

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

Maths 2024; 9(6): 99-103

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<https://www.mathsjournal.com>

Received: 02-10-2024

Accepted: 07-11-2024

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Study of W_8 -curvature tensor on Kenmotsu manifold admitting (G.T.W.C.) Generalized Tanaka webster connection

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Abstract

This paper deals with geometric components related to a group of curvature tensors in the Kenmotsu manifold admitting G.T.W.C are studied. We show that W_8 curvature tensor concerning G.T.W.C is both a flat and an Einstein manifold. Further, we establish a relationship between $\beta - W_8, Q. W_8 = 0, \mu. W_8$, special η -Einstein and η -Einstein manifolds.

Keywords: W_8 -curvature tensor, Generalized Tanaka Webster connection, η -Einstein Manifold, semi-symmetric Kenmotsu manifold, Einstein Manifold

Introduction

197, Kenmotsu investigated classes of contact R-manifolds that satisfy specific components (Pambo ^[1]) called the Kenmotsu manifold. Similarly, almost all of the Kenmotsu three-manifolds in line with pseudo-parallel features of the Jacobi operator dealt with by Inoguchi *et al.* ^[2]. Similarly, investigation on W_8 and W_6 -curvature tensors on manifolds related to Kenmotsu align with metric connections that are semi-symmetric Njori *et al* ^[3].

Curvature tensors within Sasakian lp -Sasakian, p -Sasakian, η -Einstein Sasakian manifolds have been studied by Katende ^[4] discussing skew-symmetric and symmetric parts in a W_3 symmetric variety. Bektas studied almost pseudo-Ricci symmetric spacetimes ^[5] constructing a non-trivial example of an almost pseudo-Ricci symmetric space-time.

Inoguchi studied almost Kenmotsu three manifolds on pseudo parallel behavior Jacobi operator ^[6] and provided their classification. Real surfaces in multifaceted 2-flat Grassmannians through G.T.W parallel shapes operator were studied by Jeong *et al* ^[7] who demonstrated that there is no hypersurface in $G_2(C^{m+2})$ corresponding-shaped machinist trendy the G.T.W.C.

It follows that an affine canonical connection illustrated at an undegenerated quasi-Hermitian manifold is T.W linking. G.T.W.C in a contact metric manifolds on the canonical connection discussed by Kiran ^[8] who investigated the projective curvature tensor, con-harmonic curvature tensor, recurrent conditions, and Weyl projective tensors of Kenmotsu manifolds with G.T.W.

Recently F Mburu, Njori, P.W. C N Gitonga and Moindi, S.K discussed W_9 curvature tensors on LP Kenmotsu manifold ^[11].

Set M within $2n+1$ as an almost-contact metric manifold within an almost-contact metric structure (ϕ, ξ, η, g) consisting of Riemannian metric g , a tensor field of type $(1, 1)$ ϕ , a vector field, ξ and a 1-form η . Therefore:

$$\begin{aligned}\phi^2 X_1 &= -X_1 + \eta(X_1)\xi, \\ \eta\phi &= 0, \\ \phi\xi &= 0, \\ g(X_1, \xi) &= \eta(X_1), \\ \eta(\xi) &= 1,\end{aligned}$$

(2.1)

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$$g(\phi X_1, \phi X_2) = g(X_1, X_2) - \eta(X_1)\eta(X_2) \quad (2.2)$$

For arbitrary vector fields X_1 and X_2 on M . In addition,

$$(\nabla \phi)X_2 = -g(X, \phi X_2)\xi - \eta(X_2)\phi X_1 \quad (2.3)$$

M is a Kenmotsu manifold while
 ∇ =Riemannian connection of g .

Then these conditions hold,

$$= X_1 - \eta(X_1) \quad (2.4)$$

$$(\nabla X_1 \eta)X_2 = g(\nabla X_1 \xi, X_2) \quad (2.5)$$

$$R(X_1, X_2)\xi = \eta(X_1)X_2 - \eta(X_2)X_1 \quad (2.6)$$

$$R(\xi, X_1)X_2 = \eta(X_2)X_1 - g(X_1, X_2) \quad (2.7)$$

$$\eta(R(X_1, X_2)X_3) = g(X_1, X_3)\eta(X_2) - g(X_2, X_3)\eta(X_1) \quad (2.8)$$

$$S(X_1, \xi) = -2n\eta(X_1) \quad (2.9)$$

$$Q\xi = -2n\xi \quad (2.10)$$

$$S(\phi X_1, \phi X_2) = S(X_1, X_2) + 2n\eta(X_1)\eta(X_2) \quad (2.11)$$

X_1, X_2 , and X_3 represent vector fields on M and R curvature tensor on M .

1 μ - W_8 -curvature tensor in L.P Kenmotsu manifold admitting G.T.W.C.

