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A study of deterministic and probabilistic inventory models with quadratic deterioration and exponential demand

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Abstract

This study presents a comprehensive analysis of deterministic and probabilistic inventory models considering quadratic deterioration rates and exponential demand growth. The models integrate holding and deterioration costs to evaluate total inventory expenses under both certain and uncertain conditions. The deterministic model provides insights into inventory dynamics and fixed demand and deterioration parameters, while the probabilistic model accounts for stochastic variations. Sensitivity analyses are conducted to examine the effect of deterioration coefficients, demand growth rates, and cost parameters on total costs. Results show that probabilistic models yield higher total costs due to uncertainty, emphasizing the importance of robust inventory management strategies. Heat maps and cost curves further illustrate the interaction between holding and deterioration costs. This analysis aids decision makers in selecting appropriate inventory policies to minimize costs and manage risk effectively.

Keywords: Inventory, deterioration, demand, holding cost

Introduction

Inventory management is a critical aspect of supply chain operations, influencing costs, service levels, and overall efficiency. Traditional inventory models often assume constant or linear deterioration rates and deterministic demand patterns. However, real-world inventory systems frequently exhibit more complex behaviors, including nonlinear deterioration and demand uncertainty.

In this study, we explore deterministic and probabilistic inventory models that incorporate a quadratic deterioration rate and exponential demand function to better represent practical scenarios. Quadratic deterioration accounts for situations where spoilage or degradation accelerates over time, which is common in perishable goods such as pharmaceuticals, food products, and chemicals. Similarly, exponential demand is suitable for modeling cases where demand increases due to factors like market trends, product popularity, or promotional effects.

Deterministic vs. Probabilistic Models

- **Deterministic Inventory Model:** Assumes that demand and deterioration rates are known with certainty. This model provides precise inventory control policies based on exact parameters, facilitating straightforward decision-making.
- **Probabilistic Inventory Model:** Acknowledges uncertainty in demand, deterioration, or both. It incorporates probability distributions to represent randomness, enabling more flexible and risk-aware inventory management strategies.

The study of deteriorating inventory models has been extensively explored in operations research. Ghare and Schrader (1963) introduced the concept of inventory models with exponential decay ^[1]. Following this, Covert and Philip (1973) extended the model to incorporate variable deterioration rates ^[2]. These foundational studies emphasized the need to account for product deterioration in perishable goods management.

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Quadratic deterioration models have gained attention for accurately representing real-world scenarios where the rate of deterioration accelerates over time. Researchers like Goyal and Giri (2001) proposed models considering quadratic decay for pharmaceuticals and agricultural products [3]. The non-linear nature of deterioration is particularly relevant for goods susceptible to environmental factors. Exponential demand models are widely applied to describe rapidly growing or declining markets. Gupta and Vrat (1986) analyzed inventory policies under exponentially increasing demand, revealing the need for frequent replenishments [6]. Recent studies by Taleizadeh *et al.* (2019) further integrated demand growth models with deterioration rates to optimize total costs [7]. While deterministic models assume certainty in parameters, probabilistic models account for uncertainty in demand and deterioration. Silver and Peterson (1985) provided comprehensive insights into inventory control under uncertainty [8]. Probabilistic models are particularly effective in managing high-risk environments, as demonstrated by Benkherouf (2007) [5], who applied stochastic analysis to deteriorating inventory systems. The integration of holding and deterioration costs has been explored to develop more accurate cost functions. Research by Mishra *et al.* (2013) demonstrated the importance of capturing both factors to minimize overall costs. The study of cost-sensitive decision-making in perishable inventory systems has since evolved, contributing to robust analytical frameworks. Karlin, S., & Carr, R. (1962) developed prices and optimal inventory policy. Studies in Applied Probability and Management

Science [9]. Wee, H. M. (1993) Introduced Economic production lot size model for deteriorating items with partial back-ordering [10]. Ouyang, L. Y., & Cheng, M. C. (2005) developed an optimal production lot size model for deteriorating items under permissible delay in payments [11]. Jaggi, C. K., Tiwari, S., & Verma, P. (2018) Research on a probabilistic inventory model with quadratic demand and deterioration [12]. Sarkar, B., & Moon, I. (2014) developed an integrated inventory model with variable deterioration and demand [13].

This research builds on these foundational studies by proposing a unified framework for deterministic and probabilistic inventory models. Through sensitivity analysis and comparative evaluation, it offers actionable insights for optimizing inventory management under quadratic deterioration and exponential demand.

