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Development of six sigma-based control chart for standard deviation under robust measure

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Abstract

The number of defects is frequently used to evaluate or measure quality. Quality may reduce errors, rework, and defects while increasing output from the same input. Thus, productivity increases, according to Duncan (1958). Consequently, a business that produces high-quality products will frequently witness an improvement in manufacturing productivity. When the underlying normalcy assumption is not fulfilled, a control chart based on six sigma and employing the interquartile range (IQR) for standard deviation is offered in this research paper as an alternative to the current control chart. This control chart performs better than the Shewhart (1931) control chart.

Keywords: Robust control chart, six sigma and standard deviation

1. Introduction

The notion that process behavior tends to vary over time is the basis of statistical process control. SPC uses statistical techniques to track the performance of the manufacturing process and find noteworthy variations that might result in the production of inferior items. There are two types of variability in processes.

a. Chance variation: This is the intrinsic variation that arises from a controlled process. There are chance, inherent, and random explanations for this volatility. There might be several tiny, interconnected production features at fault, such as machine, material, operator, tool, etc. It is hard to remove this sort of variant. However, with greater effort, the influence of random elements can be diminished.

b. Assignable variation: It is the variation caused by a process that is not under control and is not always present in the causal system of the process. This difference can be attributed to a few different factors. Variations in the environment, labor, equipment, materials, tooling, etc. might be the cause of the discrepancy. It is possible to locate and resolve this issue. When the only reason for any change in a process is chance, it is said to be under control. A control chart allows for the acceptance of random variation's natural dispersion while fully eliminating the impacts of assignable sources.

2. Control Limits

All those control limits are probability limits of 0.001 only. They are set up such that the likelihood of a point going over the upper control limit (UCL) and below the lower control limit, when chance reasons alone are at play, is 1 in 1000. As a result, according to Montgomery (2008), 2 out of 1000 points will slip outside the control boundaries. When there is no assignable variance, these constraints establish the risk of looking for an assignable cause. With a risk of just 2 out of 1000, 0.001 probability limitations may almost guarantee that, in the unlikely event that a point deviates from the norm, assignable reasons might be the cause. In the event that the process output has a normal distribution, the 3σ limits and the 0.001 probability limits are almost similar. This is due to the fact that the region beyond $\pm 3\sigma$ on a normal distribution is 0.27%, or roughly equivalent to 0.2% (2 out of 1000). Radhakrishnan (2009)^[6] suggested a single sample plan that was indexed to six sigma quality

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levels (SSQLs) using the weighted Poisson distribution and the intervening random effect Poisson distribution as the basis line distributions. Radhakrishnan and Sivakumaran (2009) ^[6] used the six-sigma idea to build mixed sample plans with Chsp-2 plan and attribute plan that are indexed via six sigma quality levels. They created single, double, and repeated group sampling plans that were indexed by six sigma quality levels (SSQLs) utilizing the six-sigma concept, employing the Poisson distribution as the foundation line distribution.

3. Process capability analysis

Process capability analysis is used to estimate process capabilities. It is necessary to estimate the process mean and the standard deviation of the quality characteristic. In addition, the shape of the relative frequency distribution for the relevant property is estimated. If specification limits are known, a process capacity study will also determine the proportion of nonconforming product.

In this scenario, the core of a true process capacity evaluation would be obtaining and analyzing data related to process parameters. These process characteristics include things like the depth of cut, kind of tool used, material of tool, type of fixtures, and feed rate of tool. Process parameters may therefore be linked to any issues with product qualities, allowing for the prompt identification of corrective measures.

An A quality improvement project requires frequent estimation of the process parameters. Monitoring these factors will ensure that the process runs as efficiently as possible. In the event that the process is not inherently competent, corrective action must be taken. These might involve finding out if buying new equipment or better raw materials can reduce the process spread and positioning the process average in the middle of the intended value. Analyzing the specifications' flexibility by figuring out what the client wants is an additional choice.

$$C_p = \frac{USL - LSL}{6\sigma}$$

where USL - Upper specification limit and LSL - Lower specification limit

4. Interquartile range (IQR)

The interquartile range (IQR) is a measurement of variability based on a data set's quartile partition. These quartiles split a rank-ordered data collection into four equal halves. According to Adekeye and Azubuike (2012), the values that divide each section are the first, second, and third quartiles, denoted by the letters Q_1 , Q_2 , and Q_3 , respectively.

