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Nitin Patel
B. A. College of Agriculture,
Anand Agricultural University,
Anand, Gujarat, India

Digvijaysinh Parmar
College of Agricultural
Information Technology, Anand
Agricultural University, Anand,
Gujarat, India

Xitij Shukla
College of Agricultural
Information Technology, Anand
Agricultural University, Anand,
Gujarat, India

Corresponding Author:
Xitij Shukla
College of Agricultural
Information Technology, Anand
Agricultural University, Anand,
Gujarat, India

Application of harmonic analysis in the preliminary prediction of hourly air temperature of Anand station

Nitin Patel, Digvijaysinh Parmar and Xitij Shukla

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Abstract

Air temperatures exert a marked influence on plant growth and development. Temperature has direct effect on the rate of growth of plants. Hourly Air temperature forecasting is an interesting research topic as changes in it affect our day to day lives. In crop simulation models hourly air temperature plays an important role. At a few places automatic weather stations are installed which have instrument to get the hourly air temperature. But hourly air temperature data is not available directly at every research station, which is essential for the scientist to carry out the research. In this paper, Harmonic analysis (HA) is used to apply for periodic variations of the hourly air temperatures and it is helpful to understand the regular fluctuations in the variable.

Keywords: Hourly air temperatures, harmonic analysis

Introduction

Harmonic Analysis is a branch of mathematics concerned with the representation of Functions or Signals as the superposition of basic waves and the study of generalization of the motions of Fourier series. Applications of harmonic analysis in science and engineering begin with the idea or hypothesis that a phenomenon or signal is composed of a sum of individual oscillatory components. Predicting future meteorological trends based on historical time series data is crucial for agrophysical modeling (Lamorski *et al.*, 2013; Baranowski *et al.*, 2015; Krzyszczak *et al.*, 2017) [4, 1, 2]. Shekh *et al.* (2007) [5] analyze the hourly air temperature by two different methods and four different models and they found that the average of daily extreme temperatures is overestimation of hourly air temperatures from actual hourly air temperatures with highest PAE. RMSE and PAE by William and Logan model are higher than by DFS and ANNs. Isikwue presented the Application of Harmonic Analysis in the Preliminary Prediction of Air Temperature over Lagos and Abuja, Nigeria. They observed that the variation in hourly air temperature in the two stations is dominated by the first harmonic, thus it fluctuates by one cycle with a period of 24 hours.

Harmonic analysis technique has been employed in predicting the hourly air temperatures variations at Anand station in the present study. This study consists of sourcing of the hourly air temperature data, its subsequent analysis using Harmonic Analysis method and results for corresponding Month of Year for 2017-2021.

Source of data

The observed hourly air temperatures data from the year 2017 to 2021 were collected from Automatic Weather Station (AWS) with Soil Property Monitoring System, COSMOS installed by ISRO, Ahmedabad at Agronomy Farm, BACA, AAU, Anand.

Materials and Methods

Data series of the hourly air temperatures of each month of the year 2017 to 2020 were used to find the Fourier coefficients and using their values hourly air temperatures predicted by the Harmonic Analysis for the year 2021.

Thus, following Fourier series was developed separately for month-wise using recorded hourly air temperatures.

$$T(t) = T_0 + \sum_{k=1}^N \left(C_k \sin \left(\frac{2\pi}{P} kt + A_k \right) \right)$$

Where, $t = 0, 1, 2, \dots, 23$, N is the total number of Harmonics used and C_k is the Amplitude and A_k is the Phase Angle given by

$$C_k = \sqrt{a_k^2 + b_k^2}; \quad A_k = \tan^{-1} \left(\frac{a_k}{b_k} \right)$$

Where, a_k and b_k are the Fourier Coefficients, estimated by formula

$$a_k = \frac{2}{no} \sum [T(t) \cos \left(\frac{2\pi}{P} kt \right)],$$

$$b_k = \frac{2}{no} \sum [T(t) \sin \left(\frac{2\pi}{P} kt \right)]$$

Where, $k=1,2,3$ & $P=24$ and no is the number of observations.

Results

Here, Statistical measures were found from the used data series at listed in Table 1. It shows that mean temperatures increasing from January to June & after that trend reversing and going to decrease up to December and value was 21.97°C. At Anand station time of taking observations of minimum and maximum temperature is 7.38hrs and 14.38 hrs. This table shows the air temperatures at 7 & 14 hrs nearly minimum and maximum. Variance in data series was highest in the month of April and second highest in February while variance found lowest in the rainy season in the month of July and second lowest in August.

