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Dr. Kawa M Jamal Rashid
Department of Statistics and
Informatics, College of
Administration and Economics,
University of Sylaimanyah, Iraq

Determining the approximate value of MD-Mahalanobis-Taguchi-Gram-Schmidt process by (K-MTGS) process

Kawa M Jamal Rashid

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Abstract

The main philosophy of MD calculation is a measurement scale used to discriminate and interpret the relationships among data samples (abnormal vs normal).

Mahalanobis distance is the multivariate generalization of finding how many standard deviations away a point is from the mean of the multivariate distribution.

Taguchi introduced the Mahalanobis Distance into the process of defining a reference group and measuring individual subsets.

Mahalanobis-Taguchi System MTS or Mahalanobis-Taguchi-Gram-Schmidt process (MTGS) is an alternative way of evaluating MD with the help of the Gram Schmidt orthogonalization process (GSP) is to develop and optimize a diagnostic system with a measurement scale of abnormality and for casting using multivariate data.

This paper evaluates the value of Mahalanobis Distance MD, on some different random sample sizes by using the Mahalanobis-Taguchi-Gram-Schmidt process (MTGS) and another new modification way to evaluating MD called kawa-MTGS (k-MTGS) methods then comparing the result with other.

Keywords: Mahalanobis Distance, Mahalanobis-Taguchi System, Mahalanobis-Taguchi-Gram-Schmidt process Kawa-Mahalanobis-Taguchi-Gram-Schmidt process

Introduction

Mahalanobis Distance MD used to find the “nearness” of unknown points from the mean point of a group. Mahalanobis Space (MS) is a database containing the means, standard deviations, and correlation structure of the variables in the reference group which has average unit distance.

Taguchi Methods is used the system not only for diagnosis but also for forecasting/prediction of occurrence of particular events systems also used to develop and optimize a system of multivariable diagnosis, pattern recognition.

Another alternative way of MTS to evaluating MD with the help of the Gram-Schmidt orthogonalization process (GSP) is MTGS.

Method (MTS and MTGS) have the same result, but there are a different steps to evaluate the MD value. And MS has the zero point at its mean with an average MD equal to unity ^[2, 4, 6].

The new modification method denoted by K-MTGS is suggested to calculate (MD).

In this paper evaluating the value of Mahalanobis Distance MD to compare the result of two methods, K-MTGS with (MTGS and MTS) in order to know the efficiency of this method, therefor evaluating MD by two different methods (MTGS and K-MTGS) on different random sample size with different number of variables were selected randomly and one can say that the MS, reference group is obtained using the standardized variables of the healthy or normal group data.

There are two main methods to determine the values of MD.

1. The Mahalanobis-Taguchi System (MTS).
2. Mahalanobis-Taguchi-Gram-Schmidt (MTGS).

Corresponding Author:
Dr. Kawa M Jamal Rashid
Department of Statistics and
Informatics, College of
Administration and Economics,
University of Sylaimanyah, Iraq

The main objective of his application was to make statistical judgments to distinguish one group from another. The Mahalanobis Distance is a squared distance (D^2) calculated for the i^{th} observation in a sample of size (n) with (m) variables as the following formula. [1, 3, 6, 12].

$$MD_j = D_j^2 = Z_{ij}^T C^{-1} Z_{ij} \quad (1)$$

Where: $j=1$ to n

$$Z_{ij} = (z_{1j}, z_{2j}, \dots, z_{rj})$$

Z_{ij} = Standardized vector obtained by standardized value of X_{ij} , For $i=1, \dots, m$

$$Z_{ij} = \frac{(X_{ij} - \mu_j)}{s_j}, X_{ij} = \text{Value of the } i^{\text{th}} \text{ characteristic in the } j^{\text{th}} \text{ observation,}$$

μ_i = Mean of the i^{th} characteristic, s_i = Standard deviation (SD) of the i^{th} characteristic

C^{-1} = Inverse of the correlation matrix.

The Mahalanobis-Taguchi System (MTS) [7, 2].

In the MTS, the MD obtained from equation (1) is scaled by dividing in to the number of variables (m). Therefore the equation. Taguchi introduced the Mahalanobis Distance into the process of defining a reference group and measuring individual subsets MTS was developed by Taguchi as a diagnostic and for casting using multivariate data.

Eq. (2) becomes as:

$$MD_j = \frac{1}{m} Z_{ij}^T C^{-1} Z_{ij} \quad (2)$$

$$MD = \frac{1}{m} (z_1, z_2, \dots, z_{12}) (C^{-1}) \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{pmatrix} \quad (3)$$

Where Z_i is a Vectors of $(Z_1, Z_2, Z_3, \dots, Z_m)$

And C^{-1} is inverse matrix of correlation matrix

Gram-Schmidt Orthogonalization Process [5, 12, 13].

