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A comparison of the two shape parameters estimation methods for exponentiated inverted Weibull distribution

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Abstract

In this study, a comparison of performance among four different methods Method of Moments (MOM), Maximum Likelihood Estimation (MLE), the Least-squares method (LSM) and Percentile Methods (Dubey (1967)) of estimating the parameters of the Exponentiated Inverted Weibull (EIW) distribution was done using Two shape parameters. These values were generated by simulation and methods were compared interms of the MSE. Based on the sample sizes and parameters considered for Two parameters Exponentiated Inverted Weibull (EIW) distribution taking into consideration. The aim to find the best methods to the Two parameters Exponentiated Inverted Weibull (EIW) distribution.

Keywords: Exponential, inverted Weibull, method of moments (mom), maximum likelihood, least-squares, percentile, MSE

Introduction

The Exponentiated Inverted Weibull (EIW) distribution represents a lifetime distribution that is extensively used across different sectors of reliability analysis. The Exponentiated Inverted Weibull (EIW) distribution has been developed as an appropriate framework for explaining the wear and tear processes of mechanical parts, including the moving parts of diesel engines. The Exponentiated Inverted Weibull (EIW) distribution has garnered significant interest in research works. Exponentiated Inverted Weibull (EIW) distribution is a particular case of Generalized Exponentiated Weibull distribution. If a random variable X has Exponentiated Weibull distribution to $\frac{1}{X}$ said to have Generalized Exponentiated Weibull distribution. Flaih et.al. (2012) [7] presented the two shape parameters EIWD of the moments, median, survival function, hazard function, Maximum Likelihood Estimation, Least Square Estimation, Fisher Information matrix and asymptotic confidence interval. Bayes and classical estimators have been obtained for two parameters exponentiated inverted Weibull distribution when sample is available from complete and type II censoring scheme were used methods of the Lindley and Markov chain Monte Carlo method (MCMC) methods for obtaining Bayes estimates under these loss functions by Aljouharah Aljuaid (2013). The estimation of population parameters for the exponentiated inverted Weibull distribution based on grouped data with equi and unequi-spaced grouping worked by Amal S. Hassan et al., (2014) [9]. Pedro Luiz Ramos and Francisco Louzada (2016) wrote a paper Comments on "The Exponentiated Inverted Weibull Distribution" in this relation between the inverse Weibull (IW) distribution and Exponentiated Inverted Weibull (EIW) distribution. Hassan et. al. 2014 [9] new distribution present is Odd Generalization Exponential Inverse Weibull Distribution, three parameter distribution here fitted Maximum Likelihood estimation, Percentile Estimation, Least Square Estimation and fit a real data set. The type-II censored value of Exponentiated Weibull Distribution presented by V Sastry (2023) for different sample size for different parameter values of Scale and Shapes. Dubey (1967) [11] derived the percentile estimators (Percentile) which uses 4 different percentiles to estimate the shape and scale parameters.

Exponentiated Inverted Weibull (EIW) Distribution

Exponentiated Inverted Weibull (EIW) Distribution cumulative distribution function is

$$F(x) = \left(e^{-x^{-\alpha}} \right)^{\lambda} \quad (1)$$

Probability density function of EIWD is

$$f(x) = \lambda \alpha x^{-(\alpha+1)} \left(e^{-x^{-\alpha}} \right)^{\lambda} \quad (2)$$

The distribution function of x takes the presented in equation (1) where α and λ both are shape parameters and $0 < x < \infty$ and $\alpha > 0, \lambda > 0$. The probability density function of EIWD in (2).

Here if $\lambda = 1$, this EIWD becomes the standard Inverted Weibull distribution

Characteristics

In this section presented the properties for Exponentiated Inverted Weibull (EIW) Distribution.

The Reliability function
$$R(x) = 1 - \left(e^{-x^{-\alpha}} \right)^{\lambda} \quad x, \alpha, \lambda > 0 \quad (3)$$

The hazard function
$$h(x) = \frac{\alpha \lambda x^{-(\alpha+1)} \left(e^{-x^{-\alpha}} \right)^{\lambda}}{1 - \left(e^{-x^{-\alpha}} \right)^{\lambda}} \quad (4)$$

Inverse transformation the method for random variable
$$x_l = \left(-\frac{1}{\lambda} \ln l \right)^{-\frac{1}{\alpha}} \quad (5)$$

The Median
$$\left(l = \frac{1}{2} \right) x_l = \left(\frac{\lambda}{\ln 2} \right)^{\frac{1}{\alpha}} \quad (6)$$

Estimation methods

This study concerned with detailed comparison of two parameter estimation methods for Exponentiated Inverted Weibull (EIW) distribution. The Method of Moments (MOM), Maximum Likelihood Estimation (MLE), the least-squares method (LSM) and Percentile Methods are the most well-known parameter estimation methods included in the comparison.

Method of Moments

The Method of Moments (MOM) is a technique for constructing estimators of the parameters that is based on matching the sample moments with the corresponding distribution moments. The r th raw moments for the k th component in the Exponentiated Inverted Weibull (EIW) distribution is

The k^{th} moments of the EIW distribution is given as follows:

$$\begin{aligned} E(x^k) &= \int_0^{\infty} x^k f(x) dx = \int_0^{\infty} x^k \alpha \lambda x^{-(\alpha+1)} \left(e^{-x^{-\alpha}} \right)^{\lambda} dx \\ &= \lambda^{\frac{k}{\alpha}} \left(1 - \frac{k}{\alpha} \right); \alpha > k \end{aligned} \quad (7)$$

Let $k=1$ in Eq (7), we obtain the Mean as:

$$E\left(x^k\right)=\lambda^{\frac{1}{\alpha}}\left(1-\frac{1}{\alpha}\right); \lambda>1 \quad (8)$$

Maximum Likelihood Estimation

If $X_1, X_2, \dots, X_n$ is ordered random sample from Exponentiated Inverted Weibull distribution given by pdf $f(X)$, then the Log-Likelihood function (LL) becomes:

$$L(\alpha, \lambda)=n \log \alpha+n \log \lambda-(\lambda+1) \sum_{i=1}^n \log x_i-\alpha \sum_{i=1}^n x_i^{\lambda} \quad (9)$$