- Q_1 is the "middle" value in the first half of the rank-ordered data set
- Q_2 is the median value in the set
- Q_3 is the "middle" value in the second half of the rank-ordered data set

The interquartile range is equal to Q_3 minus Q_1 .

5. Conditions for Application

- The company implements Six Sigma with strong measures quality efforts in its processes;
- There should be less human participation in the overall process/system.

6. Methods and materials

Suppose that 'N' preliminary samples are available, each of size 'n', and let S_i be the standard deviation of the i^{th} sample. The average of the 'N' standard deviation is

$$\bar{S} = \frac{\sum_{i=1}^N S_i}{N} = \frac{S_1 + S_2 + \dots + S_N}{N}.$$

If the sample size is to be vary,

$$\bar{S} = \left[\frac{\sum_{i=1}^k (n_i - 1) S_i^2}{\sum_{i=1}^k n_i - i} \right]^{1/2} = \left[\frac{(n_1 - 1) S_1^2 + (n_2 - 1) S_2^2 + \dots + (n_k - 1) S_k^2}{(n_1 - 1) + (n_2 - 2) + \dots + (n_k - k)} \right]^{1/2}$$

The three-sigma control chart for standard deviation is constructed under normal by using the probability level, $p(z \leq z_{3\sigma}) = 1 - \alpha_1, \alpha_1 = 0.0027$ as given below:

$$UCL_S^{3\sigma} = \bar{S}_S + \left(3\sqrt{1-c_4^2} \left(\frac{\bar{S}}{c_4} \right) \right)$$

$$CL_S^{3\sigma} = \bar{S}_S$$

$$LCL_S^{3\sigma} = \bar{S}_S - \left(3\sqrt{1-c_4^2} \left(\frac{\bar{S}}{c_4} \right) \right)$$

The standard deviation $\left(\sigma_{N.S}^{3\sigma-IQR} \right)$ for constructing control limits with three-sigma by using IQR for standard deviation chart under normal distribution as follows:

$$UCL_{N.S}^{3\sigma.IQR} = \bar{S}_S + \left[3\sqrt{1-c_4^2} \left(\frac{\sigma_{N.S}^{3\sigma-IQR}}{\sqrt{n}} \right) \right]$$

$$CL_{N.S}^{3\sigma.IQR} = \bar{S}_S$$

$$LCL_{N.S}^{3\sigma.IQR} = \bar{S}_S - \left[3\sqrt{1-c_4^2} \left(\frac{\sigma_{N.S}^{3\sigma-IQR}}{\sqrt{n}} \right) \right]$$

The standard deviation $\left(\sigma_{E.S}^{3\sigma-IQR} \right)$ for constructing control limits with three-sigma by using IQR for standard deviation chart under exponential distribution as follows:

$$UCL_{E.S}^{3\sigma.IQR} = \bar{S}_S + \left[3\sqrt{1-c_4^2} \left(\frac{\sigma_{E.S}^{3\sigma-IQR}}{\sqrt{n}} \right) \right]$$

$$CL_{E.S}^{3\sigma.IQR} = \bar{S}_S$$

$$LCL_{E.S}^{3\sigma.IQR} = \bar{S}_S - \left[3\sqrt{1-c_4^2} \left(\frac{\sigma_{E.S}^{3\sigma-IQR}}{\sqrt{n}} \right) \right]$$

The standard deviation control chart uses the average of subgroup Downton estimator $\left(Dn_s^{3\sigma} \right)$ to define control boundaries and the centre line based on the average of standard deviation.