MD can also be computed by using Gram-Schmidt Process. Which is discretion as follows:

Given linearly Independent Vectors $(Z_1, Z_2, Z_3, \dots, Z_m)$ there exist mutually perpendicular

Vectors $(U_1, U_2, U_3, \dots, U_m)$ with the same linear span.

Then the Gram - Schmidt vectors are constructed sequentially by setting:

$$U_1 = Z_1 \quad U_2 = Z_2 - \frac{Z_2' U_1}{U_1' U_1} U_1$$

$$U_m = Z_m - \frac{Z_m' U_1}{U_1' U_1} U_1 - \dots - \frac{Z_m' U_{m-1}}{U_{m-1}' U_{m-1}} U_{m-1} \quad (4)$$

When MD is calculated using Gram-Schmidt Process (GSP), standardized value of variables are used. Therefore in the above set of equation $Z_1, Z_2, Z_3, \dots, Z_m$ corresponds to standardized values. It is clear that the transformation process largely depends on the first variables.

Calculating of MD using the Gram-Schmidt Process (GSP) [2, 8, 10, 11].

There is the sample of size (n) and each sample contains observations on (m) variables. After standardizing the variables, we will have a set of standardized vectors:

$$Z_1 = (z_{11}, z_{12}, \dots, z_{1n})$$

$$Z_2 = (z_{21}, z_{22}, \dots, z_{2n})$$

$$Z_3 = (z_{31}, z_{32}, \dots, z_{3n})$$

$$Z_m = (z_{m1}, z_{m2}, \dots, z_{mn}) \quad (5)$$

Where $Z_1, Z_2, \dots, Z_m$ are the set of standardized vectors

And the GSP orthogonal vector as:

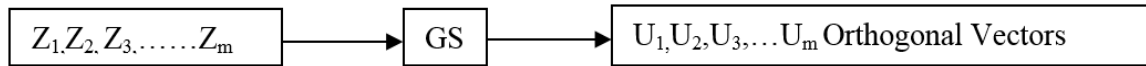


Fig 1: Gram Schmidt process

Where,

$$U_1 = (u_{11}, u_{12}, \dots, u_{1n})$$

$$U_2 = (u_{21}, u_{22}, \dots, u_{2n})$$

$$U_m = (u_{m1}, u_{m2}, \dots, u_{mn}) \quad (6)$$

The mean of vectors $(U_1, U_2, U_3, \dots, U_m)$ are zero and with $(S_1, S_2, S_3, \dots, S_m)$ respectively be the standard deviation of $(U_1, U_2, U_3, \dots, U_m)$ for sample size n , there will be (n) MD's.

The MD value be calculated as the following:

$$MD_j = \frac{1}{m} \left(\frac{u_{1j}^2}{s_1^2} + \frac{u_{2j}^2}{s_2^2} + \frac{u_{3j}^2}{s_3^2} + \dots + \frac{u_{mj}^2}{s_m^2} \right) \quad (7)$$

Where $j = 1, \dots, n$, the values of MD obtained from (2) and (7) are exactly the same.

To prove that the equation (2) and (7) are the same it is known that the Gram-Schmidt vectors $(U_1, U_2, U_3, \dots, U_m)$ are mutually perpendicular vectors, and $(S_1, S_2, S_3, \dots, S_m)$ are be the standard deviation of $(U_1, U_2, U_3, \dots, U_m)$ these vectors are obtained from standardized vectors $(Z_1, Z_2, Z_3, \dots, Z_m)$ and the mean of these vectors are equal to zero.

To compare two methods, (MTGS and MTS) with K-MTGS in order to know the efficiency of this method, samples with different sizes will be used and different number of variables were selected randomly. Read phonetically

It is clear that the new value of MD when calculated by this method is very close to the values of (MD) By (MTS & MTGS), as well as the proposed method is simpler than (MTS & MTGS) methods ^[2, 3, 6, 12].

Dictionary-VIEW DETAILED DICTIONARY

Steps in (K-MTGS) method.

