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Lot acceptance criteria for products following attribute single sampling plan and its impact towards discretization of some continuous lifetime distributions

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Abstract

In this article, the conventional formation of designing an attribute sampling plan based on continuous lifetime distributions is modified for the situation because some data are discrete in nature, even for the continuous variables. In such situation, the alternative approach attempted in this paper, which includes derivation for the discrete continuous distributions such as Discrete Exponential-logarithmic distribution, Discrete Exponential-geometric distribution, Discrete Weibull-Logarithmic distribution. As well as the acceptance sampling plan procedure is performed for the Discrete Weibull-geometric distribution with finding the minimum sample size and OC curve also performed. For better understanding, illustrations for the proposed plans are also appended at the end.

Keywords: Acceptance sampling plan, discrete exponential-logarithmic distribution, discrete exponential-geometric distribution, discrete Weibull-logarithmic distribution, discrete weibull-geometric distribution, consumer risk, producer risk

Introduction

The process of discretizing continuous models, functions, or attributes is known as discretization. In order to provide a better alternative model for describing count data sets with complex behaviour, a number of discrete distributions have been created in recent decades as analogues of continuous distributions. In actual life, it can be challenging or inconvenient to collect samples from a continuous distribution. Almost always, observed values are discrete as they are measured to a finite number of decimal places and cannot truly represent all points on a continuum. Although the measurements are conducted on a continuous scale, the observations may be documented in a fashion that lends itself to a discrete model. In some cases, due to measuring device precision or space constraints, continuous variables are measured using frequencies of non-overlapping class intervals, the union of which constitutes the entire range of random variables, and multinomial law is employed to model the situation. In categorical data analysis using the econometric technique, the existence of a continuous unobserved or latent variable under an observed categorical variable is assumed. When an unobserved continuous variable passes a threshold value, it is observed as a categorical variable. As a result, the conclusion is based on observed discrete values, which merely represent the intervals to which the unobserved continuous variable belongs, not its real values. As a result, this is an example of using a discretization of an underlying continuous variable. According to the preceding discussion, the original variables in the real world are frequently continuous in nature but discrete by observation, so it is reasonable and easy to model the situation using an appropriate discrete distribution produced from the underlying continuous models while preserving one or more important characteristics of the continuous distribution. Researchers have focused their emphasis on obtaining discrete counterparts (discretization) of continuous distributions. There are various approaches to deriving discrete distributions from continuous ones. In survival analysis, the survival function may be defined as a function of count random variables, which are discrete versions of the underlying continuous random variable.

For example, the period of residence in an observation ward is counted in days, whereas the survival time of leukemia patients is measured in weeks. These examples demonstrate that continuous life time does not always have to be measured on a continuous scale, but can also be tallied as discrete random variables. greater often than not, continuous failure time data produced by a complicated system presents greater derivational challenges than a discrete counterpart of the underlying continuous system. Despite this, discrete lifetime distributions play only a minor part in reliability analysis. As a result, more accurate discrete lifetime distributions must be considered (Rezaei Roknabadi et al. 2009)^[39]. That is, discretizing a continuous lifetime model provides an attractive and intuitive way to obtain a discrete lifetime model that corresponds to the continuous one (Lai 2013)^[28]. Almost every dependability study assumes that time is continuous. However, occasionally system lifetimes are unable to be measured using calendar time. This is the situation, for example, when an equipment functions in cycles or on demand, and the number of phases or demands before failure is determined. This is also true when an equipment is only monitored once every period, with the observation being a number of time periods completed successfully prior to failure. Furthermore, dependability statistics are frequently aggregated or reduced. In all of these cases, the system's lifetime becomes a discrete random variable. While count data sets are typically modeled using classical models like the binomial, Poisson, geometric, and negative binomial distributions, these probability models are unlikely to always match the data well. Therefore, it is necessary to create more adaptable distributions. Therefore, it is necessary to create more adaptable distributions. Because the observed measurements in lifetime modeling are typically taken to a finite value and cannot truly represent all points in a continuum, they are typically discrete in nature. Such cases, when lifetimes are assessed on a discrete scale, arise. For instance, the length of time it takes for a certain disease to get better in days, the months it takes for cancer patients to survive, etc. (see Krishna and Singh 2009)^[26]. In this case, the continuous lifetime is frequently counted as a discrete random variable rather than being measured on a continuous scale.