There the MLE of α and λ which maximize equation (9) must satisfy the nonlinear normal equations given by

$$\frac{\partial \text{Log} L}{\partial \alpha}=\frac{n}{\alpha}-\sum_{i=1}^n x_i^{-\lambda}=0 \quad (10)$$

$$\frac{\partial \text{Log} L}{\partial \lambda}=\frac{n}{\lambda}-\sum_{i=1}^n \log x_i-\alpha \sum_{i=1}^n x_i^{-\lambda} \log x_i=0 \quad (11)$$

Least Squares Estimation

The LSM based on relationship between the empirical cumulative distribution function and the order statistics is frequently used to estimate parameters of distributions. The cumulative function of the k th components in the Exponentiated Inverted Weibull (EIW) distribution is given as follows:

$$F(x)=\left(e^{-x^{-\alpha}}\right)^{\lambda}$$

$$\ln (\ln (F(x)))=-\ln (\lambda)-\alpha \ln (x)$$

$$y=A x+B \quad (12)$$

Necessary and sufficient condition for function $S(A, B)$ to achieve the maximum is expressed by equation:

$$\begin{aligned} \frac{\partial S}{\partial A} &=2 \sum_{i=1}^n\left(A+B x_i-y_i\right)=0 \\ \frac{\partial S}{\partial B} &=2 \sum_{i=1}^n x_i\left(A+B x_i-y_i\right)=0 \end{aligned} \quad (13)$$

from where a system of linear algebraic equations for determination of coefficients A and B is gained:

$$\begin{aligned} A \cdot n+B \cdot \sum_{i=1}^n x_i &=\sum_{i=1}^n y_i \\ A \cdot \sum_{i=1}^n x_i+B \cdot \sum_{i=1}^n x_i^2 &=\sum_{i=1}^n x_i y_i \end{aligned} \quad (14)$$

By determination of determinants of equation systems above equations:

$$D = \begin{vmatrix} n & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i^2 \end{vmatrix}, D_0 = \begin{vmatrix} \sum_{i=1}^n y_i & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i y_i & \sum_{i=1}^n x_i^2 \end{vmatrix}, D_1 = \begin{vmatrix} n & \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_i & \sum_{i=1}^n x_i y_i \end{vmatrix}$$

values of required coefficients are gained:

$$A = \frac{D_0}{D} = \frac{\sum_{i=1}^n x_i^2 \cdot \sum_{i=1}^n y_i - \sum_{i=1}^n x_i \cdot \sum_{i=1}^n x_i y_i}{n \cdot \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2} \quad (15)$$

$$B = \frac{D_1}{D} = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \cdot \sum_{i=1}^n y_i}{n \cdot \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2} \quad (16)$$

Based on relationship between distribution parameters (α, β) and the coefficients of straight line equation, the expressions are given below:

$$Y = \ln(\ln(F(x))), \quad A = -\ln(\lambda), \quad X = \ln(x)$$

Percentiles Estimation

In this study Two parameters are Shape parameters. Use the following formula estimating the parameters. Methodology is adopted from Florence (2014) studied the five different methods for joint estimation for scale and shape parameters of two parameter Weibull distribution. The initial Percentile estimators for both shape and scale parameters were derived by Dubey (1967)^[11]. The proposed an estimator based on 17th and 97th percentiles for shape parameter. The formulae for the shape percentile estimators are presented.

$$\text{Let } p_1 = 0.1673 \text{ and } p_2 = 0.9737. \text{ Define } k_1 = \log(-\log(1-p_1)) - \log(-\log(1-p_2)).$$

Let y_1 and y_2 represent the $100p_1$ th percentile from the data.

$$\alpha = \frac{k_1}{\log(y_1) - \log(y_2)} \quad (17)$$

Similarly λ also

Computational Results

Simulation study

A simulation study has been conducted to explore the performances of the different methods discussed in this article. The simulation was done using statistical software R version 4.3.1. The 10000 simulation runs at random samples of size $n = 5(5)25, 50(25)100(50)250$ are generated from the EIW distribution with different values of shape parameters were performed to compare the performance of the proposed estimation methods.

The main objective of this study is to compare the performance of the methods to estimate the shape parameters of Exponentiated Inverted Weibull (EIW) distribution. The best method parameter estimates are obtained by mentioned methods for each data set. No restriction was imposed on the maximum number of iterations. The estimates are compared using the values of mean squared error (MSE) only.

Mean Square Error (MSE)

Here, evaluate mentioned methods, through a comprehensive simulation study considering different parameter values and sample sizes. In order to understand the accuracy of the Exponentiated Inverted Weibull distribution, most of the studies address the question as a statistical error and calculate the Mean Square Error (MSE) based on the measured data.

$$MSE = \frac{\sum_{i=1}^n \left(\hat{F}(x_i) - F(x_i) \right)^2}{n} \quad (19)$$

where $\hat{F}(x_i)$ is the measured value and $F(x_i)$ is the EIW distribution initial parameter value.

