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Useful statistical distributions in modeling traffic flow

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Abstract

Traffic flow in, real life situation, affect the movement of people/goods from one point to another. This directly hampers efficient delivery of service in a country or region. This creates the need of studying and looking for possible ways of solving this menace. This article therefore delves in the possible statistical distributions that can be applied in modeling traffic flow. The statistical distributions that this study has looked into includes Exponential, Weibull, Gaussian and Gamma Distributions. It provides a summary of how distributions can be applied in modeling traffic.

Keywords: Traffic flow, exponential, weibull, gaussian and gamma distributions, modeling traffic

1. Introduction

Modeling traffic distributions is a critical aspect of traffic engineering and transportation research, enabling the analysis, prediction, and optimization of traffic flows, travel times, and system performance (First Capital Team, 2024, Oyala, C.O., & Otumba, E.O., 2018) [5, 2].

Here's an overview of how traffic distributions are modeled and their practical applications:

2. Traffic Distributions models

2.1 Exponential Distribution (ED)

ED is a continuous probability distribution. It is defined by the Probability Density Function(pdf) $f(x; \lambda) = \lambda e^{-\lambda x}$ $x \geq 0$ where x is the random variable. It can be time or distance. ED is widely used in modeling traffic. This distribution can easily be implemented in modeling traffic. ED models inter-arrival times of vehicles assuming a constant arrival rate (Ongiri, H.N., 2020). It is suitable in modeling Poisson process or low traffic intensity (Oyala, C.O., & Otumba, E.O., 2018) [2].

2.2 Weibull Distribution (WD)

The WD is defined by the pdf $f(x; \lambda, k) = \begin{cases} \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, & x \geq 0, \\ 0, & x < 0 \end{cases}$ where x is the random

represent time or distance variable and k is the shape parameter. WD extends the exponential distribution by adding a shape parameter to capture varying rates. This distribution is used to model travel times under different traffic conditions. It is suitable for capturing both short and long travel times (Asad Arfeen *et al.*, 2019, S. Mahmudov, 2021) [3, 5].

2.3 Gaussian Distribution (GD)

The GD is defined by its pdf $f(x; \mu, \delta^2) = \frac{1}{\sqrt{2\pi\delta^2}} e^{-\frac{(x-\mu)^2}{2\delta^2}}$ where x is the travel time, μ represents the central tendency, δ^2 represents the variance. GD is used for travel time modeling, assuming symmetric variability around the mean. It is often inappropriate for highly skewed traffic data (Meent, Remco & Mandjes, Michel & Pras, Aiko. (2006) [4].

2.4 Gamma Distribution: The Gamma Distribution is utilized in transportation engineering to model the time it takes for vehicles to travel between two points.

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This is crucial for traffic management and infrastructure planning. It is widely used in traffic analysis with variables that are positive and skewed. It is a valuable tool for capturing variability in traffic systems with both free flow and/or congested conditions (First Capital Team, 2024, S. Mahmudov, 2021) ^[5].

3. Selecting a Distribution

Factors include data characteristics (e.g., skewness, kurtosis), study objectives, and computational simplicity.

Goodness-of-fit tests, Kolmogorov-Smirnov, Chi-square and likelihood-based methods, helps in selection of the appropriate model.

4. Applications of Traffic Distributions

4.1 Vehicle Arrival Modeling: Distributions like Poisson, exponential, and Weibull are used to model the arrival of vehicles at intersections, toll booths, or highways.

Application: Poisson, exponential, and Weibull are used to optimize traffic signal timings or toll collection systems.

4.2 Travel Time Prediction

Lognormal or Weibull distributions model variability in travel times under varying traffic conditions.

Application: Lognormal or Weibull distributions are used in Real-time route planning, congestion pricing, and navigation systems.

4.3 Traffic Flow Analysis

Continuous distributions like Gamma or Normal model traffic flow rates under free-flow and congested conditions.

Application: Gamma or Normal model traffic are used in assessing roadway capacity and planning infrastructure upgrades.

4.4 Gap Acceptance

Time or space gaps between vehicles are modeled using Weibull or exponential distributions to study driver behavior at intersections or merging zones.

Application: Weibull or exponential distributions can be employed in designing safer intersections and freeway merging strategies.

4.5 Crash Frequency and Severity Modeling

Discrete distributions like Negative Binomial or Poisson are commonly used.

Application: Negative Binomial or Poisson distributions are helpful in identifying high-risk areas and developing targeted safety interventions.

4.6 Queueing Analysis

Distributions model traffic queues at intersections, toll stations, or parking facilities.

Application: Queue analysis aid in minimizing delays through improved queue management or resource allocation (Ongiri, H.N., 2020, Kendall, D. G. (1953) ^[1, 6].

4.7 Reliability Analysis: Weibull or Lognormal distributions model the reliability of transportation systems e.g., likelihood of travel time exceeding a threshold.

Application: Assessing reliability for public transit or freight logistics.

5. Data Requirements

Accurate distribution modeling requires high-quality and comprehensive traffic data.

Static distributions may not capture temporal variations or unexpected disruptions.

Integration with Advanced Technologies: Big data from GPS and connected vehicles can enhance distribution modeling. AI and machine learning offer opportunities to dynamically adjust distributions. Expanding modeling to account for diverse modes of transport (e.g., bicycles, pedestrians).

6. Conclusion (Case Study Example)

The mentioned distributions can be applied in modeling traffic

Imagine modeling vehicle arrivals at a busy urban intersection:

Data collected over peak and off-peak hours can be analyzed to fit a Weibull distribution.

Goodness-of-fit tests validate the model.

Results inform signal timing adjustments to reduce congestion and improve throughput.

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