Step 1

Compute the Standardized vectors X_{ij}

$$Z_{ij} = (z_{1j}, z_{2j}, \dots, z_{mj})$$

$$Z_1 = (z_{11}, z_{12}, \dots, z_{1n})$$

$$Z_2 = (z_{21}, z_{22}, \dots, z_{2n})$$

$$Z_3 = (z_{31}, z_{32}, \dots, z_{3n})$$

$$Z_m = (z_{m1}, z_{m2}, \dots, z_{mn})$$

Z_{ij} = Standardized vector obtained by standardized value of X_{ij} ($i = 1, \dots, m$)

Where $Z_1, Z_2, \dots, Z_m$ are the set of standardized vectors and standardized matrix as:

$$Z_{mn} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1n} \\ z_{21} & z_{22} & \dots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{m1} & z_{m2} & \dots & z_{mn} \end{bmatrix}$$

Step 2

Calculate the correlation coefficient matrix of X_{ij} observations and can be written as matrix:

$$C_{mn} = \begin{bmatrix} 1 & c_{12} & \cdots & c_{1n} \\ c_{21} & 1 & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & 1 \end{bmatrix}$$

Step 3

After performing GSP, we have the orthogonal vectors

$$\begin{aligned} K_1 &= Z_1 \\ K_2 &= Z_2 - C_{21}K_1 \\ K_3 &= Z_3 - C_{31}K_1 - C_{32}K_2 \\ &\vdots \\ K_m &= Z_m - C_{m1}K_1 - C_{m2}K_2 - C_{m3}K_3 - \cdots - C_{m,m-1}K_{m-1} \end{aligned}$$

Where,

$$C_{2,1}, C_{3,1}, C_{4,1}, \dots, C_{m,m-1} \quad (8)$$

Then K Vectors are constructed sequentially by setting

$$\begin{aligned} K_1 &= (k_{11}, k_{12}, \dots, k_{1n}) \\ K_2 &= (k_{21}, k_{22}, \dots, k_{2n}) \\ &\vdots \\ K_k &= (k_{m1}, k_{m2}, \dots, k_{mn}) \end{aligned}$$

The $K_1, K_2, \dots, K_m$ are vectors:

Step 4

Then the DM value by using the K-MTGS method:

$$MD_j = \frac{1}{m} \left(\frac{k_{1j}^2}{s_1^2} + \frac{k_{2j}^2}{s_2^2} + \frac{k_{3j}^2}{s_3^2} + \cdots + \frac{k_{mj}^2}{s_m^2} \right) \quad (9)$$

$$MD_j = \frac{1}{m} \sum_{i=1}^m \frac{k_{ij}^2}{s_i^2} \quad (10)$$

Where, (m) number of the variables or factors, j denote the number of repeating samples.

And s denote the standard deviation k_{mj} [7, 8, 9, 11, 12].

Numerical illustration**Application****Step 1**

Standardizing the normal data.

Step 2

Calculating the correlation coefficient of Standardizing data.

Step 3

Calculating the $U_1, U_2, \dots, U_m$ and $K_1, K_2, \dots, K_m$ vector as equation (7)

Step 4

Calculate the value of MD by using K-MTGS methods using the equation (11).

The average of MD (MTGS and K-MTGS) is close to one.

1. The average of $(u_{ij}, u_{2j}, \dots, u_{kj})$ is equal to zero.

2. The average of $\frac{u_{1j}^2}{s_1^2}, \frac{u_{2j}^2}{s_2^2}, \dots, \frac{u_{kj}^2}{s_k^2}$ and $\frac{k_{1j}^2}{s_1^2}, \frac{k_{2j}^2}{s_2^2}, \dots, \frac{k_{kj}^2}{s_k^2}$ is equal to one
3. The difference of between two methods MD (MTGS and K-MTGS) is very small.

Comparison between (MTGS and K-MTGS)

In this paper, we have six numerical applications to evaluate the (MTGS and K-MTGS) method then compare the result with it, each case of application contain a different data was simulated randomly with different sample size and variables.

A:

1. Example 1: Contains 8 variables with sample of size (200) observations.
2. Example 2: Contains 6 variables with sample of size (250) observations.
3. Example 3: Contains 10 variables with sample of size (248) observations.
4. Example 4: Contains 12 variables with sample of size (198) observations.
5. Example 5: Contains 4 variables with sample of size (240) observations.

The data of these examples are Shows in Table (2,3,4,5,6).

The value of (MTGS and K-MTGS) are calculated as show in tables (2,3,4,5,6),and

Fig(2,3,4,5,6) shows the charts of (MTGS and K-MTGS) .