The process mean will be determined by taking the average of mean of the subgroup mean, and the process standard deviation $\left(Dn_s^{3\sigma} \right)$ will be derived from the subgroup to lessen the impact of outliers on the control limits under normal distribution.

$$UCL_S^{3\sigma} = \bar{S}_S + \left[3\sqrt{1-c_4^2} \left(\frac{Dn_s^{3\sigma}}{\sqrt{n}} \right) \right]$$

$$CL_R^{3\sigma} = \bar{S}_S$$

$$LCL_R^{3\sigma} = \bar{S}_S - \left[3\sqrt{1-c_4^2} \left(\frac{Dn_s^{3\sigma}}{\sqrt{n}} \right) \right]$$

The six-sigma based control limits constructed based on the probability level $p(z \leq z_{6\sigma}) = 1 - \alpha_1, \alpha_1 = 3.4 \times 10^{-6}$ under normal distribution (Jayabharathi and Balamurugan, 2022) for a standard deviation are as follows:

$$UCL_S^{6\sigma} = \bar{S}_S + \left[\phi_{6\sigma} \sqrt{1-c_4^2} \left(\frac{\sigma_S^{6\sigma}}{\sqrt{n}} \right) \right]$$

$$CL_S^{6\sigma} = \bar{S}_S$$

$$LCL_S^{6\sigma} = \bar{S}_S - \left[\phi_{6\sigma} \sqrt{1-c_4^2} \left(\frac{\sigma_S^{6\sigma}}{\sqrt{n}} \right) \right]$$

The six-sigma standard deviation control chart defines control limits and the centre line based on the process standard deviation $(D_{n.S}^{6\sigma})$ by taking the average of the subgroup Downton estimator.

$$UCL_{Dn.S}^{6\sigma} = \bar{S}_S + \left[\phi_{6\sigma} \sqrt{1-c_4^2} \left(\frac{D_{n.S}^{6\sigma}}{\sqrt{n}} \right) \right]$$

$$CL_{Dn.S}^{6\sigma} = \bar{S}_S$$

$$LCL_{Dn.S}^{6\sigma} = \bar{S}_S - \left[\phi_{6\sigma} \sqrt{1-c_4^2} \left(\frac{D_{n.S}^{6\sigma}}{\sqrt{n}} \right) \right]$$

The suggested process standard deviation $(\tau_{N.6\sigma}^{S-IQR})$ for constructing standard deviation control limits with six sigma by using interquartile range (IQR) under normal distribution is used to establish the tolerance level (TL) and process capability. Using the process standard deviation $(\tau_{N.6\sigma}^{S-IQR})$ instead of the standard deviation (σ) from the control chart for standard deviation yields:

$$UCL_{N.S}^{6\sigma.IQR} = \bar{S}_S + \left[\phi_{6\sigma} \sqrt{1-c_4^2} \left(\frac{\tau_{N.6\sigma}^{S-IQR}}{\sqrt{n}} \right) \right]$$

$$CL_{N.S}^{6\sigma.IQR} = \bar{S}_S$$

$$LCL_{N.S}^{6\sigma.IQR} = \bar{S}_S - \left[\phi_{6\sigma} \sqrt{1-c_4^2} \left(\frac{\tau_{N.6\sigma}^{S-IQR}}{\sqrt{n}} \right) \right]$$

The proposed process standard deviation $(\tau_{E.6\sigma}^{S-IQR})$ for generating standard deviation control limits with six sigma using IQR under exponential distribution is used to determine the tolerance level (TL) and process capabilities. To calculate standard deviation yields, use the process standard deviation $(\tau_{E.6\sigma}^{S-IQR})$ rather than the control chart's standard deviation (σ).

$$UCL_{E.S}^{6\sigma.IQR} = \bar{S}_S + \left[\phi_{6\sigma} \sqrt{1-c_4^2} \left(\frac{\tau_{E.6\sigma}^{S-IQR}}{\sqrt{n}} \right) \right]$$

$$CL_{E.S}^{6\sigma.IQR} = \bar{S}_S$$

$$LCL_{E.S}^{6\sigma.IQR} = \bar{S}_S - \left[\phi_{6\sigma} \sqrt{1-c_4^2} \left(\frac{\tau_{E.6\sigma}^{S-IQR}}{\sqrt{n}} \right) \right]$$

7. Illustration

In the production of an electrical device operated by a thermostatic control, five control switches were tested each hour to determine the “on” temperature at which the thermostat actually operated under a given setting. Results of the test over a production period were as shown below.