Review of Literature

Many discrete distributions have been introduced using the discretization method, such as Ord (1968)^[35] proposed a discrete Student's t distribution. Lisman and van Zuylén (1972)^[29] provided and Kemp (1997)^[22] analyzed the discrete normal distribution. Salvia and Bollinger (1982)^[42] discussed the fundamental findings about discrete dependability and used discrete lifespan distributions with a single parameter to illustrate them. An alternative version of the discrete normal distribution was studied by Roy (2003)^[40]. Nakagawa and Osaki (1975)^[31] were the first who proposed a discrete analog of the Weibull distribution. The second type of discrete Weibull (DW) distribution was first presented by Stein and Dattero (1984)^[45], while Padgett and Spurrier (1985)^[36] suggested a third. Sato et al. (1999)^[43] proposed a discrete exponential distribution and used it to describe defect count in semiconductor deposition equipment as well as the distribution of defect counts per chip. Rezaei Roknabadi et al. (2009)^[39] proposed some reliability concepts for the telescopic family of distributions and compared them. Roy (2004)^[41] presented a discrete analog Rayleigh distribution to determine the dependability of a complex system as an alternative to the simulation method, which has limited use in reliability experimentation during product design.

Inusah and Kozubowski (2006)^[18] and Kozubowski and Inusah (2006)^[27] presented discrete analogs of the Laplace and skew-Laplace distributions. Krishna and Pundir (2007)^[24] introduced a discrete Maxwell distribution and investigated Bayesian analysis and Maximum likelihood estimation for it. Kemp (2008)^[23] offered a discrete version of the half-normal distribution, calculated the maximum entropy distribution, evaluated the success run models, and provided a fresh derivation as a mixture of Heine distributions. Krishna and Pundir (2009)^[25] presented a discrete Burr distribution, which resulted in the discrete Pareto distribution. Both the discrete Weibull and the discrete inverse Weibull distributions match the discrete data set, according to the discrete extended-Weibull distribution given by Jazi et al. (2010)^[20]. The discrete analog of the Lindley Distribution with compound collective risk model was suggested by Gomez-Deniz and Calderin-Ojeda (2011)^[15]. Shafaei Nooghabi et al. (2011)^[44] suggested a discrete form of the modified Weibull model, with an increasing or bathtub-shaped failure rate function. Nekoukhou et al. (2012)^[32] proposed a discrete analog of the generalized exponential distribution. Nekoukhou et al. (2013)^[33] developed a discrete generalized exponential distribution of a second type. The discrete gamma distribution was introduced by Chakraborty and Chakravarty (2012)^[10]. Chakraborty and Chakravarty (2014)^[11] constructed a discrete form of the Gumbel (Type I) extreme value distribution utilizing the general approach of discretizing a continuous distribution. Para and Jan (2014)^[37] suggested a discretization of the Burr-type III distribution and investigated associated theorems. Gomez-Deniz et al. (2014)^[16] created a discrete form of the half-normal distribution with its generalization for applications. Chakraborty S (2015a)^[8] developed a generalization of the two-parameter discrete gamma distribution that includes discrete versions of many significant continuous distributions, as well as two real-life data sets used for data modeling and likelihood ratio tests for an illustration. Chakraborty S and Chakravarty D (2015)^[12] suggested a novel discrete distribution that serves as a discrete equivalent to the two-sided power distribution. If applied to several data sets, the new discrete extended-Weibull distribution described by Hussian et al. (2016)^[17] proven to be the least loss of information model. Jayakumar and Girish Babu (2018)^[19] proposed a discrete analog of Weibull geometric distribution. A new discrete extended Weibull distribution was proposed by Jia et al. (2019)^[21], and four real-world data sets are used as examples. A natural discrete-Lindley distribution was proposed by Al-Babtain et al. (2020)^[5] as a combination of geometric and negative binomial distributions.

A discrete MO-Weibull distribution was presented by Opone et al. (2021)^[34] and they further looked at data sets that were over- and under-dispersed. Eldeeb et al. (2021)^[14] developed a discrete inverted Topp-Leone distribution and analyzed two count sets of data containing a number of deaths caused by Covid-19 in India and Pakistan. Alghamdi et al. (2022)^[4] developed a discrete power-Ailamujia distribution and examined three real-world datasets. Ramos and Louzada (2019)^[38] and Afify et al. (2021)^[3] provided a discrete Ramos-Louzada distribution. Ahmed et al., (2022)^[5] proposed a moment-exponential (MEX) distribution.