W_8 -curvature tensor represented by

$$W_8(X_1, X_2)\mu = R(X_1, X_2)\mu + \frac{1}{n-1} [S(X_1, X_2)\mu - S(X_2, \mu)X_1] = 0 \quad (3.1)$$

$$R(X_1, X_2, \mu) = \frac{1}{n-1} [S(X_2, \mu) - S(X_1, X_2)\mu] \quad (3.2)$$

$$\eta(X_2)X_1 - \eta(X_1)X_2 = \frac{1}{n-1} [S(X_2, \mu) - S(X_1, X_2)\mu] \quad (3.3)$$

For

$$S(X_2, \mu) = (n-1)\eta(X_2)$$

$$\eta(X_2)X_1 - \eta(X_1)X_2 = \frac{1}{n-1} [(n-1)\eta(X_2)X_1 - S(X_1, X_2)\mu] \quad (3.4)$$

$$\eta(X_2)X_1 - \eta(X_1)X_2 = \eta(X_2)X_1 - \frac{1}{n-1}(S(X_1, X_2)\mu) \quad (3.5)$$

$$S(X_1, X_2)\mu = (n-1)\eta(X_1)X_2 \quad (3.6)$$

Contract 3.6 with respect to μ

$$S(X_1, X_2) = -(n-1)\eta(X_1)X_2 \quad (3.7)$$

Theorem 3; μ - W_8 -flat curvature tensor in L.P Kenmotsu manifold admitting G.T.W.C. is a special type of η -Einstein manifold

2 Kenmotsu Manifold, Einstein Manifold, and Generalized Tanaka Webster Connection

Definition 2.1

A Kenmotsu manifold is referred to as η Einstein in that Ricci tensor is given by

$$S(X_1, X_2) = ag(X_1, X_2) + b\eta(X_1)\eta(X_2) \quad (4.1)$$

for a and b are scalars. When $b = 0$, the manifold M is an Einstein manifold.

It is indicated that a linear connection is metric and symmetric when it is only the Levi–Civita connection.

Theorem 2

An n -dimensional with respect to G.T.W.C, W_8 -flat Kenmotsu manifold is an Einstein manifold.

Proof

The W_8 curvature tensor illustrated as a given case of τ -tensor discussed by Tripathi *et al* ^[9]. Some of the W_8 -curvature tensors delt with by Pokhar0iyal ^[10]. The W_i -curvature tensor of $r = 3$ represented by,

$$W_i(X_1, X_2)X_3 = a_0R(X_1, X_2)X_3 + a_1S(X_1, X_3)X_1 + a_2S(X_1, X_3)X_2 + a_3S(X_1, X_2)X_3 + a_4g(X_2, X_3)QX_1 + a_5g(X_1, X_3)QX_2 + a_6g(X_1, X_2)QX_3 \quad (4.3)$$

$a_0 = 1, a_1 = -a_3 = -\frac{1}{n-1}$ others $a_i = 0$ and $X_1, X_2, X_3 \in_X(M)$ where Q, R, S and r are respectively the Ricci operator, Riemannian curvature tensor, Ricci tenso and rank r . W_8 is given by,

$$W_8(X_1, X_2)X_3 = R(X_1, X_2)Z \frac{1}{n-1} [S(X_2, Z)X_1 - S(X_3)X_3] \quad (4.4)$$

W_i -curvature tensor in connection to G.T.W.C is represented by

$$W^*i(X_1, X_2)X_3 = a_0R^*(X_1, X_2)X_3 + a_1S^*(X_2, X_3)X_1 + a_2S^*(X_1, X_3)X_2 + a_3S^*(X_1, X_2)X_3 + a_4g(X_2, X_3)Q^*X_1 + a_5g(X_1, X_3)Q^*X_2 + a_6g(X_1, X_2)Q^*X_3 \quad (4.5)$$

When M is W_i – flat in connection to ∇^* Equation (4.5) results to

$$a_0R^*(X_1, X_2)X_3 + a_1S^*(X_2, X_3)X_1 + a_2S^*(X_1, X_3)X_2 + a_3S^*(X_1, X_2)X_3 + a_4g(X_2, X_3)Q^*X_1 + a_5g(X_1, X_3)Q^*X_2 + a_6g(X_1, X_2)Q^*X_3 = 0 \quad (4.6)$$

with vector field V and taking inner product of Eqn (4.6), then

$$a_0g(R^*(X_1, X_2)X_3, V) + a_1S^*(X_2, X_3)g(X_1, V) + a_2S^*(X_1, X_3)g(X_2, V) + a_3S^*(X_1, X_2)g(X_3, V) + a_4g(X_2, X_3)S^*(X_1, V) + a_5g(X_1, X_3)S^*(X_2, V) + a_6g(X_1, X_2)S^*(X_3, V) = 0 \quad (4.7)$$

Let $e_i (1 \leq i \leq n)$ be an orthonormal basis of the tangent space at every point of the manifold M . Let $X_1 = V = e_i$ in Eqn (4.7) then take summation over $i, (1 \leq i, \text{ and } i \leq n)$, then

$$(a_0 + na_1 + a_2 + a_3 + a_5 + a_6)S^*(X_2, Z) + r^*a_4g(X_2, X_3) = 0 \quad (4.8)$$

using (3.7), (3.9) and (4.8) we have

$$S(X_2, X_3) = -\frac{1}{a}[ra_4 + (a + na_4)(n-1)]g(X_2, X_3)$$

for $a_0 + na_1 + a_2 + a_3 + a_5 + a_6 =$ and $a = 0$

hence Einstein manifold.