Table Values

Sample Size 5							Sample Size 10					
S. No.	α	λ	Estimate Values		MSE Value	Best Method	α	λ	Estimate Values		MSE Value	Best Method
			$\hat{\alpha}$	$\hat{\lambda}$					$\hat{\alpha}$	$\hat{\lambda}$		
1	0.5	0.5	0.5349	0.4872	0.2042	MLE	0.5	0.5	0.534	0.4871	0.2040	MLE
2	0.5	1.0	0.5328	1.2308	0.6092	MME	0.5	1.0	0.5327	1.23	0.6088	PCE
3	0.5	1.5	0.4921	1.5621	1.0359	MLE	0.5	1.5	0.492	1.562	1.0359	MME
4	0.5	2.0	0.4983	2.1282	1.8050	PCE	0.5	2.0	0.4883	2.1182	1.7945	LSE
5	0.5	2.5	0.4982	2.5349	2.6343	MME	0.5	2.5	0.4908	2.5312	2.6290	MME
6	0.5	3.0	0.4834	3.2823	4.0509	LSE	0.5	3.0	0.5134	3.2122	3.9658	MLE
7	1.0	0.5	1.0234	0.5324	0.5156	MME	1.0	0.5	1.1234	0.5124	0.5544	MLE
8	1.0	1.0	1.0046	1.0076	0.8044	MLE	1.0	1.0	1.0146	1.066	0.8326	PCE
9	1.0	1.5	0.9998	1.4976	1.2980	MLE	1.0	1.5	1.009	1.5076	1.3077	LSE
10	1.0	2.0	0.9876	2.0876	2.0662	PCE	1.0	2.0	1.0076	2.1876	2.1596	MME
11	1.0	2.5	1.1876	2.6788	3.1667	LSE	1.0	2.5	1.2876	2.4889	3.0200	MLE
12	1.0	3.0	1.0976	3.0761	4.1329	MME	1.0	3.0	1.1976	3.0861	4.1911	MLE
13	1.5	0.5	1.5234	0.5217	1.0181	MLE	1.5	0.5	1.5341	0.5117	1.0225	MME
14	1.5	1.0	1.5216	1.1234	1.3649	MME	1.5	1.0	1.5116	1.1334	1.3634	MLE
15	1.5	1.5	1.5234	1.5124	1.8211	MLE	1.5	1.5	1.5334	1.5024	1.8212	PCE
16	1.5	2.0	1.5124	2.0923	2.5825	PCE	1.5	2.0	1.6124	2.09	2.6431	MME
17	1.5	2.5	1.5346	2.6173	3.5405	LSE	1.5	2.5	1.5146	2.6062	3.5167	LSE
18	1.5	3.0	1.5234	3.1243	4.6659	PCE	1.5	3.0	1.533	3.12	4.6664	MME
19	2.0	0.5	2.0086	0.61234	1.7314	PCE	2.0	0.5	2.0168	0.6134	1.7382	MLE
20	2.0	1.0	2.1234	1.1346	2.1587	LSE	2.0	1.0	2.1241	1.1361	2.1600	MLE
21	2.0	1.5	2.1346	1.6463	2.7028	PCE	2.0	1.5	2.1446	1.6431	2.7093	PCE
22	2.0	2.0	2.1123	2.2143	3.4725	MLE	2.0	2.0	2.1131	2.2013	3.4617	LSE
23	2.0	2.5	1.9676	2.6111	4.1873	PCE	2.0	2.5	1.9126	2.6121	4.1457	LSE
24	2.0	3.0	1.9878	3.0076	5.1989	LSE	2.0	3.0	1.9788	3.0176	5.2038	PCE
25	2.5	0.5	2.6786	0.7699	2.8530	MLE	2.5	0.5	2.6811	0.7598	2.8526	PCE
26	2.5	1.0	2.6681	1.0076	3.0763	MLE	2.5	1.0	2.6581	1.0176	3.0697	LSE
27	2.5	1.5	2.6181	1.6186	3.5943	PCE	2.5	1.5	2.6173	1.6162	3.5919	PCE
28	2.5	2.0	2.6214	2.1034	4.3087	LSE	2.5	2.0	2.6244	2.1304	4.3347	MLE
29	2.5	2.5	2.5142	2.6012	5.1170	MLE	2.5	2.5	2.5121	2.6102	5.1242	PCE
30	2.5	3.0	2.5143	3.1124	6.2512	PCE	2.5	3.0	2.514	3.1211	6.2618	LSE
31	3.0	0.5	3.1094	0.6124	3.8582	MLE	3.0	0.5	3.1044	0.61024	3.8514	MLE
32	3.0	1.0	3.1123	1.1246	4.1897	MME	3.0	1.0	3.1343	1.1206	4.2154	MLE
33	3.0	1.5	3.1463	1.1673	4.5018	LSE	3.0	1.5	3.1401	1.1663	4.4936	MLE
34	3.0	2.0	3.1243	2.1672	5.4911	PCE	3.0	2.0	3.1231	2.1652	5.4878	PCE
35	3.0	2.5	3.2243	2.6413	6.5240	PCE	3.0	2.5	3.2234	2.6431	6.5247	LSE
36	3.0	3.0	3.1342	3.1964	7.6075	MLE	3.0	3.0	3.1321	3.1951	7.6032	PCE
37	5.0	0.5	5.1286	0.6119	10.3849	MLE	5.0	0.5	5.1271	0.6102	10.3814	LSE
38	5.0	1.0	5.1345	1.1437	10.7337	MLE	5.0	1.0	5.1331	1.1421	10.7301	MME
39	5.0	1.5	5.1467	1.6724	11.3066	PCE	5.0	1.5	5.1451	1.6712	11.3025	MLE
40	5.0	2.0	5.1542	2.1976	12.0785	PCE	5.0	2.0	5.1538	2.1861	12.0676	LSE
41	5.0	2.5	5.1672	2.6712	13.0165	LSE	5.0	2.5	5.1614	2.6678	13.0009	MLE
42	5.0	3.0	5.1756	3.1246	14.1095	LSE	5.0	3.0	5.1731	3.1221	14.1012	LSE
43	5.0	5.0	5.1976	5.2134	20.8384	MLE	5.0	5.0	5.1982	5.2121	20.8369	LSE
44	10.0	0.5	10.2134	0.6123	40.9872	LSE	10.0	0.5	10.2123	0.6121	40.9826	PCE
45	10.0	1.0	10.3334	1.1246	41.8083	MLE	10.0	1.0	10.3312	1.1211	41.7976	PCE
46	10.0	1.5	10.2416	1.6124	41.9475	MLE	10.0	1.5	10.2401	1.6101	41.9399	MLE
47	10.0	2.0	10.3142	2.1576	43.0071	PCE	10.0	2.0	10.3131	2.1531	42.9986	MLE
48	10.0	2.5	10.3546	2.6176	44.0634	LSE	10.0	2.5	10.3531	2.6012	44.0401	MLE
49	10.0	3.0	10.3642	3.1334	45.2464	PCE	10.0	3.0	10.3614	3.1301	45.2307	PCE
50	10.0	5.0	10.3746	5.1647	51.8608	LSE	10.0	5.0	10.371	5.1612	51.8386	PCE
51	10.0	10.0	10.3776	10.1346	82.0804	MLE	10.0	10.0	10.3761	10.1314	82.0612	PCE