Derivation of Discrete Lifetime distributions

Continuous distributions are noted with numerous features like probability densityfunction, survival function or reliability function, etc, which augments its nature. So in order to discretize a continuous distribution one of its features are used. Methodologies adapted for the discretizing the continuous distributions involve,

- Discretizing the continuous distribution's cdf function
- Discretizing its hazard rate function
- Discretizing using the probability density function or any other function of it.

The analogue form of Weibull-Geometric (WG) distribution had been introduced from the survival function of this distribution by Jayakumar and Babu (2018) by following the steps given below,

1. A continuous random variable X is considered.
2. The discrete form of this variable can be determined from the following as,

$$P(Y = y) = P(X \geq y) - P(X \geq y + 1) = S_X(y) - S_X(y + 1); y = 0, 1, 2, \dots \quad (1)$$

In similar manner, by using the survival function of exponential-logarithmic distribution, exponential-geometric distribution, Weibull-logarithmic distribution and Weibull-geometric distribution their discretized versions are derived in this section.

Discrete Exponential-Logarithmic distribution

Exponential-Logarithmic (EL) distribution was proposed by Tahmasbi and Rezaei (2008)^[46], a compound distribution formed from the exponential a continuous distribution and logarithmic, a discrete distribution. It is characterized with many salient features and its discretized version is derived here from its survival function which is given as follows,

The cumulative distribution function of discrete form of Exponential-Logarithmic distribution is derived as follows,

$$\begin{aligned} F(y) &= 1 - S_X(y) + P(Y = y) \\ &= 1 - S_X(y) + S_X(y) - S_X(y + 1) \\ &= 1 - S_X(y + 1) \\ &= 1 - \frac{\ln(1 - (1 - \gamma)\rho^{y+1})}{\ln \gamma} \\ &= \frac{\ln \gamma - \ln(1 - (1 - \gamma)\rho^{y+1})}{\ln \gamma} \\ &= \frac{1}{\ln \gamma} (\ln \gamma - \ln(1 - (1 - \gamma)\rho^{y+1})) \end{aligned} \quad (3)$$

Where $y = 0, 1, 2, \dots$; $0 < \gamma < 1$; $0 < \rho < 1$

Discrete Exponential-Geometric distribution

Discrete Exponential-geometric distribution is derived from the survival function of exponential-geometric distribution introduced by Adamidis and Loukas (1998)^[1] and is given as follows,

$$S(x, \kappa, \theta) = \frac{(1 - \kappa)e^{-\theta x}}{(1 - \kappa e^{-\theta x})} \quad (4)$$

Thus, Re-parameterizing we get,

$$\rho = e^{-\theta}$$

$$\begin{aligned} P_Y(\rho, \kappa) &= P(Y = y) = S_X(y) - S_X(y + 1) = \frac{(1 - \kappa)\rho^y}{(1 - \kappa\rho^y)} - \frac{(1 - \kappa)\rho^{y+1}}{(1 - \kappa\rho^{y+1})} \\ P_Y(y, \rho, \kappa) &= \frac{(1 - \kappa)\rho^y - \rho^{y+1}}{(1 - \kappa\rho^y)(1 - \kappa\rho^{y+1})} \end{aligned} \quad (5)$$

The cumulative distribution function of discrete form of exponential-geometric distribution is derived below,

$$F(y) = 1 - S_X(y) + P(Y = y)$$

$$S(y, \kappa, \theta) = \frac{(1 - \kappa)\rho^y}{(1 - \kappa\rho^y)}$$

$$\begin{aligned}
F(y) &= 1 - \frac{(1-\kappa)\rho^y}{(1-\kappa\rho^y)} + \frac{(1-\kappa)\rho^y - \rho^{y+1}}{(1-\kappa\rho^y)(1-\kappa\rho^{y+1})} \\
&= \frac{1 - \kappa\rho^y - \rho^y + \kappa\rho^y}{(1-\kappa\rho^y)} + \frac{(1-\kappa)(\rho^y + \rho^{y+1})}{(1-\kappa\rho^y)(1-\kappa\rho^{y+1})} \\
&= \frac{(1-\rho^y)(1-\kappa\rho^{y+1}) + \rho^y - \kappa\rho^y - \rho^{y+1} + \kappa\rho^{y+1}}{(1-\kappa\rho^y)(1-\kappa\rho^{y+1})} \\
&= \frac{1 - \kappa\rho^{y+1} - \rho^y + \kappa\rho^y\rho^{y+1} + \rho^y - \kappa\rho^y - \rho^{y+1} + \kappa\rho^{y+1}}{(1-\kappa\rho^y)(1-\kappa\rho^{y+1})}
\end{aligned}$$