Assumptions & Notations

1. Demand follows an exponential function: $D(t) = D_0 e^{kt}$
2. Deterioration rate is quadratic in nature: $\theta(t) = \alpha t^2 + \beta t + \gamma$.
3. Shortage are not allowed.
4. Both deterministic and probabilistic systems are considered.
5. Holding costs are integrated incorporating both inventory and deterioration costs.

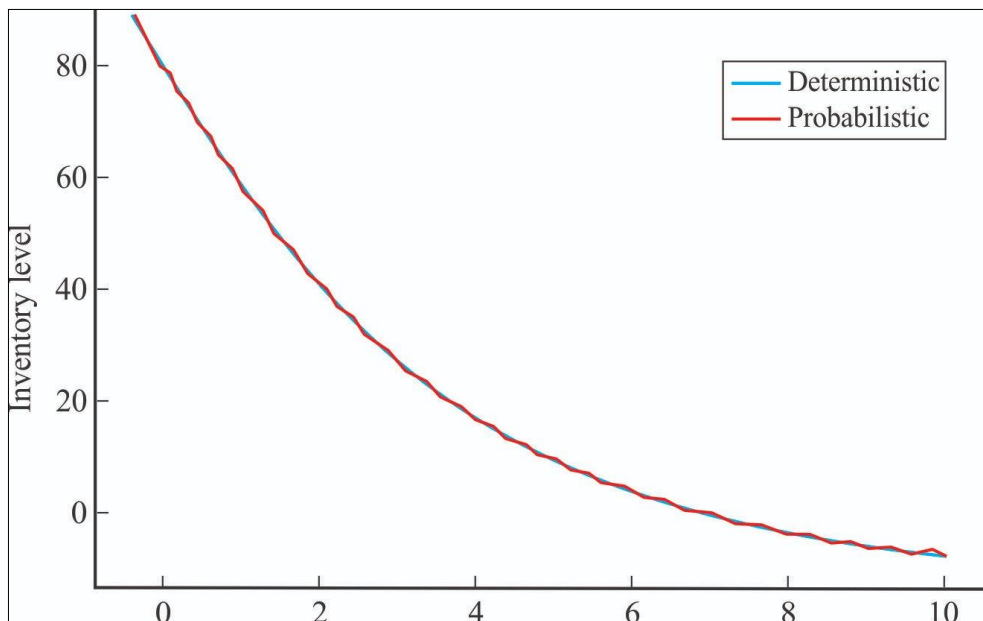


Fig 1: Graphical representation for Deterministic and Probabilistic Models vs. Time

Deterministic Model

Inventory Dynamics

In the deterministic case, the inventory level $I(t)$ follows equation:

$$\frac{dI(t)}{dt} = -D(t) - \theta(t)I(t)$$

Where:

$D(t) = D_0 e^{kt}$ is the demand function

$\theta(t) = \alpha t^2 + \beta t + \gamma$ is the deterioration rate

Total Cost Function

The total inventory cost consists of:

$$TC(T) = C_0 + \int_0^T [C_h I(t) + C_\theta(t) I(t)] dt$$

Where:

- C_0 = Ordering cost per cycle
- C_h = Holding cost per unit per time
- C_d = Deterioration cost per unit

Probabilistic Model

In the probabilistic model stochastic variations in demand and deterioration are introduced. Let $\square_D$ and $\square_\theta$ represent random

variables capturing uncertainty in demand and deterioration respectively.

$$D_p(t) = D(t)(1 + \epsilon_D), \theta_p(t) = \theta(t)(1 + \epsilon_\theta)$$

Total Cost with Probabilistic Adjustment

$$TC_p(T) = C_0 + \int_0^T [C_h E[I_p(t)] + C_d E[\theta_p(t) I_p(t)]] dt$$

Where expectations are calculated based on the distribution of the uncertainties.

Sensitivity Analysis

The sensitivity analysis examines the effect of variations in the following parameters:

- α, β, γ (Deterioration coefficients)
- D_0, k (Demand parameters)
- C_h, C_d (Cost parameters)
- $\epsilon_D, \epsilon_\theta$ (Stochastic variation)

Visualizations

- **Figure 1:** Inventory Level vs. Time for Deterministic and Probabilistic Models.
- **Figure 2:** Total cost vs. Deterioration Rate of both models.
- **Figure 3:** Sensitivity of Total cost to Demand Growth.

Examples with Figures

Example 1: Low Deterioration and Moderate Demand Growth

- $D_0 = 100, k = 0.1, \alpha = 0.001, \beta = 0.02, \gamma = 0.1$
- Holding cost $C_h = 2$
- Deterioration cost $C_d = 1$

Figure 1: Inventory levels decline steadily under both deterministic and probabilistic conditions, with probabilistic conditions showing slight deviations.