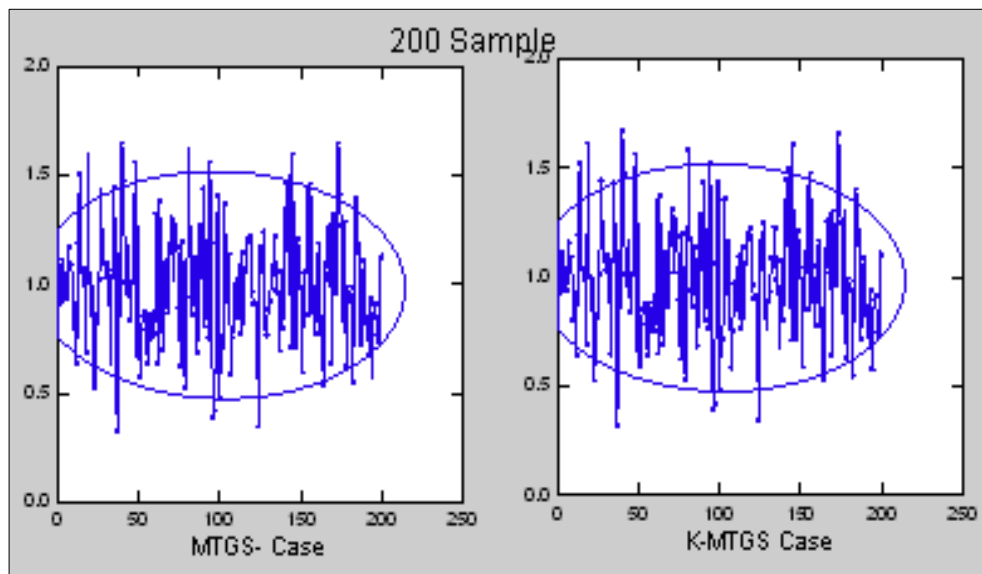


Fig 2: (MTGS and K-MTGS) chart with sample size (200)

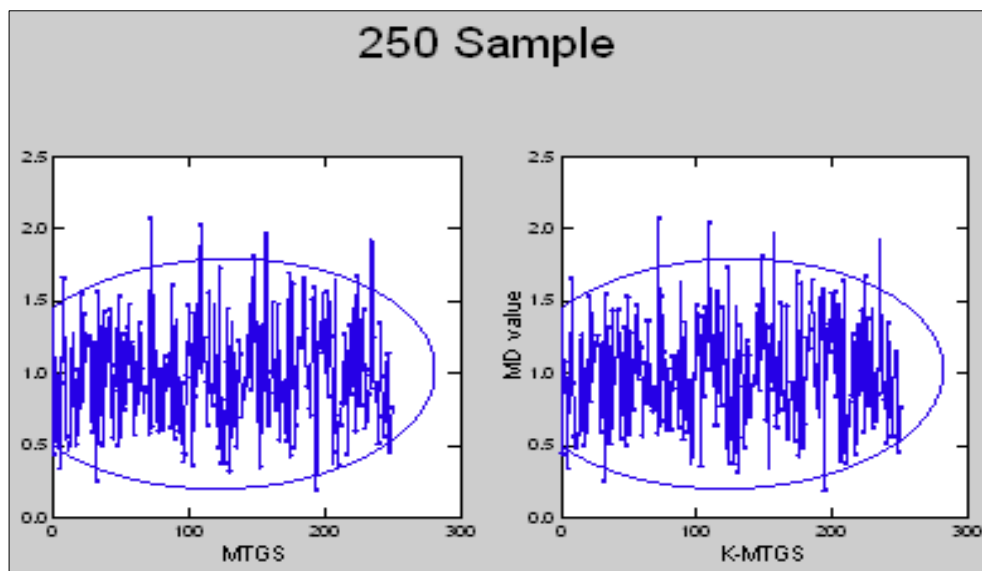
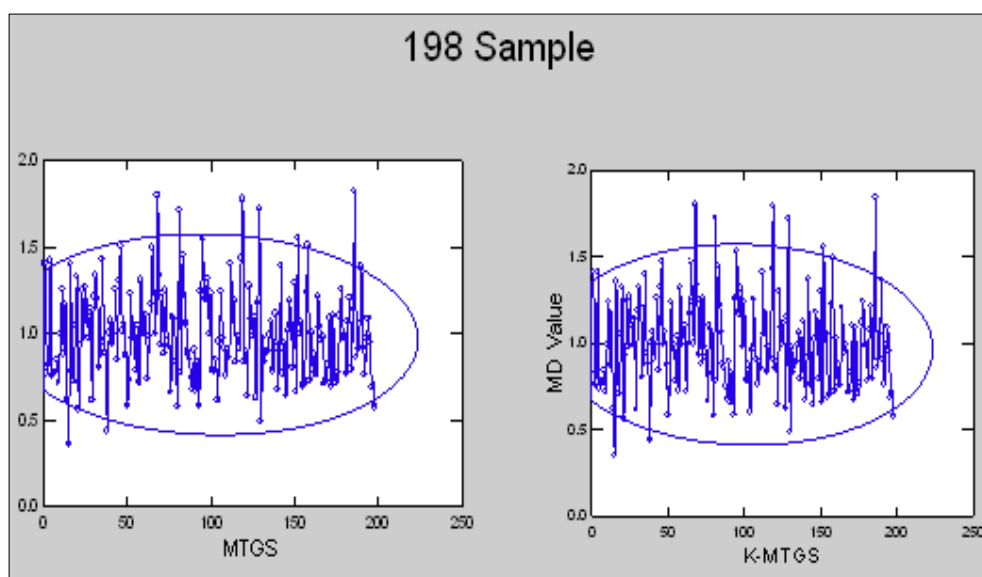
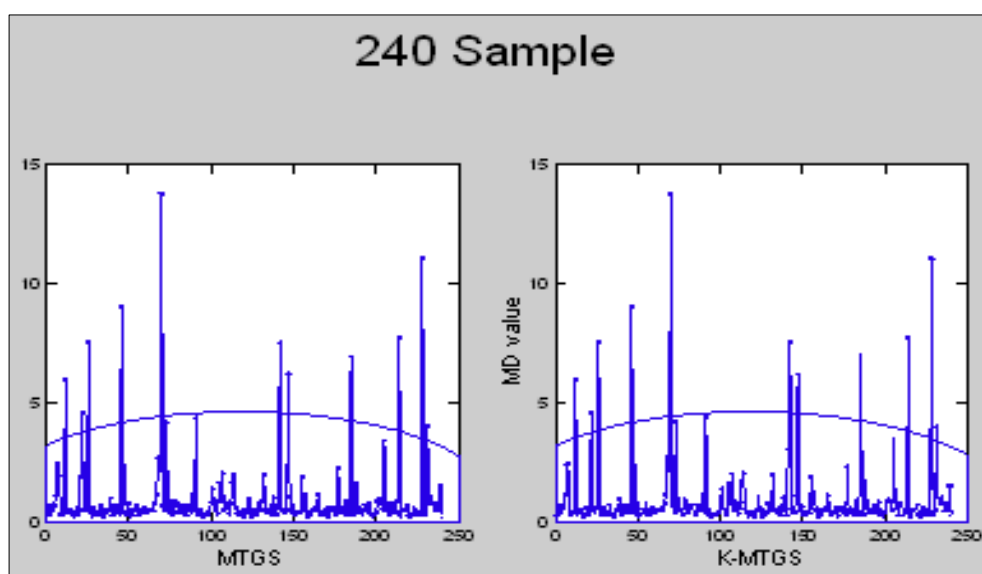