Table 1: Measurement on production of an electrical device operated by thermostatic control

Sub group number	I	II	III	IV	V	Standard deviation	IQR _{Norm}	IQR _{Exp}	Downton Statistic
1	390	393	395	405	420	12.22	8.90	10.92	12.76
2	376	381	381	383	401	9.63	1.48	1.82	9.22
3	380	387	395	397	407	10.26	7.41	9.10	11.35
4	377	383	387	390	393	6.24	5.19	6.37	6.91
5	393	395	403	405	414	8.43	7.41	9.10	9.22
6	376	388	395	397	400	9.58	6.67	8.19	10.11
7	387	400	400	403	410	8.34	2.22	2.73	8.69
8	391	392	394	397	405	5.63	3.71	4.55	5.85
9	390	391	395	401	405	6.47	7.41	9.10	7.09
10	379	391	393	394	410	11.06	2.22	2.73	11.52
11	390	397	400	406	428	14.50	6.67	8.19	15.07
12	380	382	389	391	399	7.60	6.67	8.19	8.33
13	375	383	392	395	404	11.17	8.90	10.92	12.41

14	387	390	398	400	408	8.35	7.41	9.10	9.22
15	390	395	395	397	403	4.69	1.48	1.82	4.96
16	382	399	401	406	406	9.88	5.19	6.37	9.75
17	390	395	395	400	410	7.58	3.71	4.55	7.98
18	381	390	394	397	399	7.12	5.19	6.37	7.62
19	387	389	398	401	415	11.18	8.90	10.92	12.06
20	372	378	396	400	405	14.39	16.31	20.03	15.60
21	387	389	391	391	400	4.98	1.48	1.82	4.96
22	376	380	391	406	412	15.75	19.27	23.67	17.37
23	395	396	397	400	400	2.30	2.97	3.64	2.48
24	392	394	397	399	400	3.36	3.71	4.55	3.72
25	385	390	390	392	392	2.86	1.48	1.82	2.84
26	388	395	395	400	415	10.11	3.71	4.55	10.46
27	385	391	400	407	425	15.55	11.86	14.56	17.02
28	379	390	392	411	414	14.86	15.57	19.12	16.13
29	390	399	402	406	411	7.89	5.19	6.37	8.69
30	392	395	400	400	406	5.37	3.71	4.55	5.85
Average						8.91	6.40	7.86	9.51

Source: Eugene L. Grant and Richard S. Leavenworth (1952)

(i) The control chart for standard deviation based on 3σ , then the control limits are as follows

$$UCL_s^{3\sigma} = \bar{S}_s + \left(3\sqrt{1-c_4^2} \left(\frac{\bar{S}}{c_4} \right) \right) = 8.91 + \left(3\sqrt{1-0.94^2} \left(\frac{8.91}{0.94} \right) \right) = 18.62$$

$$CL_s^{3\sigma} = \bar{S}_s = 8.91$$

$$LCL_s^{3\sigma} = \bar{S}_s - \left(3\sqrt{1-c_4^2} \left(\frac{\bar{S}}{c_4} \right) \right) = 8.91 - \left(3\sqrt{1-0.94^2} \left(\frac{8.91}{0.94} \right) \right) = -0.79 < 0$$

(ii) The control chart for standard deviation by using interquartile range (IQR) based on 3σ under normal distribution, then the control limits are as follows:

$$UCL_{N.S}^{3\sigma.IQR} = \bar{S}_s + \left[3\sqrt{1-c_4^2} \left(\frac{\sigma_{N.S}^{3\sigma-IQR}}{\sqrt{n}} \right) \right] = 8.91 + \left(3\sqrt{1-0.94^2} \left(\frac{6.40}{\sqrt{5}} \right) \right) = 11.84$$

$$CL_{N.S}^{3\sigma.IQR} = \bar{S}_s = 8.91$$

$$LCL_{N.S}^{3\sigma.IQR} = \bar{S}_s - \left[3\sqrt{1-c_4^2} \left(\frac{\sigma_{N.S}^{3\sigma-IQR}}{\sqrt{n}} \right) \right] = 8.91 - \left(3\sqrt{1-0.94^2} \left(\frac{6.40}{\sqrt{5}} \right) \right) = 5.98$$