The original observed data of hourly air temperatures for two consecutive days of the year 2021 is plotted in the Figure 1. The patterns show sinusoidal curve. This shows the periodicity in the data with period of 24 hrs. Thus, one can employ the tools of harmonic analysis to predict the hourly air temperatures.

Table 1: Statistical Measures of data series (Year 2017-2021)

Month	Time (Hours)	Air Temp. Observed (°C)	Mean in data series (°C)	Variance in data series (°C)
January	7	14.2	21.67	55.88
	14	29.15		
	21	21.65		
February	7	13.75	22.65	100.37
	14	33.5		
	21	20.7		
March	7	17.7	26.48	89.50
	14	36.5		
	21	25.25		
April	7	21.05	30.03	103.10
	14	41.05		
	21	28		
May	7	27.9	33.27	24.16
	14	37.55		
	21	34.35		
June	7	28.1	32.92	19.88
	14	36.9		
	21	33.75		
July	7	28	31.78	11.13
	14	34.3		
	21	33.05		
August	7	27	30.88	16.72
	14	35.15		
	21	30.5		
September	7	20.85	28.33	55.88
	14	35.8		
	21	28.35		
October	7	17.9	25.88	78.33
	14	35.4		
	21	24.35		
November	7	15.5	24.15	96.76
	14	34.85		
	21	22.1		
December	7	14.55	21.97	56.04
	14	29.52		
	21	21.85		

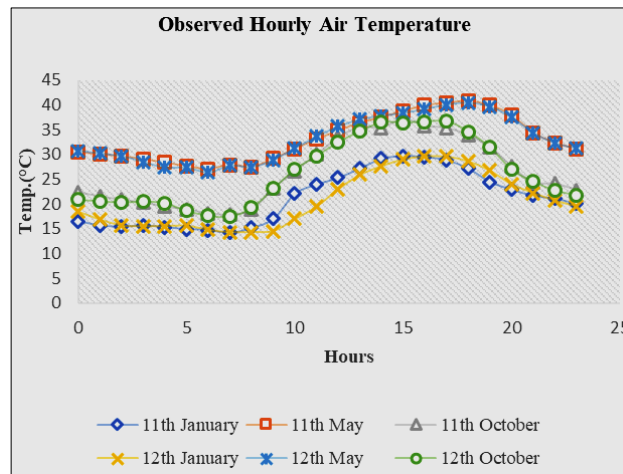


Fig 1: Observed Hourly Air Temperatures of the year 2021

The different harmonics are treated as an independent entity, each harmonic may have a different physical cause. Complete cycle of hourly air temperatures is fairly well described by the first harmonic in most of the cases. It is always not possible to account the complete variations in hourly air temperatures at once but the individual harmonics can be explained variance in parts.

The first harmonic has the fundamental period of $P=24$ hrs. The second harmonic has a period of half of the fundamental period that is $P/2=12$ hrs. and third harmonic have $P/3=8$ hrs.

Predicted hourly air temperatures of 7, 14 & 21th hrs by harmonic analysis for 11th day of each month for the year

2021 with its observed values and calculated prediction percentage error and mean percentage error calculated given in Table 2. It is observed that the highest prediction percentage error is 1.64 at 21 hrs for month of October and the lowest prediction percentage error is 0.15 at 14 hrs for the month of February.

The less mean prediction percentage error is 0.98 of the February months followed by 1.20, 1.27, 1.28, 1.29, 1.35, 1.38, 1.39 and 1.42 which were less than 10 %. Thus, it shows that the predicted air temperatures are significant to observed air temperatures. The predicted air temperatures are showed in Figures 2 to 13.

Table 2: Observed& Predicted Hourly Air Temperatures of 11th day of each month for the year 2021