Fig 3: (MTGS and K-MTGS) chart with sample size (250)

**Fig 4:** (MTGS and K-MTGS) chart with sample size (248)**Fig 5:** (MTGS and K-MTGS) chart with sample size (198)**Fig 6:** (MTGS and K-MTGS) chart with sample size (240)

The average of $(\mu_{1j}, \mu_{2j}, \dots, \mu_{kj})$ are equal to zero, and the mean of MD-values is Closed to one and the average of $\frac{\mu_{ij}^2}{s_i^2}$ and $\frac{k_{ij}^2}{s_k^2}$ are

equal one,

From Basic statistics table for all case it seen that the different between MD-MTGS and K-MTGS is very small or there is no difference exactly (mean, standard deviation, and Standard Error of Kurtosis,

Table 1: Basic statistics of (MTGS and K-MTGS) for five example

	Example 1		Example 2		Example 3		Example 4		Example 5	
	MTGS	K-MTGS	MTGS	K-MTGS	MTGS	K-MTGS	MTGS	K-MTGS	MTGS	K-MTGS
Subjects										
Minimum	0.245	0.245	0.191	0.191	0.275	0.278	0.361	0.354	0.235	0.235
Maximum	2.033	2.015	2.078	2.081	1.946	1.938	1.818	1.847	13.759	13.745
Range	1.788	1.770	1.887	1.890	1.671	1.659	1.457	1.493	13.524	13.510
Sum	199	199	249	249	247	247	197	197	239	239
Arithmetic Mean	0.995	0.995	0.996	0.996	0.996	0.996	0.995	0.995	0.996	0.996
Trimmed Mean (10%, Two Sided)	0.993	0.993	0.986	0.986	0.987	0.987	0.978	0.979	0.590	0.590
Standard Deviation	0.322	0.321	0.369	0.369	0.283	0.283	0.268	0.268	1.680	1.680
Variance	0.104	0.103	0.136	0.136	0.080	0.080	0.072	0.072	2.822	2.821
Standard Error of Skewness	0.172	0.172	0.154	0.154	0.155	0.155	0.173	0.173	0.157	0.157
Standard Error of Kurtosis	0.342	0.342	0.307	0.307	0.308	0.308	0.344	0.344	0.313	0.313
Sum of Difference between MTGS and K-MTGS	-5.246E-05		-7.515E-03		2.270E-14		-3.397E-14		1.732E-14	