(iii) The control chart for standard deviation by using interquartile range (IQR) based on 3σ under exponential distribution, then the control limits are as follows

$$UCL_{E.S}^{3\sigma.IQR} = \bar{S}_s + \left[3\sqrt{1-c_4^2} \left(\frac{\sigma_{E.S}^{3\sigma-IQR}}{\sqrt{n}} \right) \right] = 8.91 + \left(3\sqrt{1-0.94^2} \left(\frac{7.86}{\sqrt{5}} \right) \right) = 12.51$$

$$CL_{E.S}^{3\sigma.IQR} = \bar{S}_s = 8.91$$

$$LCL_{E.S}^{3\sigma.IQR} = \bar{S}_s - \left[3\sqrt{1-c_4^2} \left(\frac{\sigma_{E.S}^{3\sigma-IQR}}{\sqrt{n}} \right) \right] = 8.91 - \left(3\sqrt{1-0.94^2} \left(\frac{7.86}{\sqrt{5}} \right) \right) = 5.31$$

(iv) The control chart for standard deviation by using Downton statistic based on 3σ , then the control limits are as follows

$$UCL_s^{3\sigma} = \bar{S}_s + \left[3\sqrt{1-c_4^2} \left(\frac{Dn_s^{3\sigma}}{\sqrt{n}} \right) \right] = 8.91 + \left(3\sqrt{1-0.94^2} \left(\frac{9.51}{\sqrt{5}} \right) \right) = 13.26$$

$$CL_R^{3\sigma} = \bar{S}_s = 8.91$$

$$LCL_R^{3\sigma} = \bar{S}_s - \left[3\sqrt{1-c_4^2} \left(\frac{Dn_s^{3\sigma}}{\sqrt{n}} \right) \right] = 8.91 - \left(3\sqrt{1-0.94^2} \left(\frac{9.51}{\sqrt{5}} \right) \right) = 4.56$$

(v) The six-sigma based control chart for standard deviation, then the control limits are as follows

$$UCL_{S}^{6\sigma} = \bar{S}_S + \left[\phi_{6\sigma} \sqrt{1 - c_4^2} \left(\frac{\sigma_S^{6\sigma}}{\sqrt{n}} \right) \right] = 8.91 + \left(4.831 \sqrt{1 - 0.94^2} \left(\frac{1.12}{\sqrt{5}} \right) \right) = 9.74$$

$$CL_{S}^{6\sigma} = \bar{S}_S = 8.91$$

$$LCL_{S}^{6\sigma} = \bar{S}_S - \left[\phi_{6\sigma} \sqrt{1 - c_4^2} \left(\frac{\sigma_S^{6\sigma}}{\sqrt{n}} \right) \right] = 8.91 - \left(4.831 \sqrt{1 - 0.94^2} \left(\frac{1.12}{\sqrt{5}} \right) \right) = 8.09$$

(vi) The six-sigma based control chart for standard deviation by using Downton statistic, then the control limits are as follows

$$UCL_{Dn.S}^{6\sigma} = \bar{S}_S + \left[\phi_{6\sigma} \sqrt{1 - c_4^2} \left(\frac{D_{n.S}^{6\sigma}}{\sqrt{n}} \right) \right] = 8.91 + \left(4.831 \sqrt{1 - 0.94^2} \left(\frac{1.24}{\sqrt{5}} \right) \right) = 9.83$$

$$CL_{Dn.S}^{6\sigma} = \bar{S}_S = 8.91$$

$$LCL_{Dn.S}^{6\sigma} = \bar{S}_S - \left[\phi_{6\sigma} \sqrt{1 - c_4^2} \left(\frac{D_{n.S}^{6\sigma}}{\sqrt{n}} \right) \right] = 8.91 - \left(4.831 \sqrt{1 - 0.94^2} \left(\frac{1.24}{\sqrt{5}} \right) \right) = 8.00$$