Cancelling like terms we get,

$$\begin{aligned}
&= \frac{1 + \kappa\rho^y\rho^{y+1} - \kappa\rho^y - \rho^{y+1}}{(1-\kappa\rho^y)(1-\kappa\rho^{y+1})} \\
&= \frac{1 - \kappa\rho^y + \rho^{y+1}(\kappa\rho^y - 1)}{(1-\kappa\rho^y)(1-\kappa\rho^{y+1})} \tag{6}
\end{aligned}$$

Where $y = 0, 1, 2, \dots$; $0 < \kappa < 1$; $0 < \rho < 1$

Discrete Weibull-Logarithmic distribution

Discrete form of Weibull-Logarithmic distribution is obtained from the survival function of Weibull-logarithmic distribution propounded by Roxana Ciumara and Vasile preda (2009) ^[13] and the survival function of Weibull-Logarithmic distribution is given as,

$$S(x) = \frac{\ln(1 - (1-\kappa)e^{-\theta x^\gamma})}{\ln \kappa} \tag{7}$$

By re-parameterizing $\rho = e^{-\theta}$, the survival function can be re-written as follows,

$$S(x) = \frac{\ln(1 - (1-\kappa)\rho^{x^\gamma})}{\ln \kappa}$$

Thus probability mass function of discrete form of Weibull-logarithmic distribution is given below as,

$$\begin{aligned}
P_y(\gamma, \kappa, \rho) &= P(Y = y) = S_X(y) - S_X(y+1) \\
&= \frac{\ln(1 - (1-\kappa)\rho^{y^\gamma})}{\ln \kappa} - \frac{\ln(1 - (1-\kappa)\rho^{(y+1)^\gamma})}{\ln \kappa} \tag{8}
\end{aligned}$$

Cumulative distribution function of discrete Weibull-Logarithmic distribution is derived as follows,

$$F(y) = 1 - S_X(y) + P(Y = y)$$

$$F(y) = 1 - S_X(y) + S_X(y) - S_X(y+1)$$

Cancelling like terms we get,

$$F(y) = 1 - S_X(y+1)$$

$$1 - \frac{\ln(1 - (1-\kappa)\rho^{(y+1)^\gamma})}{\ln \kappa}$$

$$\frac{\ln \kappa - \ln(1 - (1-\kappa)\rho^{(y+1)^\gamma})}{\ln \kappa}$$

$$\frac{1}{\kappa} (\ln \kappa - \ln(1 - (1-\kappa)\rho^{(y+1)^\gamma})) \tag{9}$$

Where $y = 0, 1, 2, \dots$; $0 < \kappa < 1$; $0 < \rho < 1$; $\gamma > 0$

Discrete Weibull-Geometric Distribution

The pdf and cdf of the Weibull-Geometric (WG) distribution can be obtained, respectively, as follow

$$f(x) = \frac{\alpha\beta^\alpha x^{\alpha-1}(1-p)e^{-(\beta x)^\alpha}}{[1 - pe^{-(\beta x)^\alpha}]^2}; x > 0 \quad (10)$$

and

$$F(x) = \frac{1 - e^{-(\beta x)^\alpha}}{1 - pe^{-(\beta x)^\alpha}}; x > 0 \quad (11)$$

for the parameters α and β is positive, and $0 < p < 1$. α, β and p are the shape, scale and mixing parameters, respectively. The survival and hazard functions of the WG distribution are given:

$$S(x) = \frac{(1-p)e^{-(\beta x)^\alpha}}{1 - pe^{-(\beta x)^\alpha}}; x > 0 \quad (12)$$

And

$$h(x) = \frac{\alpha\beta^\alpha x^{\alpha-1}}{1 - pe^{-(\beta x)^\alpha}}; x > 0 \quad (13)$$

Formation of Discrete Weibull-Geometric distribution

Discrete form of Weibull-Geometric distribution is derived by Jayakumar and Babu (2018) ^[19] from the survival function of Weibull-Geometric distribution which was studied by Barreto-Souza, De Moraes and Cordeiro (2011) ^[6] and is obtained as follows,

$$S(x; \gamma, \theta, \kappa) = \frac{(1-\kappa)e^{-(\theta x)^\gamma}}{1 - \kappa e^{-(\theta x)^\gamma}}; x > 0 \quad (14)$$