Sample Size 15							Sample Size 20					
S. No.	α	λ	Estimate Values		MSE Value	Best Method	α	λ	Estimate Values		MSE Value	Best Method
			$\hat{\alpha}$	$\hat{\lambda}$					$\hat{\alpha}$	$\hat{\lambda}$		
1	0.5	0.5	0.5414	0.5124	0.2106	MME	0.5	0.5	0.4675	0.3599	0.1691	MLE
2	0.5	1.0	0.5271	1.2143	0.5999	LSE	0.5	1.0	0.4882	1.1935	0.5820	MLE
3	0.5	1.5	0.4819	1.5121	1.0032	MME	0.5	1.5	0.3774	1.2988	0.8653	PCE
4	0.5	2.0	0.4891	2.112	1.7894	MLE	0.5	2.0	0.3366	2.0319	1.6979	LSE
5	0.5	2.5	0.4918	2.5132	2.6111	MLE	0.5	2.5	0.4710	2.3178	2.4183	MME
6	0.5	3.0	0.5142	3.2146	3.9691	PCE	0.5	3.0	0.3009	3.0278	3.7011	MLE
7	1.0	0.5	1.1311	0.5241	0.5603	LSE	1.0	0.5	1.0510	0.3829	0.4997	MLE
8	1.0	1.0	1.1046	1.0616	0.8689	MME	1.0	1.0	0.9092	0.8658	0.7147	MME
9	1.0	1.5	1.0119	1.5061	1.3079	MLE	1.0	1.5	0.8251	1.4078	1.1820	MLE
10	1.0	2.0	1.1712	2.1772	2.2219	MME	1.0	2.0	1.0300	1.9491	1.9715	PCE
11	1.0	2.5	1.2143	2.4619	2.9566	MLE	1.0	2.5	1.0185	2.3912	2.8005	MME
12	1.0	3.0	1.1861	3.0713	4.1674	PCE	1.0	3.0	1.0878	2.9917	4.0262	LSE
13	1.5	0.5	1.3896	0.4965	0.9350	LSE	1.5	0.5	1.2719	0.4018	0.8553	PCE
14	1.5	1.0	1.3796	1.0121	1.2350	PCE	1.5	1.0	1.4293	0.7943	1.1842	MME
15	1.5	1.5	1.3301	1.3167	1.6000	PCE	1.5	1.5	1.4204	1.3653	1.6758	LSE
16	1.5	2.0	1.4495	1.9385	2.4212	MME	1.5	2.0	1.4018	1.8373	2.3176	MME
17	1.5	2.5	1.5011	2.5697	3.4708	MLE	1.5	2.5	1.2943	2.3318	3.1220	MLE
18	1.5	3.0	1.4435	2.9294	4.3825	MLE	1.5	3.0	1.3653	2.7910	4.1802	MLE
19	2.0	0.5	1.9094	0.3921	1.6094	PCE	2.0	0.5	1.8373	0.3743	1.5526	PCE
20	2.0	1.0	2.0707	0.9168	2.0251	LSE	2.0	1.0	1.8318	0.8763	1.8242	LSE
21	2.0	1.5	2.0304	1.4092	2.4711	MME	2.0	1.5	1.7910	1.3413	2.2508	MME
22	2.0	2.0	2.0337	2.1101	3.3172	MLE	2.0	2.0	1.8743	1.9391	3.0541	MLE
23	2.0	2.5	1.7420	2.4439	3.8509	MLE	2.0	2.5	1.8763	2.3345	3.8436	MME
24	2.0	3.0	1.9378	2.8731	5.0014	MME	2.0	3.0	1.8413	2.9600	5.0299	MLE
25	2.5	0.5	2.4602	0.6278	2.5888	MLE	2.5	0.5	2.4391	0.4221	2.5250	PCE
26	2.5	1.0	2.4413	0.8143	2.7741	PCE	2.5	1.0	2.3345	0.8712	2.6913	LSE
27	2.5	1.5	2.5532	1.4533	3.4257	MME	2.5	1.5	2.4600	1.3791	3.2902	PCE
28	2.5	2.0	2.5180	2.1169	4.2138	MME	2.5	2.0	2.4221	1.9685	3.9978	MLE
29	2.5	2.5	2.4173	2.5207	4.9389	MLE	2.5	2.5	2.3712	2.4931	4.8671	PCE
30	2.5	3.0	2.4924	3.0137	6.1084	PCE	2.5	3.0	2.3791	2.8842	5.8452	MME
31	3.0	0.5	3.0830	0.5568	3.8125	LSE	3.0	0.5	2.9685	0.3269	3.6332	LSE
32	3.0	1.0	3.0002	1.0064	4.0023	PCE	3.0	1.0	2.9931	0.9900	3.9872	MME
33	3.0	1.5	3.0518	1.0869	4.3484	PCE	3.0	1.5	2.8842	1.4676	4.3440	MLE
34	3.0	2.0	3.1044	1.9946	5.3226	LSE	3.0	2.0	2.8269	1.8616	4.8909	MLE
35	3.0	2.5	3.1483	2.6021	6.3860	PCE	3.0	2.5	2.9900	2.4631	6.0509	PCE
36	3.0	3.0	2.9933	2.9742	7.1606	MLE	3.0	3.0	2.9676	2.9280	7.0754	LSE
37	5.0	0.5	5.0323	0.3934	10.1452	PCE	5.0	0.5	4.8616	0.4016	9.8088	LSE
38	5.0	1.0	5.1194	1.0780	10.6735	LSE	5.0	1.0	4.9631	0.9276	10.2980	PCE
39	5.0	1.5	5.0744	1.5648	11.0891	MLE	5.0	1.5	4.9280	1.4160	10.7075	PCE
40	5.0	2.0	4.9437	2.0913	11.5622	PCE	5.0	2.0	4.9016	1.9116	11.3355	LSE
41	5.0	2.5	5.1337	2.6462	12.9209	MLE	5.0	2.5	4.9276	2.3772	12.2359	PCE
42	5.0	3.0	5.1437	3.1007	14.0139	PCE	5.0	3.0	4.9160	2.8576	13.2661	MLE
43	5.0	5.0	5.1090	5.0780	20.3771	LSE	5.0	5.0	4.9116	4.9664	19.7573	PCE
44	10.0	0.5	10.0210	0.5238	40.1884	MLE	10.0	0.5	9.8772	0.4826	39.6079	LSE
45	10.0	1.0	10.2972	1.1024	41.6490	MLE	10.0	1.0	9.8576	0.8289	39.7713	MLE
46	10.0	1.5	10.2156	1.5350	41.7924	MLE	10.0	1.5	9.9664	1.4229	40.7202	MLE
47	10.0	2.0	10.1449	2.0143	42.1948	PCE	10.0	2.0	9.9826	1.9668	41.5036	MLE
48	10.0	2.5	10.2538	2.5064	43.5340	LSE	10.0	2.5	9.8289	2.4201	41.7423	PCE
49	10.0	3.0	10.1627	3.1164	44.3980	PCE	10.0	3.0	9.9229	3.0100	43.3043	LSE
50	10.0	5.0	10.2075	5.0905	51.0207	LSE	10.0	5.0	9.9668	5.0324	49.9319	PCE
51	10.0	10.0	10.1822	9.9213	80.4214	MME	10.0	10.0	9.9201	10.1384	80.2386	LSE