(vii) The six-sigma based control chart for standard deviation by using interquartile range (IQR) under normal distribution, then the control limits are as follows

$$UCL_{N.S}^{6\sigma.IQR} = \bar{S}_S + \left[\phi_{6\sigma} \sqrt{1 - c_4^2} \left(\frac{\tau_{N.6\sigma}^{S-IQR}}{\sqrt{n}} \right) \right] = 8.91 + \left(4.831 \sqrt{1 - 0.94^2} \left(\frac{1.48}{\sqrt{5}} \right) \right) = 10.00$$

$$CL_{N.S}^{6\sigma.IQR} = \bar{S}_S = 8.91$$

$$LCL_{N.S}^{6\sigma.IQR} = \bar{S}_S - \left[\phi_{6\sigma} \sqrt{1 - c_4^2} \left(\frac{\tau_{N.6\sigma}^{S-IQR}}{\sqrt{n}} \right) \right] = 8.91 - \left(4.831 \sqrt{1 - 0.94^2} \left(\frac{1.48}{\sqrt{5}} \right) \right) = 7.82$$

(viii) The six-sigma based control chart for standard deviation by using interquartile range (IQR) under exponential distribution, then the control limits are as follows

$$UCL_{E.S}^{6\sigma.IQR} = \bar{S}_S + \left[\phi_{6\sigma} \sqrt{1 - c_4^2} \left(\frac{\tau_{E.6\sigma}^{S-IQR}}{\sqrt{n}} \right) \right] = 8.91 + \left(4.831 \sqrt{1 - 0.94^2} \left(\frac{1.82}{\sqrt{5}} \right) \right) = 10.25$$

$$CL_{E.S}^{6\sigma.IQR} = \bar{S}_S = 8.91$$

$$LCL_{E.S}^{6\sigma.IQR} = \bar{S}_S - \left[\phi_{6\sigma} \sqrt{1 - c_4^2} \left(\frac{\tau_{E.6\sigma}^{S-IQR}}{\sqrt{n}} \right) \right] = 8.91 - \left(4.831 \sqrt{1 - 0.94^2} \left(\frac{1.82}{\sqrt{5}} \right) \right) = 7.57$$

Table 2: Summary of three-sigma and six-sigma control charts for standard deviation

Sl. No	Control chart	UCL	LCL	CLI
1	Shewhart three-Sigma	18.62	-0.79	19.41
2	Three-sigma with IQR under Exponential distribution	12.51	5.31	7.19
3	Three-sigma with IQR under Normal distribution	11.84	5.98	5.86
4	Three-sigma using Downton	13.26	4.56	8.70
5	Six-sigma using Downton	9.83	8.00	1.83
6	Six-sigma	9.74	8.09	1.65
7	Six-sigma with IQR under Exponential distribution	10.25	7.57	2.68
8	Six-sigma with IQR under Normal distribution	10.00	7.82	2.19

Because only five sample points fall inside the control limits and the control limit interval (CLI) for the sample size $n=5$ is 1.65 when compared to other control limits, the resulting Table-2, Figure-1, and Figure-2 demonstrate that the process is out of control based on six sigma for standard deviation.

The six-sigma control chart for standard deviation based on average run duration is more successful than the current three-sigma and other control charts, as shown by Table-3, Figure-3, and Figure-4.