Discrete version of Weibull-Geometric distribution is obtained below from the following formula,

$$p_y = S_X(y) - S_X(y+1)$$

$$p_y = \frac{(1-\kappa)e^{-(\theta y)^\gamma}}{1 - \kappa e^{-(\theta y)^\gamma}} - \frac{(1-\kappa)e^{-(\theta(y+1))^\gamma}}{1 - \kappa e^{-(\theta(y+1))^\gamma}}$$

With reparameterization we get $\rho = e^{-\theta^\gamma}$

$$p_y(y; \kappa, \rho, \gamma) = P(Y = y) = \frac{(1-\kappa)(\rho^{y^\gamma} - \rho^{(y+1)^\gamma})}{[1 - \kappa\rho^{y^\gamma}][1 - \kappa\rho^{(y+1)^\gamma}]} \quad (15)$$

$$y = 0, 1, 2, \dots; \gamma > 0, 0 < \kappa < 1; 0 < \rho < 1$$

The cumulative distribution function of DWG (κ, ρ, γ) is obtained as,

$$\begin{aligned} F(y) &= P(Y \leq y) = 1 - S_X(y) + P(Y = y) \\ &= 1 - \frac{(1-\kappa)\rho^{y^\gamma}}{1 - \kappa\rho^{y^\gamma}} + \frac{(1-\kappa)(\rho^{y^\gamma} - \rho^{(y+1)^\gamma})}{[1 - \kappa\rho^{y^\gamma}][1 - \kappa\rho^{(y+1)^\gamma}]} \\ &= \frac{1 - \kappa\rho^{y^\gamma} - (1-\kappa)\rho^{y^\gamma}}{1 - \kappa\rho^{y^\gamma}} + \frac{(1-\kappa)(\rho^{y^\gamma} - \rho^{(y+1)^\gamma})}{[1 - \kappa\rho^{y^\gamma}][1 - \kappa\rho^{(y+1)^\gamma}]} \\ &= \frac{1 - \kappa\rho^{y^\gamma} - \rho^{y^\gamma} + \kappa\rho^{y^\gamma}}{1 - \kappa\rho^{y^\gamma}} + \frac{\rho^{y^\gamma} - \rho^{(y+1)^\gamma} - \kappa\rho^{y^\gamma} + \kappa\rho^{(y+1)^\gamma}}{[1 - \kappa\rho^{y^\gamma}][1 - \kappa\rho^{(y+1)^\gamma}]} \\ &= \frac{(1 - \kappa\rho^{(y+1)^\gamma})(1 - \rho^{y^\gamma}) + \rho^{y^\gamma} - \rho^{(y+1)^\gamma} - \kappa\rho^{y^\gamma} + \kappa\rho^{(y+1)^\gamma}}{[1 - \kappa\rho^{y^\gamma}][1 - \kappa\rho^{(y+1)^\gamma}]} \end{aligned}$$

$$\begin{aligned}
&= \frac{1 - \rho^{y^y} - \kappa \rho^{(y+1)^y} + \kappa \rho^{(y+1)^y} \rho^{y^y} + \rho^{y^y} - \rho^{(y+1)^y} - \kappa \rho^{y^y} + \kappa \rho^{(y+1)^y}}{[1 - \kappa \rho^{y^y}][1 - \kappa \rho^{(y+1)^y}]} \\
&= \frac{1 + \kappa \rho^{(y+1)^y} \rho^{y^y} - \rho^{(y+1)^y} - \kappa \rho^{y^y}}{[1 - \kappa \rho^{y^y}][1 - \kappa \rho^{(y+1)^y}]} \\
&= \frac{(1 - \rho^{(y+1)^y}) - \kappa \rho^{y^y} (-\rho^{(y+1)^y} + 1)}{[1 - \kappa \rho^{y^y}][1 - \kappa \rho^{(y+1)^y}]} \\
&= \frac{(1 - \rho^{(y+1)^y})(1 - \kappa \rho^{y^y})}{[1 - \kappa \rho^{y^y}][1 - \kappa \rho^{(y+1)^y}]} \\
F(y) &= \frac{(1 - \rho^{(y+1)^y})}{(1 - \kappa \rho^{(y+1)^y})} \tag{16}
\end{aligned}$$