Month	Time (hours)	Observed (°C)	Predicted (°C)	Prediction Percentage Error	Mean (24 hrs) Percentage Error
January	7	14.2	14.05	1.06	1.28
	14	29.15	29.34	0.65	
	21	21.65	21.45	0.92	
February	7	13.75	13.60	1.09	0.98
	14	33.5	33.55	0.15	
	21	20.7	20.52	0.87	
March	7	17.7	17.48	1.24	1.35
	14	36.5	36.15	0.96	
	21	25.25	25.65	1.58	
April	7	21.05	21.34	1.38	1.27
	14	41.05	40.85	0.49	
	21	28	28.32	1.14	
May	7	27.9	27.69	0.75	1.38
	14	37.55	37.98	1.15	
	21	34.35	34.85	1.46	
June	7	28.1	28.29	0.67	1.20
	14	36.9	36.55	0.95	
	21	33.75	33.50	0.74	
July	7	28	27.70	1.07	1.39
	14	34.3	33.94	1.05	
	21	33.05	33.35	0.91	
August	7	27	26.66	1.25	1.42
	14	35.15	34.75	1.14	
	21	30.5	30.95	1.48	
September	7	20.85	21.08	1.10	1.38
	14	35.8	35.39	1.15	
	21	28.35	28.70	1.23	
October	7	17.9	17.68	1.23	1.35
	14	35.4	35.15	0.71	
	21	24.35	24.75	1.64	
November	7	15.5	15.71	1.35	1.39
	14	34.85	34.55	0.86	
	21	22.1	22.35	1.13	
December	7	14.55	14.39	1.10	1.29
	14	29.52	29.34	0.61	
	21	21.85	21.58	1.23	