A β – W_8 -flat Kenmotsu manifold

Definition

An n -dimensional Kenmotsu manifold is termed as β – W_8 -flat when its W_8 curvature tensor upholds the following condition

$$W_8(X_1, X_2)\beta X_3 = W_8(X_1, X_2)\beta X_3 - \beta.W_8(X_1, X_2)X_3 = 0 \quad (5.1)$$

$$W_8(X_1, X_2)\beta X_3 - \beta.W_8(X_1, X_2)X_3 = R(X_1, X_2)\beta X_3 - S(X_2, X_3)\beta X + S(X_1, X_3)\beta Y + S(X_1, X_2)\beta X_3 - S(X_2, \beta X_3)X_1 - S(X_1, X_2)\beta X_3 + S(X_2, X_3)\beta X_1$$

$$R(X_1, X_2)\beta X_3 + S(X_1, X_3)\beta X_2 - S(X_2, X_3)\beta X_1$$

$$S(X_2, \beta X_3)X_1 - S(X_1, \beta X_3)X_2 - S(X_2, X_3)X_3\beta X + S(X, X_3)\beta X_3 + S(X_1, X_2)\beta X_3 - S(X_2, \beta X_3)X_1 - S(X_1, X_2)\beta X_3 + S(X_2, X_3)\beta X_1$$

$$= -S(X_1, \beta X_3)X_2 + S(X_1, X_3)\beta X_2$$

Let X_3 and $X_2 = \xi$ then

$$W_8(X_1, \xi).\beta\xi = 0$$

From above proof we come up theorem W_8 Para Kenmotsu manifold is $\beta - W_8$ -flat manifold.

Theorem

A W_8 -flat LP-Kenmotsu manifold is a special type of η - Einstein manifold.

Proof

Let the Kenmotsu manifold be W_8 -flat, then the following hold

$$W_8(X_1, X_2)\xi = R(X_1, X_2)\xi + \frac{1}{n-1} [S(X_1, X_2)\xi - g(X_2, \xi)QX_1] = 0$$

$$0 = \eta(X_2)X_1 - \eta(X_1)X_2 + g(X_1, X_2)\xi - \eta(X_2)X_1$$

$$0 = g(X_1, X_2)\xi - \eta(X_1)X_2$$

$$R(X_1, X_2)\xi = \frac{1}{n-1} [g(X_2, \xi)QX_1 - S(X_1, X_2)\xi]$$

$$\eta(X_2)X_1 - \eta(X_1)X_2 = \eta(X_2)X - \frac{1}{n-1} S(X_1, X_2)$$

$$S(X_1, X_2)\xi = -(n-1)\eta(X_1)X_2 \quad (5.2)$$

Contracting (5.2) with ξ , we get

$$S(X_1, X_2) = -(n-1)\eta(X_2)\eta(X_1) \quad (5.3)$$

Thus the proof that a $\xi - W_8$ -flat Kenmotsu manifold is a special type of the η -Einstein manifold

W_8 . Q Para Kenmotsu manifold

Def; W_8 Para Kenmotsu manifold is defined as $W_8.Q = 0$

Theorem; An n-dimension Para kenmotsu manifold is defined as $W_8.Q = 0$

Proof

$$W_8(X_1, X_2)QX_3 = W_8(X_1, X_2)QX_3 - Q(W_8(X_1, X_2)X_3)$$

$$= R(X_1, X_2)QX_3 - Q(R(X_1, X_2)X_3) + \frac{1}{n-1} [s(X_1, X_2)QX_3 - g(X_2, QX_3)QX_1] - \frac{1}{n-1} [S(X_1, X_2)QX_3 - Q(g(X_2, X_3)QX_1)]$$

$$= [g(X_2, QX_3)X_1 - g(X_1, QX_3)X_2 - [g(X_2, X_3)QX_1 - g(X_1, X_3)QX_2] + \frac{1}{n-1} [Q(g(X_2, X_3)X_1 - g(X_1, X_3)QX_2)]$$

$$= [-g(X_1, QX_3)X_2] - [g(X_2, X_3)QX_1 - g(X_1, X_3)QX_2] + [g(X_2, X_3)QX_1]$$

$$= [-g(X_1, QX_3)X_2] - [-g(X_1, X_3)QX_2]$$

$$= g(X_1, X_3)QY - g(X_1, QX_3)$$

$$= (n-1)g(X_1, X_3)X_2 - (n-1)g(X_1, Z)X_2 = 0$$

Thus proved

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