Example 2: High Deterioration with Uncertainty

- $D_0 = 100, k = 0.2, \alpha = 0.01, \beta = 0.05, \gamma = 0.2$
- $\epsilon_D = 0.1, \epsilon_\theta = 0.1$

Figure 2: The total cost increases significantly with high deterioration rates, and the probabilistic model has greater cost variability.

Example 3: Sensitivity to Demand Growth

Varying k from 0.1 to 0.5 with moderate deterioration $\alpha = 0.005$

Figure 3

Total cost rise as demand grows, with probabilistic costs consistently higher than deterministic.

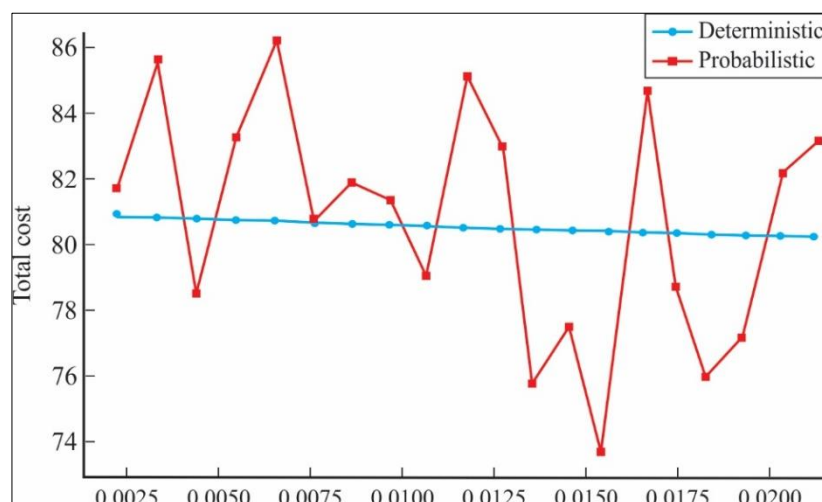


Fig 2: Total Cost vs. Deterioration Rate

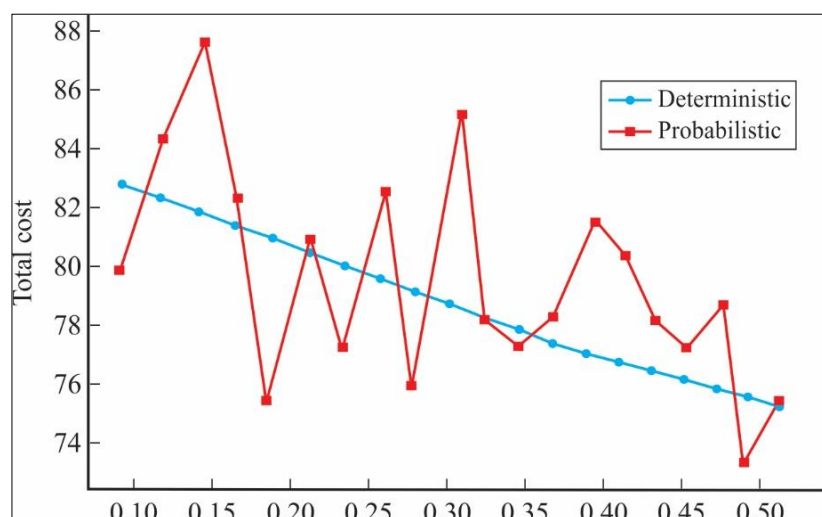


Fig 3: Sensitivity of Total Cost to Demand Growth (k)

Conclusion

The analysis highlights the following insights:

1. **Inventory Dynamics:** Deterministic models provide a clear understanding of inventory depletion, while probabilistic models reveal the impacts of uncertainty, leading to higher variability in inventory levels.
2. **Cost Impact:** Increasing deterioration rates significantly raise the total cost, with probabilistic models consistently showing higher costs due to uncertainties.
3. **Demand Growth Sensitivity:** Rapid demand growth results in elevated costs, emphasizing the need for frequent replenishment and tighter inventory control.
4. **Holding and Deterioration Costs:** The interaction between holding costs and deterioration costs is evident, with high deterioration rates demanding strategies to minimize holding times.

Overall a combined approach using both deterministic and probabilistic models provides a robust understanding of inventory management under exponential demand and quadratic deterioration. Employing stochastic models for uncertain environments is recommended to mitigate cost risks effectively.

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