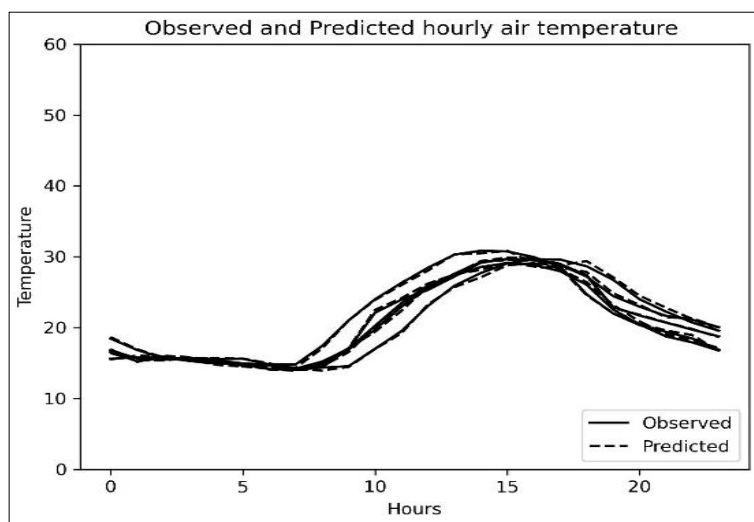


Fig 2: Observed & Predicted Hourly Air Temperatures of 11 to 15th January 2021

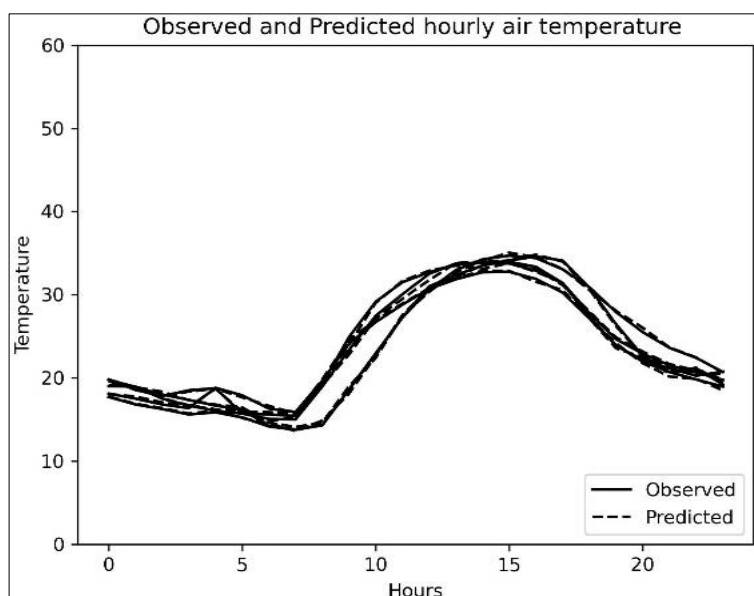


Fig 3: Observed & Predicted Hourly Air Temperatures of 11 to 15th February 2021

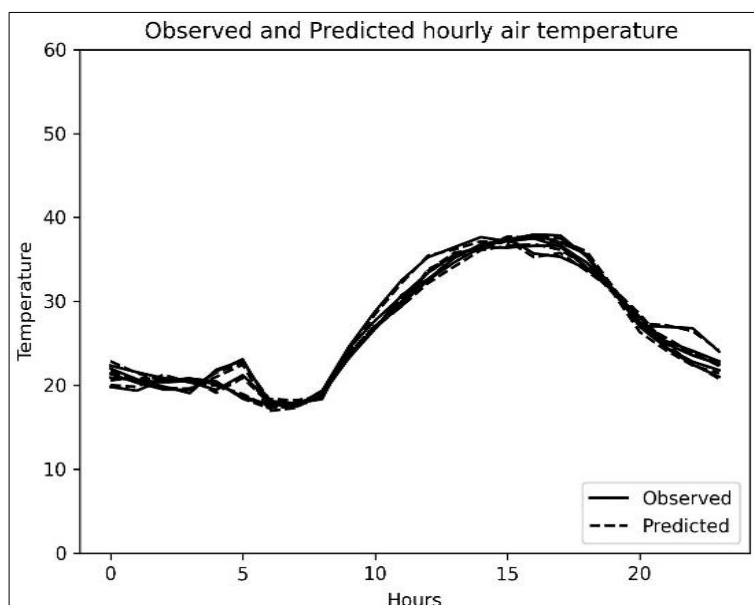


Fig 4: Observed & Predicted Hourly Air Temperatures of 11 to 15th March 2021