Where $y = 0, 1, 2, \dots$; $0 < \kappa < 1$; $0 < \rho < 1$; $\gamma > 0$

Survival function of DWG(κ, ρ, γ) is given below as,

$$\begin{aligned}
S(y) &= 1 - P(Y \leq y) = 1 - \frac{(1 - \rho^{(y+1)^y})}{(1 - \kappa \rho^{(y+1)^y})} \\
S(y) &= \frac{(1 - \kappa \rho^{(y+1)^y}) - (1 - \rho^{(y+1)^y})}{(1 - \kappa \rho^{(y+1)^y})}
\end{aligned}$$

Taking $\rho^{(y+1)^y}$ as a common term in the numerator, we get the survival function of DWG(κ, ρ, γ) as follows,

$$S(y) = \frac{(1 - \kappa) \rho^{(y+1)^y}}{(1 - \kappa \rho^{(y+1)^y})} \tag{17}$$

Hazard function of DWG(κ, ρ, γ) is given as follows,

$$\begin{aligned}
h(y) &= \frac{P(Y = y)}{P(Y \geq y)} = \frac{\frac{(1 - \kappa)(\rho^{y^y} - \rho^{(y+1)^y})}{[1 - \kappa \rho^{y^y}][1 - \kappa \rho^{(y+1)^y}]}}{\frac{(1 - \kappa) \rho^{(y+1)^y}}{(1 - \kappa \rho^{(y+1)^y})}} \\
h(y) &= \frac{(1 - \kappa)(\rho^{y^y} - \rho^{(y+1)^y})}{(1 - \kappa) \rho^{(y+1)^y}} * \frac{(1 - \kappa \rho^{(y+1)^y})}{[1 - \kappa \rho^{y^y}][1 - \kappa \rho^{(y+1)^y}]}
\end{aligned}$$

By cancelling the like terms reduced form of Hazard function of DWG(κ, ρ, γ) is given as follows,

$$h(y) = \frac{1 - \rho^{(y+1)^y - y^y}}{1 - \rho^{(y+1)^y}} \tag{18}$$

Properties of Discrete Weibull-Geometric distribution

➤ When $\gamma=1$, this distribution pmf reduces to pmf of discrete exponential-geometric distribution as,

$$p_y(y; \kappa, \rho) = \frac{(1 - \kappa)(\rho^{y^y} - \rho^{(y+1)^y})}{(1 - \kappa \rho^{y^y})(1 - \kappa \rho^{(y+1)^y})}$$

➤ As $\kappa \rightarrow 0$, $p_y(y; \kappa, \alpha) = (\rho^{y^y} - \rho^{(y+1)^y})$ is obtained which is the probability mass function of discrete Weibull distribution (Nakagawa and Osaki 1975) [31]

➤ When $\rightarrow 0$ and $\gamma \rightarrow 2$, we get $p_y(y; \rho) = (\rho^{y^2} - \rho^{(y+1)^2})$ the probability mass function of discrete Rayleigh distribution.

➤ The geometric distribution is obtained when, $\kappa \rightarrow 0$ and $\gamma \rightarrow 1$ as $p_y(y; \rho) = (\rho^{y^y} - \rho^{(y+1)^y})$ with the parameter ρ .

Reliability Acceptance Sampling Plans for Discrete Weibull-Geometric distribution

In this section, reliability acceptance sampling plan has been drafted for the discrete Weibull-geometric (WG) distribution by equating its cdf top, the proportion of product failing before time t and for sketching these plans, pivotal parameters are the assumed distribution's scale and shape parameter. Let us take t to be a lifetime random variable that is used to model certain electronic components which is assumed to follow the discrete Weibull-geometric distribution. In certain cases, the life time

random variable t has to be represented in the form of discrete variable and so in such cases discrete distributions takes the main role. In order to design the reliability acceptance sampling plans for this discrete distribution, we equate the cumulative distribution function of the distribution to p , proportion of product failing before time t

$$p = P(T \leq t) = F(t)$$

Here we take the lifetime random variable t as y as it is represented in the discrete form and so p is equated as follows,

$$p = \frac{(1 - \rho^{(y+1)^y})}{(1 - \kappa\rho^{(y+1)^y})}$$

Single Sampling Plan

Acceptance Sampling plans have the following plans, but not limited to that alone; some of them are single sampling plans (SSP), double sampling plans, sequential sampling plans, etc. In connection with the title ASP, it is clear that its main objective is to check the quality of products with some criteria and decide whether to accept or reject.