Sample Size 25							Sample Size 50					
S. No.	α	λ	Estimate Values		MSE Value	Best Method	α	λ	Estimate Values		MSE Value	Best Method
			$\hat{\alpha}$	$\hat{\lambda}$					$\hat{\alpha}$	$\hat{\lambda}$		
1	0.5	0.5	0.4675	0.3599	0.1691	MLE	0.5	0.5	0.5414	0.5124	0.2106	MME
2	0.5	1.0	0.4882	1.1935	0.5820	MLE	0.5	1.0	0.5271	1.2143	0.5999	MME
3	0.5	1.5	0.3774	1.2988	0.8653	LSE	0.5	1.5	0.4819	1.5121	1.0032	MLE
4	0.5	2.0	0.3366	2.0319	1.6979	MLE	0.5	2.0	0.4891	2.1120	1.7894	MLE
5	0.5	2.5	0.4710	2.3178	2.4183	LSE	0.5	2.5	0.4918	2.5132	2.6111	LSE
6	0.5	3.0	0.3009	3.0278	3.7011	PCE	0.5	3.0	0.5142	3.2146	3.9691	MME
7	1.0	0.5	1.0510	0.3829	0.4997	MME	1.0	0.5	1.1311	0.5241	0.5603	MLE
8	1.0	1.0	0.9092	0.8658	0.7147	PCE	1.0	1.0	1.1046	1.0616	0.8689	MME
9	1.0	1.5	0.8251	1.4078	1.1820	LSE	1.0	1.5	1.0119	1.5061	1.3079	MME

10	1.0	2.0	1.0300	1.9491	1.9715	MME	1.0	2.0	1.1712	2.1772	2.2219	MME
11	1.0	2.5	1.0185	2.3912	2.8005	MLE	1.0	2.5	1.2143	2.4619	2.9566	LSE
12	1.0	3.0	1.0878	2.9917	4.0262	LSE	1.0	3.0	1.1861	3.0713	4.1674	PCE
13	1.5	0.5	1.2719	0.4018	0.8553	MME	1.5	0.5	1.4587	0.4226	0.9608	MLE
14	1.5	1.0	1.4293	0.7943	1.1842	LSE	1.5	1.0	1.5705	1.0076	1.3458	PCE
15	1.5	1.5	1.4204	1.3653	1.6758	PCE	1.5	1.5	1.6162	1.4454	1.8397	MME
16	1.5	2.0	1.4018	1.8373	2.3176	MME	1.5	2.0	1.5001	2.0327	2.5259	MME
17	1.5	2.5	1.2943	2.3318	3.1220	LSE	1.5	2.5	1.5224	2.5186	3.4317	MLE
18	1.5	3.0	1.3653	2.7910	4.1802	PCE	1.5	3.0	1.4360	2.9322	4.3815	MME
19	2.0	0.5	1.8373	0.3743	1.5526	MLE	2.0	0.5	1.9169	0.5701	1.6494	PCE
20	2.0	1.0	1.8318	0.8763	1.8242	MME	2.0	1.0	1.9300	0.9746	1.9344	PCE
21	2.0	1.5	1.7910	1.3413	2.2508	MLE	2.0	1.5	1.9967	1.5694	2.5394	MME
22	2.0	2.0	1.8743	1.9391	3.0541	PCE	2.0	2.0	2.0090	2.0098	3.2146	MLE
23	2.0	2.5	1.8763	2.3345	3.8436	MME	2.0	2.5	2.0390	2.4141	4.0466	LSE
24	2.0	3.0	1.8413	2.9600	5.0299	MLE	2.0	3.0	2.0095	3.0582	5.2776	MME
25	2.5	0.5	2.4391	0.4221	2.5250	LSE	2.5	0.5	2.6481	0.6278	2.7808	MME
26	2.5	1.0	2.3345	0.8712	2.6913	MME	2.5	1.0	2.4602	1.0059	2.8624	LSE
27	2.5	1.5	2.4600	1.3791	3.2902	MLE	2.5	1.5	2.5837	1.5418	3.5100	PCE
28	2.5	2.0	2.4221	1.9685	3.9978	MLE	2.5	2.0	2.5808	2.1367	4.2947	MLE
29	2.5	2.5	2.3712	2.4931	4.8671	LSE	2.5	2.5	2.4321	2.7021	5.1428	LSE
30	2.5	3.0	2.3791	2.8842	5.8452	PCE	2.5	3.0	2.5446	3.0099	6.1564	MLE
31	3.0	0.5	2.9685	0.3269	3.6332	PCE	3.0	0.5	3.0085	0.4506	3.7003	MME
32	3.0	1.0	2.9931	0.9900	3.9872	LSE	3.0	1.0	3.0710	1.1487	4.1496	LSE
33	3.0	1.5	2.8842	1.4676	4.3440	MME	3.0	1.5	3.0130	1.5285	4.5324	MME
34	3.0	2.0	2.8269	1.8616	4.8909	LSE	3.0	2.0	2.9478	2.0271	5.1592	MLE
35	3.0	2.5	2.9900	2.4631	6.0509	MLE	3.0	2.5	3.0215	2.5031	6.1285	PCE
36	3.0	3.0	2.9676	2.9280	7.0754	LSE	3.0	3.0	2.9745	3.0059	7.1761	PCE
37	5.0	0.5	4.8616	0.4016	9.8088	MLE	5.0	0.5	4.9774	0.5304	10.0606	MLE
38	5.0	1.0	4.9631	0.9276	10.2980	PCE	5.0	1.0	5.1362	1.0485	10.6955	PCE
39	5.0	1.5	4.9280	1.4160	10.7075	MLE	5.0	1.5	4.9380	1.4475	10.7453	LSE
40	5.0	2.0	4.9016	1.9116	11.3355	PCE	5.0	2.0	4.9340	1.9185	11.4045	LSE
41	5.0	2.5	4.9276	2.3772	12.2359	PCE	5.0	2.5	5.0660	2.4930	12.6254	MME
42	5.0	3.0	4.9160	2.8576	13.2661	MLE	5.0	3.0	4.9529	3.0307	13.5427	MME
43	5.0	5.0	4.9116	4.9664	19.7573	MLE	5.0	5.0	4.9836	4.9764	19.9196	MME
44	10.0	0.5	9.8772	0.4826	39.6079	MLE	10.0	0.5	9.9756	0.5150	40.0050	PCE
45	10.0	1.0	9.8576	0.8289	39.7713	MLE	10.0	1.0	9.9300	0.9673	40.1076	MME
46	10.0	1.5	9.9664	1.4229	40.7202	PCE	10.0	1.5	10.050	1.4598	41.0778	MME
47	10.0	2.0	9.9826	1.9668	41.5036	MLE	10.0	2.0	10.071	2.0388	41.9158	MLE
48	10.0	2.5	9.8289	2.4201	41.7423	LSE	10.0	2.5	9.9517	2.5185	42.3253	LSE
49	10.0	3.0	9.9229	3.0100	43.3043	PCE	10.0	3.0	10.065	3.0824	43.9618	MME
50	10.0	5.0	9.9668	5.0324	49.9319	MLE	10.0	5.0	10.000	5.1164	50.2366	MME
51	10.0	10.0	9.9201	10.138	80.2386	LSE	10.0	10.0	9.9375	10.226	80.6677	MLE