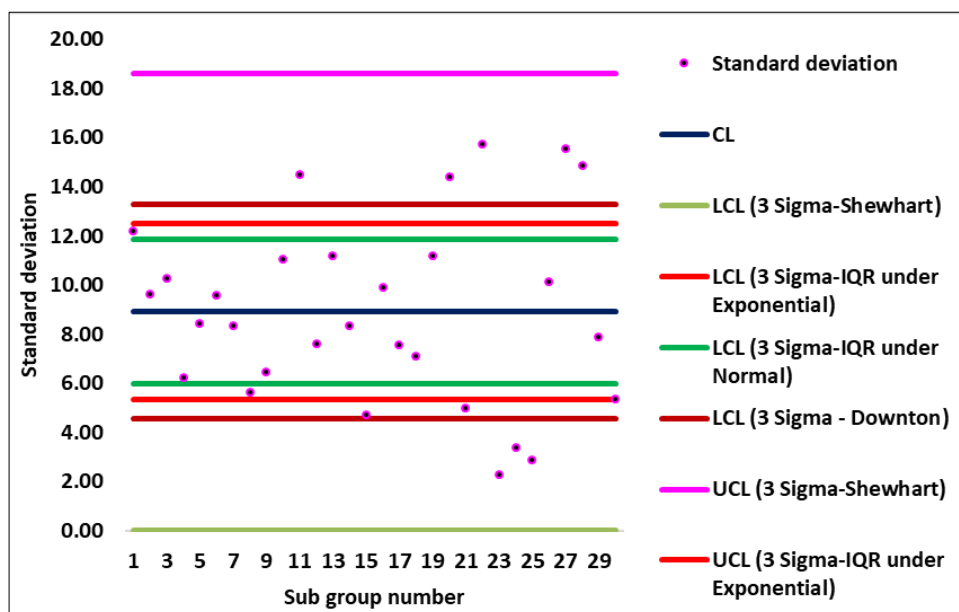


Fig 1: Three-sigma control charts for standard deviation

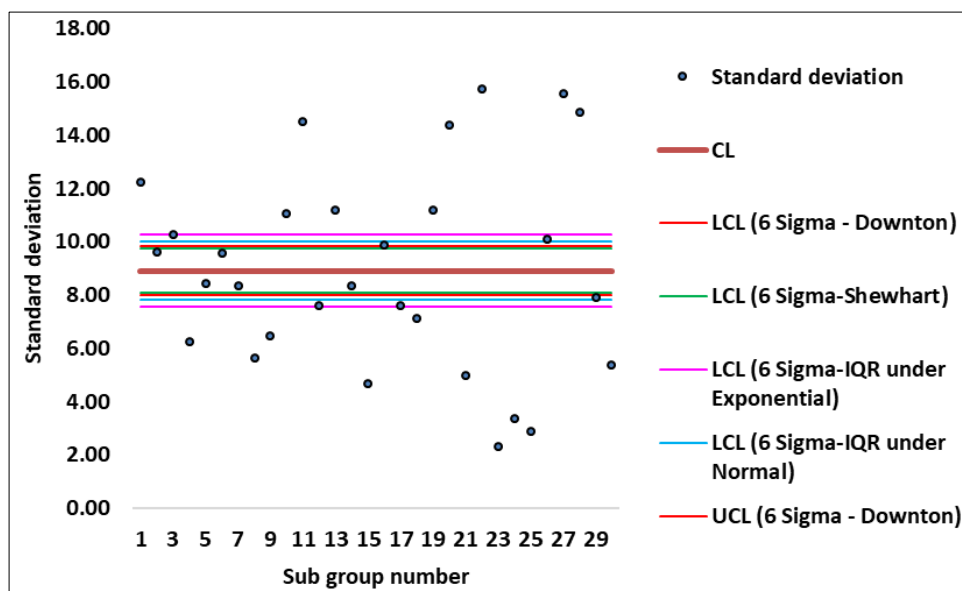


Fig 2: Six-sigma control charts for standard deviation

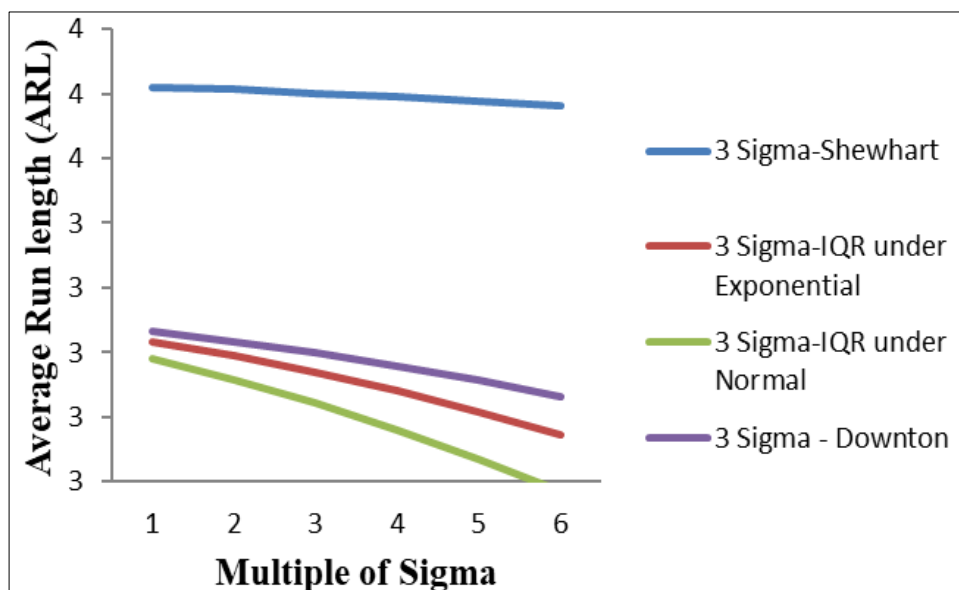


Fig 3: ARL for Three-sigma control charts for standard deviation

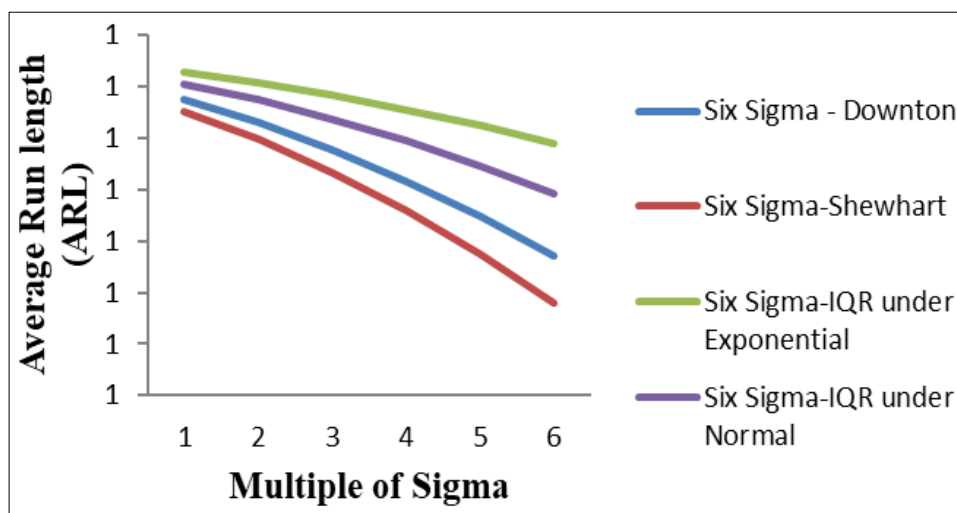


Fig 4: ARL for Six-sigma control charts for standard deviation

Table 3: ARL of three-sigma and six-sigma control charts for standard deviation

Sl. No	Multiple of σ	0.5	0.6	0.7	0.8	0.9	1.0
	Control chart						
1	Shewhart three-Sigma	3.61045	3.60613	3.60103	3.59517	3.58856	3.58120
2	Three-sigma with IQR under Exponential distribution	3.21606	3.19411	3.16864	3.13985	3.10798	3.07328
3	Three-sigma with IQR under Normal distribution	3.19076	3.15843	3.12123	3.07961	3.03405	2.98505
4	Three-sigma using Downton	3.23210	3.21688	3.19912	3.17893	3.15641	3.13169
5	Six-sigma using Downton	1.09815	1.09806	1.09796	1.09783	1.09770	1.09754
6	Six-sigma	1.09810	1.09799	1.09787	1.09772	1.09755	1.09736
7	Six-sigma with IQR under Exponential distribution	1.09826	1.09822	1.09817	1.09811	1.09805	1.09798
8	Six-sigma with IQR under Normal distribution	1.09821	1.09815	1.09807	1.09799	1.09789	1.09779

8. Conclusion

The results mentioned above demonstrate that the process is out of statistical control and that the interval between the six-sigma based control chart and the control limits of Shewhart and IQR is smaller when Shewhart 3-Sigma, IQR, and six-sigma based IQR control limits for standard deviation under normal and exponential distributions are used. It is obvious that the system or procedure has to be changed since the product or service is not up to grade.

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