B:

Testing the example from Mahalanobis-Taguchi Strategy book

To more compare between (MTGS and K-MTGS) methods used the data from Mahalanobis-Taguchi Strategy book page 51: ^[11]. Calculating the MD value of the data by two methods (MTGS and k-MTGS) as shows in table (7) and fig (6), it seen that the result of two methods are simile equal.

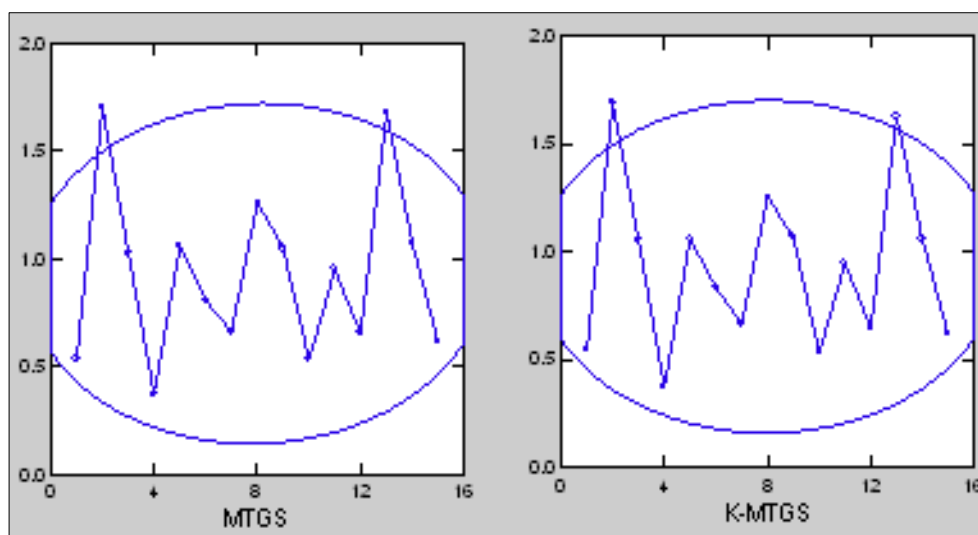


Fig 7: MD Chart (MTGS and K-MTGS)

Discussion

There are two ways (MTS and MTGS) to determine the value of MD, both have the same result, but there are a different steps to calculate the MD value

1. Determining the MD-MTS methods is very difficult, the mathematical calculation is not easy, exactly finding the inverse of correlation matrix, so that the value of the inverted leads to incorrect results.
2. The MD-MTGS method have complex steps to determine the MD with the knowledge that the two methods have the same complications and they have the same results. We conclude that the two methods have the same results but applying the procedures are not easy.
3. But the K-MTGS method is an alternative method to use instead of MTS and MTGS methods.
4. From all Cases it clears that:
 - i) The difference between MD value by (MTGS and K-MTGS) is very small.
 - ii) The average of MD value by using K-MTGS is equal to one or close to one the same as MTS and MTGS.
 - iii) The average of $(u_{ij}, u_{2j}, \dots, u_{kj})$ are equal to zero and $\frac{u_{1j}^2}{s_1^2}, \frac{u_{2j}^2}{s_2^2}, \dots, \frac{u_{kj}^2}{s_k^2}, \frac{k_{1j}^2}{s_1^2}, \frac{k_{2j}^2}{s_2^2}, \dots, \frac{k_{kj}^2}{s_k^2}$, are equal to one this satisfies the basic properties of MTGS.

With these results obtained by using the modified method (K-MTGS) needs further investigation to study in the future.