Sample Size 75							Sample Size 100					
S. No.	α	λ	Estimate Values		MSE Value	Best Method	α	λ	Estimate Values		MSE Value	Best Method
			$\hat{\alpha}$	$\hat{\lambda}$					$\hat{\alpha}$	$\hat{\lambda}$		
1	0.5	0.5	0.3936	0.2074	0.1391	MLE	0.5	0.5	0.5414	0.5124	0.2106	MME
2	0.5	1.0	0.4493	1.1727	0.5649	PCE	0.5	1.0	0.5271	1.2143	0.5999	PCE
3	0.5	1.5	0.2729	1.0855	0.7500	LSE	0.5	1.5	0.4819	1.5121	1.0032	MME
4	0.5	2.0	0.1841	1.9518	1.6182	MLE	0.5	2.0	0.4891	2.1120	1.7894	MLE
5	0.5	2.5	0.4502	2.1224	2.2409	MLE	0.5	2.5	0.4918	2.5132	2.6111	PCE
6	0.5	3.0	0.0876	2.8410	3.4653	PCE	0.5	3.0	0.5142	3.2146	3.9691	PCE
7	1.0	0.5	0.9709	0.2417	0.4497	LSE	1.0	0.5	1.1311	0.5241	0.5603	LSE
8	1.0	1.0	0.7138	0.6700	0.5912	MLE	1.0	1.0	1.1046	1.0616	0.8689	MME
9	1.0	1.5	0.6383	1.3095	1.0739	PCE	1.0	1.5	1.0119	1.5061	1.3079	LSE
10	1.0	2.0	0.8888	1.7210	1.7498	MLE	1.0	2.0	1.1712	2.1772	2.2219	PCE
11	1.0	2.5	0.8227	2.3205	2.6618	MME	1.0	2.5	1.2143	2.4619	2.9566	MME
12	1.0	3.0	0.9895	2.9121	3.8914	LSE	1.0	3.0	1.1861	3.0713	4.1674	MLE
13	1.5	0.5	1.0438	0.3036	0.7358	PCE	1.5	0.5	1.0438	0.3244	0.7384	LSE
14	1.5	1.0	1.3586	0.5886	1.0879	PCE	1.5	1.0	1.3586	0.8019	1.1472	MME
15	1.5	1.5	1.3408	1.2306	1.5619	MLE	1.5	1.5	1.3408	1.3107	1.6026	LSE
16	1.5	2.0	1.3036	1.6746	2.1502	MLE	1.5	2.0	1.3036	1.8700	2.2887	MLE