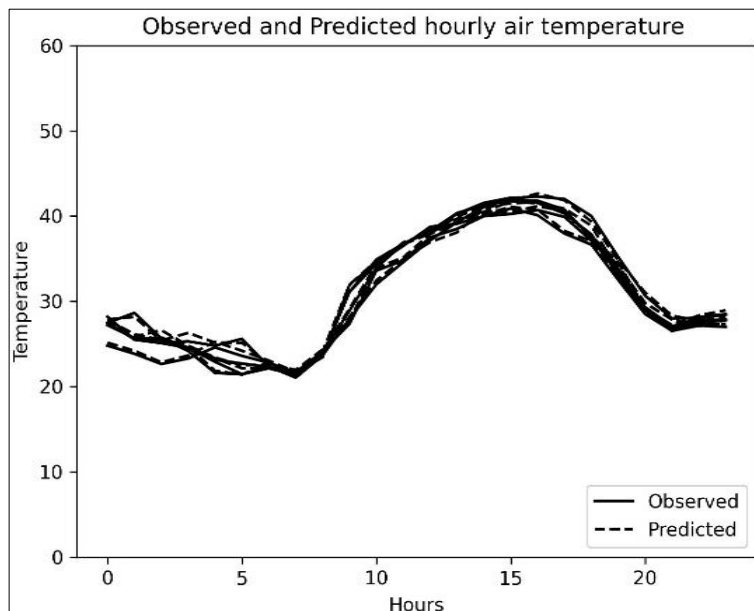


Fig 5: Observed & Predicted Hourly Air Temperatures of 11 to 15th April 2021

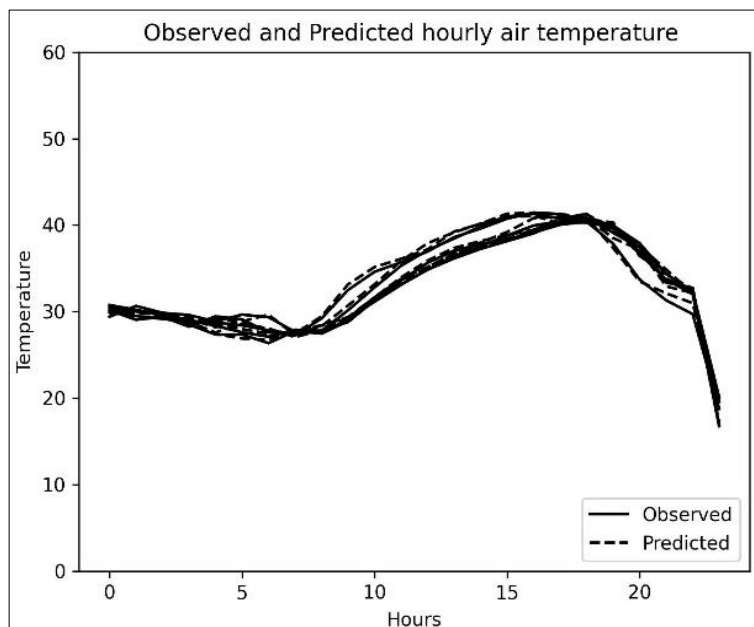


Fig 6: Observed & Predicted Hourly Air Temperatures of 11 to 15th May 2021

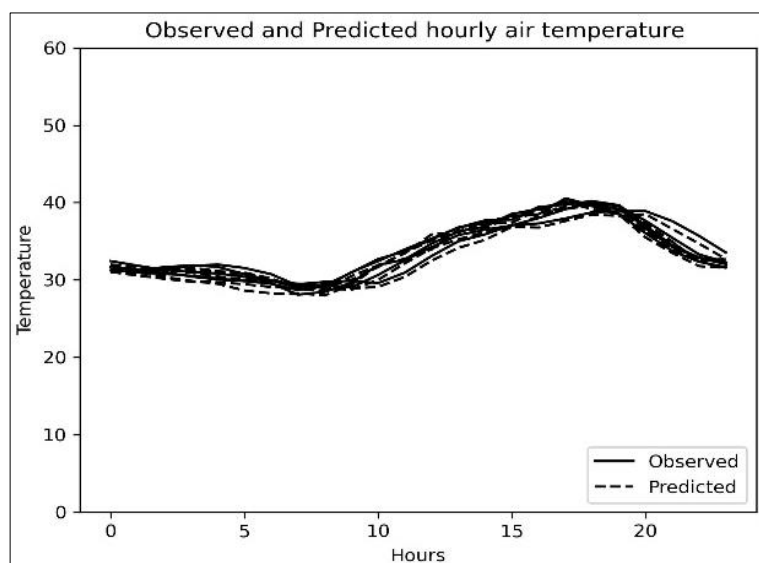


Fig 7: Observed & Predicted Hourly Air Temperatures of 11 to 15th June 2021

