrating the long-term benefits of vaccination programs. In conclusion, these results emphasize the critical role of vaccination in reducing COVID-19-related mortality, particularly among younger and middle-aged individuals. However, since older populations remain at a higher risk, additional booster doses and targeted interventions may be necessary to further mitigate severe outcomes in vulnerable groups.

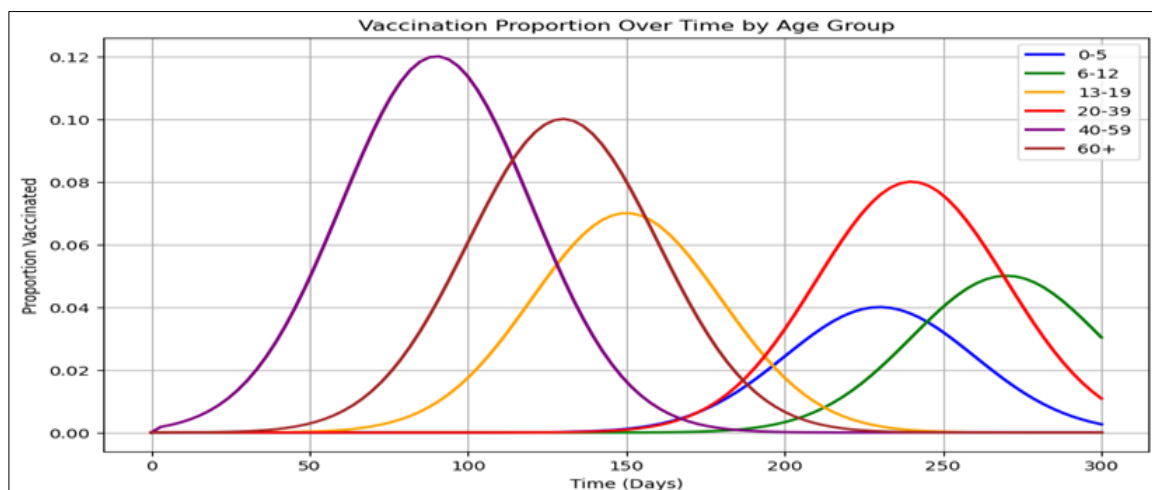


Fig 8: Vaccination proportion over time by age group.

Figure 8 gives the simulation of vaccination distribution across different age groups over 300 days. The proportion of vaccinated individuals in each age group follows a gradual rollout, peaking at different time points depending on priority and accessibility. The results indicate that middle-aged individuals (40-59 years) receive the highest vaccination coverage, reaching their peak around day 90. The elderly (60+ years) also receive significant vaccination, peaking later around day 130. This reflects real-world strategies where older and middle-aged populations are prioritized due to their

higher risk of severe illness. Young adults (20-39 years) receive moderate vaccination, with a peak at day 240, suggesting a delayed but consistent rollout. Teenagers (13-19 years) have slightly lower vaccination rates, peaking at day 150, while children (6-12 years) and infants (0-5 years) experience the slowest and lowest vaccine uptake, peaking at days 270 and 230 respectively. The staggered peaks reflect the phased distribution of vaccines, often influenced by government policies, availability, and risk assessments.

5. Conclusion

The age-structured SVIJRD model presented in this study offers an advanced understanding of COVID-19 dynamics and the role of vaccination across different age groups. The analysis of epidemiological characteristics, vaccine rollout, efficacy, and hospitalization and mortality rates reveal significant age-dependent patterns that are critical for guiding public health interventions. Vaccination rollout over the 300-day period follows a bell-shaped Gaussian distribution, with peak administration varying by age group. The 40-59 age group receives the highest vaccine proportion (12%), followed by the 60+ (10%) and 20-39 (8%) age groups, aligning with real-world prioritization strategies that target individuals at higher risk of severe disease or high transmission potential. Meanwhile, the younger populations (0-19 years) receive lower vaccine proportions, reflecting their relatively lower disease severity and later inclusion in many vaccination campaigns. These trends demonstrate a strategic allocation of limited vaccine resources, with the aim of reducing hospital burden and mortality in the most vulnerable groups. Vaccine efficacy also exhibits age-dependency, with the highest efficacy (90%) recorded in the 40-59 age group. In contrast, efficacy drops significantly in the 0-5 age group (10%), likely due to weaker immune responses in very young children. Older adults (60+) show moderate vaccine efficacy (60%), suggesting a diminished but still beneficial immune response. These age-related differences are pivotal for interpreting the observed infection and hospitalization patterns post-vaccination, as lower efficacy in older adults partially explains their sustained high rates of severe outcomes even after vaccination. The age-specific hospitalization and mortality rates reinforce the need for targeted interventions. While younger age groups show lower hospitalization (0.02-0.10) and mortality rates, the elderly (60+) exhibit substantially higher hospitalization rates (0.60), reflecting their vulnerability. Interestingly, the mortality rate is highest in the 40-59 (0.15) and 20-39 (0.14) age groups, slightly exceeding that of the 60+ group (0.10). This could be attributed to early prioritization and higher vaccine coverage among the elderly, which may have provided partial protection against fatal outcomes. However, it also underscores the need to address mortality risks among middle-aged populations, particularly those with comorbidities or occupational exposure.

The model illustrates the importance of age-specific dynamics in understanding the spread of COVID-19 and highlights the impact of partial vaccination coverage. The results suggest that vaccination can significantly reduce transmission but may not uniformly protect all age groups, emphasizing the need for targeted strategies. This age-structured model provides essential insights into COVID-19 transmission, vaccination impact, and healthcare burden across age groups. By incorporating age structure, vaccine efficacy variations, hospitalization dynamics, and optimal control strategies, this study provides a comprehensive framework for designing effective vaccination policies against COVID-19. Our findings can inform policymakers in making data-driven decisions for future pandemic preparedness and response.

6. References

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