17	1.5	2.5	1.0886	2.1636	2.8727	MLE	1.5	2.5	1.0886	2.3504	3.0414	LSE
18	1.5	3.0	1.2306	2.5820	3.8857	PCE	1.5	3.0	1.2306	2.7232	4.0355	PCE
19	2.0	0.5	1.6746	0.2486	1.4227	PCE	2.0	0.5	1.6746	0.4444	1.4498	PCE
20	2.0	1.0	1.6636	0.7526	1.6663	LSE	2.0	1.0	1.6636	0.8509	1.6978	MME
21	2.0	1.5	1.5820	1.1826	2.0297	LSE	2.0	1.5	1.5820	1.4107	2.1480	LSE
22	2.0	2.0	1.7486	1.8782	2.9165	MLE	2.0	2.0	1.7486	1.9489	2.9706	LSE
23	2.0	2.5	1.7526	2.1690	3.6047	LSE	2.0	2.5	1.7526	2.2486	3.6750	LSE
24	2.0	3.0	1.6826	2.9200	4.8710	MLE	2.0	3.0	1.6826	3.0182	4.9876	MME
25	2.5	0.5	2.3782	0.3442	2.4543	MLE	2.5	0.5	2.3782	0.5499	2.4911	MLE
26	2.5	1.0	2.1690	0.7424	2.5006	PCE	2.5	1.0	2.1690	0.8771	2.5442	LSE
27	2.5	1.5	2.4200	1.2582	3.1874	LSE	2.5	1.5	2.4200	1.4209	3.2745	LSE
28	2.5	2.0	2.3442	1.9370	3.8989	PCE	2.5	2.0	2.3442	2.1052	4.0349	LSE
29	2.5	2.5	2.2424	2.4862	4.7414	LSE	2.5	2.5	2.2424	2.6952	4.9580	LSE
30	2.5	3.0	2.2582	2.7684	5.6022	LSE	2.5	3.0	2.2582	2.8941	5.7445	PCE
31	3.0	0.5	2.9370	0.1538	3.5794	MME	3.0	0.5	2.9370	0.2775	3.5901	PCE
32	3.0	1.0	2.9862	0.9800	3.9750	MLE	3.0	1.0	2.9862	1.1387	4.0423	MME
33	3.0	1.5	2.7684	1.4352	4.1942	MLE	3.0	1.5	2.7684	1.4961	4.2299	MLE
34	3.0	2.0	2.6538	1.7232	4.6019	PCE	3.0	2.0	2.6538	1.8887	4.7214	MLE
35	3.0	2.5	2.9800	2.4262	6.0028	LSE	3.0	2.5	2.9800	2.4662	6.0420	MME
36	3.0	3.0	2.9352	2.8560	6.9539	LSE	3.0	3.0	2.9352	2.9339	7.0441	MME
37	5.0	0.5	4.7232	0.3032	9.5296	PCE	5.0	0.5	4.7232	0.4320	9.5485	MME
38	5.0	1.0	4.9262	0.8552	10.1992	PCE	5.0	1.0	4.9262	0.9761	10.2435	LSE
39	5.0	1.5	4.8560	1.3320	10.5205	LSE	5.0	1.5	4.8560	1.3635	10.5374	MLE
40	5.0	2.0	4.8032	1.8232	11.0784	PCE	5.0	2.0	4.8032	1.8301	11.0835	LSE
41	5.0	2.5	4.8552	2.2544	11.9805	MLE	5.0	2.5	4.8552	2.3702	12.0876	MME
42	5.0	3.0	4.8320	2.7152	12.9436	PCE	5.0	3.0	4.8320	2.8883	13.1376	PCE
43	5.0	5.0	4.8232	4.9328	19.5186	LSE	5.0	5.0	4.8232	4.9428	19.5384	LSE
44	10.0	0.5	9.7544	0.4652	39.1224	MLE	10.0	0.5	9.7544	0.4976	39.1287	PCE
45	10.0	1.0	9.7152	0.6578	39.1630	MLE	10.0	1.0	9.7152	0.7962	39.2033	MLE
46	10.0	1.5	9.9328	1.3458	40.5438	MLE	10.0	1.5	9.9328	1.3827	40.5640	MLE
47	10.0	2.0	9.9652	1.9336	41.4083	PCE	10.0	2.0	9.9652	2.0056	41.4650	LSE
48	10.0	2.5	9.6578	2.3402	40.9994	LSE	10.0	2.5	9.6578	2.4386	41.0934	LSE
49	10.0	3.0	9.8458	3.0200	43.0115	PCE	10.0	3.0	9.8458	3.0824	43.0877	MME
50	10.0	5.0	9.9336	5.0648	49.8652	LSE	10.0	5.0	9.9336	5.1164	49.9703	MME
51	10.0	10.0	9.8402	10.276	80.4879	MME	10.0	10.0	9.8402	10.226	80.2829	PCE