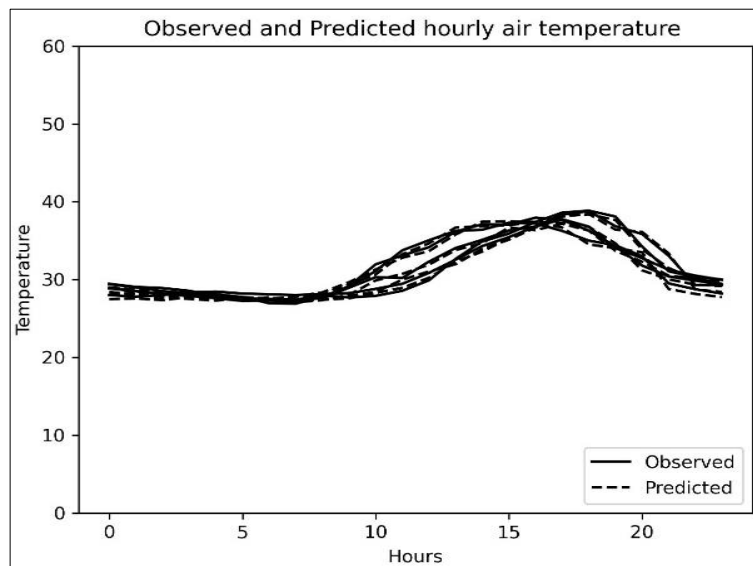


Fig 8: Observed & Predicted Hourly Air Temperatures of 11 to 15th July 2021

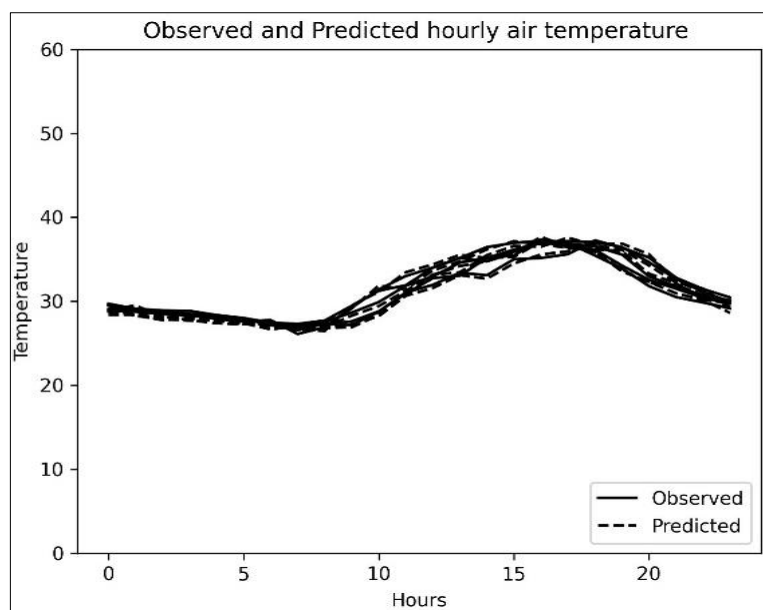


Fig 9: Observed & Predicted Hourly Air Temperatures of 11 to 15th August 2021

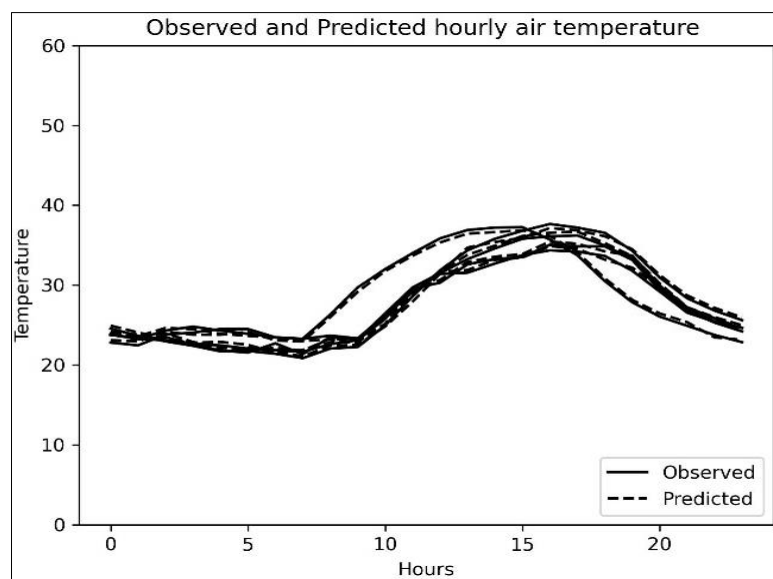


Fig 10: Observed & Predicted Hourly Air Temperatures of 11 to 15th September 2021