Table 2: Example (1) MD-values by (MTGS and K-MTGS) METHODS

MD										
#	X1	X2	X3	X4	X5	X6	X7	X8	MTGS	k-MTGS
1	9.463	5.275	6.503	15.333	0.152	53.527	784.509	79.236	1.016	1.012
2	5.963	46.526	3.740	13.209	0.892	44.958	440.115	42.801	0.511	0.516
3	0.679	55.991	3.689	13.975	0.926	67.860	474.921	35.979	0.771	0.777
4	1.199	65.836	6.361	22.416	0.050	34.083	149.877	22.240	0.936	0.931
5	5.466	96.323	9.503	89.751	0.267	0.370	698.184	43.500	1.464	1.471
6	2.193	71.837	9.945	8.111	0.771	73.475	567.497	67.569	1.097	1.105
7	1.255	89.066	2.398	97.558	0.708	62.158	275.443	65.121	1.059	1.060
8	6.617	80.021	5.839	13.034	0.825	33.700	958.155	18.000	1.065	1.077
9	3.971	57.131	2.150	75.280	0.834	93.455	859.323	2.294	1.187	1.165
10	2.936	98.823	7.683	45.325	0.680	35.723	749.514	4.423	0.992	1.000
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196	1.640	40.249	5.515	43.831	0.554	14.756	659.116	72.564	0.554	0.553
197	1.464	56.333	2.866	70.106	0.123	26.467	162.000	1.306	1.128	1.138
198	9.064	51.919	4.186	88.433	0.464	24.780	286.634	81.676	0.868	0.868
199	0.042	68.922	7.537	11.660	0.456	57.018	344.441	98.176	1.246	1.235
200	3.798	52.617	5.653	3.563	0.960	47.516	405.988	12.565	0.874	0.885

Table 3: Example (2) MD-Values By (MTGS and K-MTGS) Methods

			6 x 250				MD			
	X1	X2	X3	X4	X5	X6	MTGS	k-MTGS		
1	44.645	433	886	31.042	746	478	0.441	0.440		
2	50.493	805	202	8.054	926	596	1.093	1.094		
3	81.256	325	193	88.580	546	664	0.870	0.871		
4	65.401	924	550	70.632	117	98	0.975	0.966		
5	69.779	643	449	22.452	386	612	0.339	0.340		
6	71.747	677	903	5.246	772	751	1.091	1.083		
7	96.518	731	46	76.168	254	541	1.112	1.112		
8	97.846	592	15	61.171	961	778	1.660	1.659		
9	25.583	535	721	5.221	555	916	0.931	0.928		
10	95.422	612	232	59.250	443	550	0.563	0.563		
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246	48.286	5	770	86.014	505	246	1.079	1.080		
247	39.148	507	623	74.219	990	39	1.134	1.146		
248	69.224	557	784	61.997	541	789	0.454	0.453		
249	42.291	623	97	50.548	751	507	0.542	0.544		
250	91.891	453	791	85.071	479	540	0.761	0.761		

Table 4: Example (3) MD-Values By(MTGS and K-MTGS) Methods

				10 X 248						MD		
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	MTGS	k-MTGS
1	0.267	0.018	0.195	0.804	0.986	0.301	0.848	0.673	0.337	0.163	1.228	1.206
2	0.118	0.908	0.900	0.127	0.329	0.215	0.597	0.333	0.929	0.418	1.161	1.125
3	0.846	0.921	0.858	0.125	0.948	0.566	0.070	0.713	0.593	0.023	1.399	1.376
4	0.528	0.599	0.182	0.288	0.531	0.938	0.348	0.107	0.652	0.971	0.849	0.852
5	0.728	0.200	0.911	0.077	0.074	0.414	0.920	0.623	0.181	0.913	1.293	1.312
6	0.765	0.729	0.572	0.827	0.957	0.053	0.388	0.881	0.014	0.307	1.113	1.128
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244	0.136	0.646	0.022	0.584	0.875	0.612	0.643	0.807	0.823	0.123	1.088	1.100
245	0.631	0.742	0.022	0.603	0.285	0.091	0.163	0.964	0.011	0.610	1.251	1.264
246	0.178	0.906	0.473	0.484	0.003	0.283	0.826	0.067	0.681	0.052	1.247	1.259
247	0.073	0.498	0.406	0.290	0.510	0.005	0.340	0.608	0.259	0.991	0.872	0.879
248	0.145	0.071	0.520	0.767	0.462	0.910	0.709	0.164	0.289	0.520	0.756	0.749