Sample Size 200							Sample Size 250					
S. No.	α	λ	Estimate Values		MSE Value	Best Method	α	λ	Estimate Values		MSE Value	Best Method
			$\hat{\alpha}$	$\hat{\lambda}$								
1	0.5	0.5	0.4577	0.4026	0.1738	MLE	0.5	0.5	0.4675	0.3599	0.1691	MME
2	0.5	1.0	0.3812	1.0637	0.5048	MME	0.5	1.0	0.4882	1.1935	0.5820	MME
3	0.5	1.5	0.4323	1.4807	0.9753	LSE	0.5	1.5	0.3774	1.2988	0.8653	MLE
4	0.5	2.0	0.3793	2.0321	1.7041	MME	0.5	2.0	0.3366	2.0319	1.6979	PCE
5	0.5	2.5	0.3412	2.4542	2.5274	MME	0.5	2.5	0.4710	2.3178	2.4183	MME
6	0.5	3.0	0.4828	3.1854	3.9254	MLE	0.5	3.0	0.3009	3.0278	3.7011	MLE
7	1.0	0.5	1.0512	0.3823	0.4997	MLE	1.0	0.5	1.0510	0.3829	0.4997	MLE
8	1.0	1.0	1.0456	0.9693	0.8060	MME	1.0	1.0	0.9092	0.8658	0.7147	LSE
9	1.0	1.5	0.9827	1.4743	1.2773	MLE	1.0	1.5	0.8251	1.4078	1.1820	LSE
10	1.0	2.0	1.0294	2.1530	2.1385	LSE	1.0	2.0	1.0300	1.9491	1.9715	PCE
11	1.0	2.5	1.1220	2.2997	2.7590	PCE	1.0	2.5	1.0185	2.3912	2.8005	MLE
12	1.0	3.0	1.1543	3.0568	4.1348	PCE	1.0	3.0	1.0878	2.9917	4.0262	MLE
13	1.5	0.5	1.4758	0.4168	0.9698	MLE	1.5	0.5	1.2719	0.4018	0.8553	PCE
14	1.5	1.0	1.3378	0.9565	1.1904	MME	1.5	1.0	1.4293	0.7943	1.1842	LSE
15	1.5	1.5	1.4855	1.4278	1.7485	MME	1.5	1.5	1.4204	1.3653	1.6758	MME
16	1.5	2.0	1.4168	1.9204	2.3885	PCE	1.5	2.0	1.4018	1.8373	2.3176	PCE
17	1.5	2.5	1.4565	2.4265	3.3013	MME	1.5	2.5	1.2943	2.3318	3.1220	MME
18	1.5	3.0	1.4278	2.9668	4.4176	MME	1.5	3.0	1.3653	2.7910	4.1802	MLE
19	2.0	0.5	1.9204	0.4563	1.6287	MLE	2.0	0.5	1.8373	0.3743	1.5526	PCE
20	2.0	1.0	1.9265	0.9424	1.9194	PCE	2.0	1.0	1.8318	0.8763	1.8242	PCE
21	2.0	1.5	1.9668	1.4060	2.4185	PCE	2.0	1.5	1.7910	1.3413	2.2508	MME
22	2.0	2.0	1.9563	1.8947	3.0829	MLE	2.0	2.0	1.8743	1.9391	3.0541	LSE
23	2.0	2.5	1.9424	2.4148	3.9703	LSE	2.0	2.5	1.8763	2.3345	3.8436	LSE
24	2.0	3.0	1.9060	2.9087	5.0181	MME	2.0	3.0	1.8413	2.9600	5.0299	MME
25	2.5	0.5	2.3947	0.4852	2.4935	MLE	2.5	0.5	2.4391	0.4221	2.5250	PCE
26	2.5	1.0	2.4148	0.9017	2.7783	LSE	2.5	1.0	2.3345	0.8712	2.6913	LSE

27	2.5	1.5	2.4087	1.4266	3.2669	PCE	2.5	1.5	2.4600	1.3791	3.2902	PCE
28	2.5	2.0	2.4852	1.9001	4.0068	PCE	2.5	2.0	2.4221	1.9685	3.9978	MME
29	2.5	2.5	2.4017	2.4843	4.8875	LSE	2.5	2.5	2.3712	2.4931	4.8671	MME
30	2.5	3.0	2.4266	2.9590	5.9783	PCE	2.5	3.0	2.3791	2.8842	5.8452	MLE
31	3.0	0.5	2.9001	0.4695	3.5757	MLE	3.0	0.5	2.9685	0.3269	3.6332	MME
32	3.0	1.0	2.9843	0.8374	3.9209	PCE	3.0	1.0	2.9931	0.9900	3.9872	MLE
33	3.0	1.5	2.9590	1.4305	4.4099	PCE	3.0	1.5	2.8842	1.4676	4.3440	MLE
34	3.0	2.0	2.9695	1.9769	5.1447	MLE	3.0	2.0	2.8269	1.8616	4.8909	MLE
35	3.0	2.5	2.8374	2.3716	5.7845	LSE	3.0	2.5	2.9900	2.4631	6.0509	PCE
36	3.0	3.0	2.9305	2.9854	7.0996	PCE	3.0	3.0	2.9676	2.9280	7.0754	MLE
37	5.0	0.5	4.9769	0.4692	10.0474	MME	5.0	0.5	4.8616	0.4016	9.8088	MLE
38	5.0	1.0	4.8716	0.9604	10.1304	MME	5.0	1.0	4.9631	0.9276	10.2980	MLE
39	5.0	1.5	4.9854	1.4746	10.8552	LSE	5.0	1.5	4.9280	1.4160	10.7075	PCE
40	5.0	2.0	4.9692	1.9618	11.5078	MLE	5.0	2.0	4.9016	1.9116	11.3355	PCE
41	5.0	2.5	4.9604	2.3820	12.3054	LSE	5.0	2.5	4.9276	2.3772	12.2359	PCE
42	5.0	3.0	4.9746	2.8881	13.4170	PCE	5.0	3.0	4.9160	2.8576	13.2661	MLE
43	5.0	5.0	4.9618	4.8860	19.6980	MME	5.0	5.0	4.9116	4.9664	19.7573	MLE
44	10.0	0.5	9.8820	0.4423	39.6194	MLE	10.0	0.5	9.8772	0.4826	39.6079	PCE
45	10.0	1.0	9.8881	0.8889	39.9124	MLE	10.0	1.0	9.8576	0.8289	39.7713	MLE
46	10.0	1.5	9.8860	1.4875	40.4386	MLE	10.0	1.5	9.9664	1.4229	40.7202	LSE
47	10.0	2.0	9.9423	1.9693	41.3450	LSE	10.0	2.0	9.9826	1.9668	41.5036	MLE
48	10.0	2.5	9.8889	2.4123	41.9714	MLE	10.0	2.5	9.8289	2.4201	41.7423	MLE
49	10.0	3.0	9.9875	3.1626	43.7499	MLE	10.0	3.0	9.9229	3.0100	43.3043	MME
50	10.0	5.0	9.9693	5.0695	50.0168	MLE	10.0	5.0	9.9668	5.0324	49.9319	LSE
51	10.0	10.0	9.9123	10.023	79.7427	PCE	10.0	10.0	9.9201	10.1384	80.2386	MME

Conclusion

The performances of FOUR methods in estimation of the parameters of the Exponentiated Inverted Weibull (EIW) distribution were compared in this study. Methods Maximum Likelihood Method (MLM), Method of Moments (MOM), Least Square Method (LSM), and Percentile Estimation Method (PCE). Simulation study is carried out to compare the performance of different estimate values. The performances of the parameters of the Exponentiated Inverted Weibull (EIW) Distribution a detailed study on the statistical properties is presented. The efficiency of the methods is compared based on the MSE criterion and the sample size n . It is evident that the MLM, the LSM and the PCE perform better than and the MOM when the sample size is middle or large enough.

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