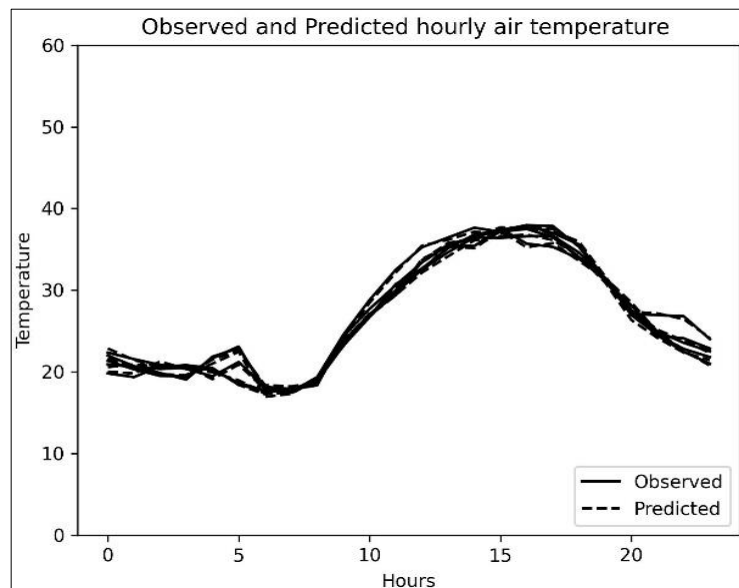


Fig 11: Observed & Predicted Hourly Air Temperatures of 11 to 15thOctober 2021

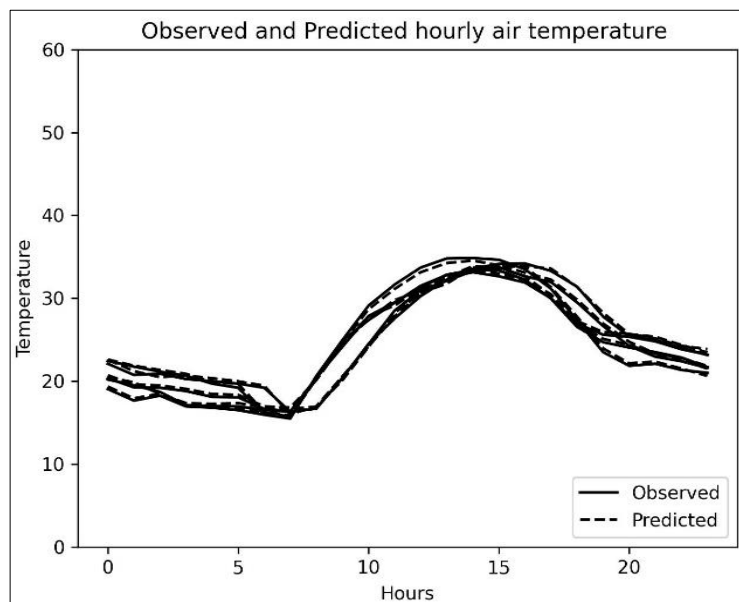


Fig 12: Observed & Predicted Hourly Air Temperatures of 11 to 15thNovember 2021

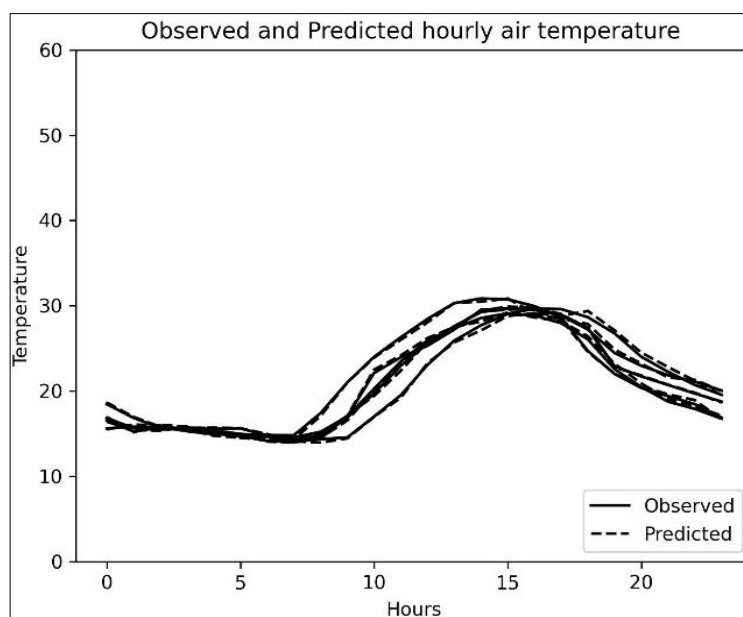


Fig 13: Observed & Predicted Hourly Air Temperatures of 11 to 15thDecember 2021

Variations accounted by each number of harmonics from $k=1$ to 3 are computed by formula $C_k^2/2S^2$ where S^2 is the total variance in observed data series. There computed values for selected there days 11, 12 & 13th of each month are listed in Table 3.

Total accounted perctange of variance of different months have a different values. The highest value is 112.14% of 12th day of February months and the lowest values is 79 % of 13th

day for month of August as this months is of rainy season and this model does not take in account of it's cloudy weather effect. First harmonic has biggest amplitude, it's period is one fundamental period of 24 hrs and during this time period second harmonic oscillates two times and so on for other harmonics. Also, the higher harmonics contribute less in the covering of variance. All the values of accounted variations were ranges from 79 to 100%. This shows the high accuracy of predicted hourly air temperatures.

Table 3: Variations accounted by three numbers of harmonics in the data series

Month	Days	Variations accounted by each harmonic (%)	Total Variations accounted by three Harmonics (%)
January	11	(1) 84.90(2) 3.64(3) 6.52	95.07
	12	(1) 83.97(2) 8.14(3) 0.22	92.33
	13	(1) 77.63(2) 15.74(3) 3.66	97.03
February	11	(1) 94.32(2) 4.64(3) 1.32	100.28
	12	(1) 108.31(2) 3.72(3) 0.11	112.14
	13	(1) 102.83(2) 5.03(3) 0.70	108.55
March	11	(1) 83.33(2) 4.84(3) 1.93	90.10
	12	(1) 82.48(2) 10.85(3) 3.30	96.63
	13	(1) 90.16(2) 12.56(3) 3.47	106.19
April	11	(1) 95.31(2) 5.53(3) 3.81	104.65
	12	(1) 93.35(2) 4.81(3) 1.59	99.75
	13	(1) 89.48(2) 4.2(3) 2.54	96.73
May	11	(1) 92.64(2) 2.14(3) 2.02	96.80
	12	(1) 90.65(2) 4.99(3) 3.29	98.93
	13	(1) 91.03(2) 5.97(3) 2.38	99.38
June	11	(1) 75.84(2) 7.04(3) 1.14	84.30
	12	(1) 82.48(2) 2.96(3) 1.11	86.55
	13	(1) 70.32(2) 10.43(3) 0.88	81.63
July	11	(1) 74.05(2) 5.08(3) 1.15	80.28
	12	(1) 69.70(2) 9.90(3) 0.11	79.72
	13	(1) 80.71(2) 7.83(3) 0.18	88.72
August	11	(1) 72.18(2) 7.29(3) 0.33	79.81
	12	(1) 73.16(2) 6.54(3) 0.17	79.87
	13	(1) 71.02(2) 7.82(3) 0.16	79.00
September	11	(1) 80.32(2) 9.07(3) 0.19	89.57
	12	(1) 73.41(2) 9.70(3) 0.03	83.14
	13	(1) 77.10(2) 9.66(3) 1.15	87.90
October	11	(1) 89.14(2) 5.90(3) 0.27	95.30
	12	(1) 87.33(2) 8.03(3) 0.51	95.87
	13	(1) 89.22(2) 6.67(3) 0.48	96.36
November	11	(1) 78.33(2) 8.77(3) 0.33	87.43
	12	(1) 77.82 (2) 10.37(3) 0.46	88.65
	13	(1) 80.36(2) 7.70(3) 0.10	88.16
December	11	(1) 86.10(2) 10.00(3) 0.64	96.74
	12	(1) 81.80(2) 15.01(3) 0.34	97.15
	13	(1) 81.32(2) 12.70(3) 0.20	94.23