The procedure for the SSP is as follows:

- At the initial stage, choose a random sample n from the lot and subject it to inspection.
- Note down the number of defectives d from the sample of size n , where c is the acceptance number.
- If $d \leq c$, then accept the lot, otherwise reject it.

Minimum Sample Size

Designing an acceptance sampling plan's foremost requirement is finding the appropriate sample size n for conducting life tests, which is obtained with optimality considering two different kind of situations such as finite or infinite lot sizes modelled with the Hypergeometric distribution in the first case and Binomial or Poisson distribution in the latter case. In this contributory work, prime importance has been given for designing sampling plans with optimum sample size n and acceptance number c which are obtained by verifying the conditions given below in equations (23) and (24).

Producer and Consumer risk

The probability of rejecting a good lot is a risk for the producer, so it is named as Producer risk (α) and here the producer risk is fixed as $\alpha=0.05$. Probability of accepting a bad lot is termed as Consumer risk in terms of acceptance sampling plans and it is taken as $\beta = 0.10$ for which the sampling plans in this contributory work are drafted by satisfying both the producer and consumer's risks for the probability of acceptance while the Binomial distribution is used for modelling as given in equations (19) and (20).

$$P_a(p_1) \Rightarrow \binom{n}{x} p^x q^{n-x} \geq 1 - \alpha \quad (19)$$

$$P_a(p_2) \Rightarrow \binom{n}{x} p^x q^{n-x} \leq \beta \quad (20)$$

Operating Characteristic function for designing the reliability acceptance sampling plan for Discrete Weibull-Geometric distribution

The operating characteristic curve plays a significant role for assessing the strength of a sampling plan which is mainly obtained from the operating characteristic function evaluated using the discrete models such as the Hypergeometric, Binomial and the Poisson distributions where the first one is used when the lot is of finite size and the other two models are used when the lot is of infinite sizes or formed from the process. For designing the reliability acceptance sampling plan, Binomial or Poisson distributions are used and their probability mass functions are as follows,

$$p(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, 2, \dots, n \quad (21)$$

$$p(x) = e^{-np} \frac{(np)^x}{x!}, x = 0, 1, 2, \dots \quad (22)$$

Where, n is the sample size, p is the proportion defective, $0 \leq p \leq 1$, q is the proportion effective, $q = 1 - p$ and x is the number of occurrences. The procedure for designing the reliability acceptance sampling plan for the discrete Weibull-geometric distribution involves the distribution parameters and the sampling plan parameters such as sample size n and the acceptance number c . In order to design a single sampling plan, the OC curve should pass through two vital points namely (p_1, α) and (p_2, β) satisfying the following conditions in terms of probability of acceptance modelled using the discrete distributions such as the Binomial or Poisson distribution equations given above. Table 1 displays the reliability acceptance sampling plans of optimum n and c values by satisfying the conditions modelled using the Binomial distribution.