Table 5: Example (4) MD-values by (MTGS and K-MTGS) methods

						12 x 198							MD	
	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	MTGS	-MTGS
1	0.261	0.719	0.928	0.993	0.867	0.061	0.747	0.343	0.972	0.060	0.578	0.558	1.404	1.409
2	0.061	0.933	0.238	0.391	0.542	0.484	0.560	0.546	0.917	0.256	0.138	0.268	0.769	0.764
3	0.425	0.755	0.431	0.120	0.117	0.441	0.356	0.146	0.108	0.397	0.552	0.866	0.815	0.830
4	0.243	0.594	0.937	0.135	0.037	0.855	0.299	0.577	0.512	0.081	0.897	0.464	1.417	1.417
5	0.386	0.049	0.449	0.420	0.453	0.813	0.640	0.636	0.721	0.184	0.829	0.220	0.754	0.742
6	0.088	0.848	0.198	0.289	0.505	0.201	0.281	0.426	0.846	0.255	0.227	0.521	0.771	0.765
7	0.258	0.849	0.440	0.460	0.931	0.317	0.779	0.755	0.195	0.404	0.617	0.724	0.779	0.783
8	0.667	0.960	0.111	0.387	0.854	0.615	0.314	0.837	0.376	0.404	0.281	0.193	0.850	0.843
	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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194	0.454	0.131	0.529	0.031	0.427	0.620	0.861	0.262	0.985	0.735	0.830	0.074	1.085	1.094
195	0.893	0.277	0.113	0.417	0.659	0.943	0.121	0.258	0.624	0.897	0.823	0.340	0.947	0.956
196	0.515	0.234	0.112	0.357	0.227	0.543	0.925	0.675	0.133	0.181	0.396	0.587	0.687	0.688
197	0.537	0.590	0.005	0.799	0.529	0.305	0.190	0.349	0.489	0.722	0.607	0.099	0.702	0.715
198	0.184	0.539	0.584	0.334	0.123	0.603	0.579	0.034	0.730	0.565	0.577	0.439	0.575	0.578

Table 6: Example (5) MD-Values By(MTGS and K-MTGS) Methods

	X1	X2	X3	X4	MTGS	k-MTGS
1	1	1.5	93.8	1.48	0.311	0.311
2	1	2.02	88.42	5.47	0.683	0.682
3	1	1.64	73.86	1.41	0.358	0.357
4	1	1.86	112.9	2.1	0.527	0.527
5	1	1.1	76.27	0.22	0.698	0.696
6	1	2.93	99.6	2.48	1.131	1.13
7	1	3.27	136	0.05	2.44	2.441
8	1	1.53	144	0.05	1.972	1.973
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
236	1	2.38	93.23	2.02	0.499	0.499
237	1	0.93	94.01	0.45	0.82	0.818
238	10	1.56	67.37	1.03	0.505	0.504
239	10	2.12	47.68	0.07	1.529	1.531
240	1	1.44	81.3	2.92	0.27	0.27

Table 7: (MTGS and K-MTGS) value and Basic statistics of example “Mahalanobis-Taguchi Strategy book page 51”

MTGS	K-MTGS	Difference	#	Basic Statistics	MTGS	K-MTGS
0.537377	0.545207	-0.0078293	1	N of Cases	15	15
1.703119	1.692693	0.0104255	2	Minimum	0.37418	0.3753
1.028316	1.057755	-0.0294385	3	Maximum	1.70312	1.69269
0.374178	0.375304	-0.0011262	4	Range	1.32894	1.31739
1.061257	1.060896	0.0003615	5	Median	0.95124	0.94776
0.807145	0.837215	-0.0300702	6	Arithmetic Mean	0.93333	0.93333
0.664241	0.665509	-0.0012673	7	Standard Error of Arithmetic Mean	0.10262	0.10029
1.264483	1.258943	0.0055394	8	95.0% Lower Confidence Limit	0.71323	0.71823
1.048261	1.067666	-0.0194049	9	95.0% Upper Confidence Limit	1.15344	1.14844
0.533830	0.535030	-0.0011996	10	Trimmed Mean (10%, Two Sided)	0.88268	0.8881
1.679398	1.627839	0.0515584	13	Standard Deviation	0.39745	0.38843
1.073739	1.060116	0.0136233	14	Coefficient of Variation	0.42584	0.41618
0.617223	0.619719	-0.0024968	15	Skewness(G1)	0.70877	0.62767
Average	Average	Sum of Difference	16	Standard Error of Skewness	0.58012	0.58012
0.933333	0.933333	-1.66533E-15	17	Kurtosis(G2)	-0.05662	-0.17862
			18	Standard Error of Kurtosis	1.1209	1.1209

Conclusion

This study evaluated the efficiency of the Mahalanobis Distance (MD) using two methods: Mahalanobis-Taguchi-Gram-Schmidt (MTGS) and the proposed K-MTGS method. Both methods yielded similar results, demonstrating that K-MTGS is a viable alternative to MTGS and MTS for calculating MD. The analysis showed that the average MD values from both methods were close to one, indicating consistency in results. Additionally, the K-MTGS method simplifies the calculation process while maintaining accuracy. Future research should further explore the K-MTGS method's applications and its potential advantages in various statistical analyses, particularly in multivariate diagnostics and pattern recognition.

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