The incoming solar radiation flux which is a radiant energy, is the sum of the diffused (the incoming shortwave in the shade) and direct radiations incident on the earth's surface in the form of shortwave radiation. The sunshine hour is an important factor to determine solar irradiance and air temperatures. The sunshine hour is higher in the summer and least in the winter season. Daily extreme temperatures appear to be very much related to solar irradiation. The average monthly maximum temperature is highest in May and lowest in January.

The formula to derive the Solar Irradiance is given as below,

$$R_s = R_a \left[a + b \frac{n}{N} \right]$$

where a and b are empirically derived coefficient from Anand

$$a = 0.25, b = 0.50$$

n = Actual sunshine hours measured by sunshine recorder

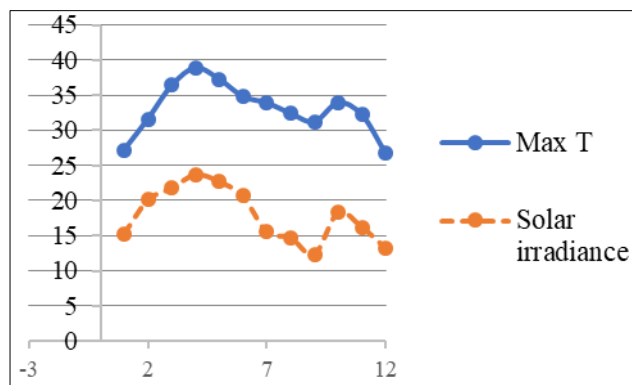
N = Maximum possible sunshine hours for a given station and latitude for a given month

R_a = Radiation at the top of the atmosphere in $\text{mj m}^{-2} \text{d}^{-1}$

Computed Solar Irradiance by above formula are listed with the values of maximum air temperature in Table 4. From the Table 4 it is observed that the highest solar irradiance is 23.74 in April month and the lowest is 12.33 in the September month. There was a strong positive correlation of Maximum air temperature and Solar irradiance i.e. 0.81 at 5% level. It is due to the fact that air temperatures are directly related to the solar irradiance.

Table 4: Monthly Maximum Air Temperature and Solar Irradiance at Anand Station

Month	Maximum air Temp. (°C)	Solar Irradiance (MJ / m ² / day)
January	27.2	15.26
February	31.6	20.13
March	36.4	21.8
April	38.9	23.74
May	37.3	22.74
June	34.8	20.65
July	33.9	15.52
August	32.4	14.74
September	31.2	12.33
October	33.9	18.30
November	32.2	16.23
December	26.7	13.24

**Fig 14:** Monthly Maximum Air Temperature and Solar Irradiance of the year 2021

Conclusion

Observed hourly air temperatures has sinusoidal curve and its periodic so harmonic analysis can be used for prediction of hourly air temperatures at Anand station. Predicted hourly air temperatures by harmonic analysis were accurate and at par with as computed Mean Percentage Error for 24 hrs. in each month were less than 10 %. Solar irradiance varies with maximum air temperature. This variation is liner and correlation coefficient are 0.81. The monthly variation pattern of solar irradiance and air temperature demonstrates that they have the highest value at summer and lowest in winter. More number of data can improve the accuracy of the predicting task.

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