$$P_a(p_1) \geq 1 - \alpha \quad (23)$$

And

$$P_a(p_2) \leq \beta \quad (24)$$

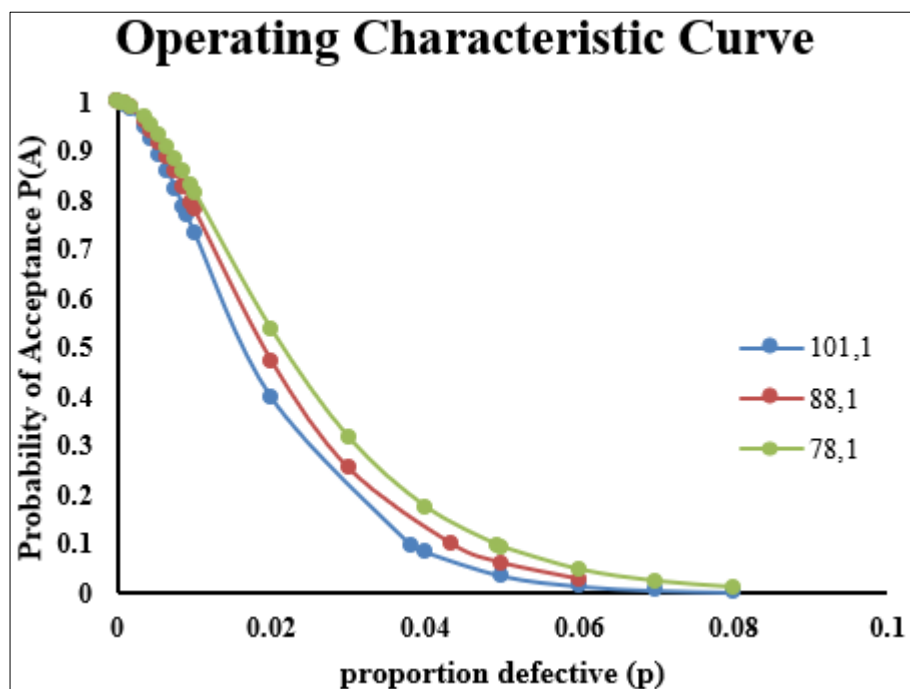


Fig 1: Operating Characteristic Curve (For varying sample sizes n)

The above operating characteristic curve is a depiction of different OC curves with varying sample size n and a constant acceptance number c obtained from the sampling plan table given below. OC curve 1 is graphed for sample size 101 and acceptance number 1, while the OC curve 2 represents the sample size 88 and acceptance number 1 and OC curve 3 is graphed for a sample size of 78 and acceptance number 1. This picture is a visual representation that as the sample size decreases keeping the acceptance number as constant value, the OC curve bends towards the left.

Numerical Illustration

An experimenter conducts a life test experiment for his manufactured products and their lifetime are represented in terms of number of days and the products are assumed to follow the discrete Weibull-Geometric (WG) distribution. The proportion defective is got from the table 1 as while the sample size and acceptance number for this kind of experiment are obtained as $n=160$ and $c=2$ from the table 1 when $p_1=0.0032$ and $p_2=0.0330$ values are taken as 0.0060 and 0.0364

- For a lot inspection, the sample size of $n=145$ is taken and the life test is performed for a certain number of days.
- The number of failures observed are compared to the acceptance number $c=2$ that occurs within the specified number of days or months or years according to the product taken for testing.
- If the number of failures is less than or equal to the acceptance number c , then the respective lot is accepted, otherwise the lot is rejected.

In particular, we can relate these kinds of discrete distributions with several applications in real life where the lifetime is taken as a discrete case such as for instance, the medical equipment that are used in the hospitals specifically mentioning thermometer used for measuring the body temperature of the patient can be tested for its life and come to a conclusion about its functioning strength for a specified number of years which is a discrete case. This is a valid example of where the discrete form distributions take a major role in real time. Not only limited to this, but the expiry date of tablets is mentioned either by number of days or years which is also a best example of discrete case that can be modelled using this type of distributions and also certain kind of devices has its lifetime recorded in number of weeks, a discrete case.

Table 1: Reliability Acceptance Sampling Plans for discrete Weibull-Geometric (WG) distribution when the shape parameter takes the values $\gamma = 0.7$ and values $\kappa = 0.8$ (n, c) (Producer Risk $\alpha = 0.05$, Consumer Risk $\beta = 0.10$)

$p_1 \backslash p_2$	0.0002	0.0008	0.0018	0.0032	0.0050	0.0072	0.0097	0.0127
0.0197	116,0	196,1	196,1	338,3	469,50	657,8	1195,17	2725,44
0.0238	96,0	163,1	163,1	223,2	335,4	441,6	696,11	1230,22
0.0282	81,0	137,1	137,1	188,2	236,3	327,5	459,8	711,14
0.0330	69,0	117,1	117,1	160,2	160,2	241,4	318,6	429,9
0.0381	60,0	60,0	101,1	101,1	139,2	174,3	242,5	307,7
0.0435	52,0	52,0	88,1	88,1	121,2	152,3	182,4	240,6
0.0493	46,0	46,0	78,1	78,1	107,2	107,2	134,3	186,5
0.0554	41,0	41,0	69,1	69,1	69,1	69,1	119,3	143,4
0.0618	37,0	37,0	62,1	62,1	62,1	62,1	107,3	107,3
0.0685	33,0	33,0	56,1	56,1	56,1	56,1	77,2	96,3

Conclusion

In this contributory work, the probability mass function and the cumulative distribution functions are derived for the discrete versions of the Exponential-Logarithmic distribution, Exponential-Geometric distribution and Weibull-Logarithmic distributions. Reliability acceptance sampling plans has been designed considering the assumption that the products follow discrete Weibull-Geometric distribution. As a future scope, the derivations related to mean, skewness, kurtosis, and other measures for the above said discrete distributions are to be derived, and the plans related to them are extended.

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