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New exact wave solution of KMN equation by the exponential rational function method

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Abstract

In this paper, we employ the exponential rational function method to construct precise solutions for a nonlinear equation. First of all, we apply the traveling wave transformation to transform the original nonlinear equation into an ordinary differential equation. Secondly, employing the principles of homogeneous equilibrium, we can obtain the wave solution for Kundu-Mukherjee-Naskar (KMN) equation. Finally, using Maple software, we analyze the properties of both bright and dark wave solutions.

Keywords: The exponential rational function method, KMN equation, Wave solution

Introduction

Nonlinear evolution equations, which are holding substantial importance in both theoretical frameworks and practical applications, constitute a significant area within contemporary mathematics. The Kundu-Mukherjee-Naskar (KMN) equation ^[1] reveals quite important optical solitons of its (2+1) -dimension. The intricate KMN equation appears in a range of disciplines within nonlinear mathematical physics, encompassing fields such as hydrodynamics, condensed matter physics, nonlinear optics, plasma physics, and so on. The key aspect of this equation lies in that it represents a novel extension of the nonlinear Schrödinger equation, incorporating various forms of nonlinearity in relation to both Kerr and non-Kerr law nonlinearities. This extension is particularly significant for the analysis of soliton pulses in (2+1)-dimensional spaces. The dynamics of solitons in (1+1) dimensions have been extensively studied by numerous researchers, whereas investigations into solitons in (2+1) dimensions have been undertaken by only a limited number of scholars ^[2-6].

So, we studied the KMN equation, and focused on finding the exact solutions to display the competence of the exponential rational function method ^[7] to KMN

$$iu_x + au_{xy} + ibu(uu_x^* - u^*u_x) = 0 \quad (1)$$

The temporal evolution of the pulses is governed by the first term, while the profile of the optical solitons is characterized by the corresponding function $u(x, y, t)$. Furthermore, the dispersion and nonlinearity effects are maintained by their respective coefficients.

The exponential rational function method

Suppose KMN has the following form

$$G(u, u_t, u_x, u_{tt}, u_{xx}, \dots) = 0, \quad (2)$$

where G stands for a polynomial function.

By using the exponential rational function method, the steps to solve the exact wave solution of the KMN equation can be given.

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Step 1: Considering wave transformation to convert multiple independent variables into a single variable involves the application of specific mathematical techniques that simplify the analysis of complex systems

$$u(x, t) = U(\xi) \quad , \quad \xi = hx + lt \quad , \quad (3)$$

where h, l are wave length and i frequency. By using Equation (3), Equation (2) can be converted into ODE form

$$P(U, U', U'', U''', \dots) = 0 \quad (4)$$

Step 2: Solution of Equation (4) has the form

$$\psi(\xi) = \frac{c_0 + c_1 \phi(\xi) + \dots + c_N \phi^N(\xi)}{d_0 + d_1 \phi(\xi) + \dots + d_N \phi^N(\xi)} \quad (5)$$

With

$$\phi(\xi) = \frac{p_1 e^{q_1 \xi} + p_2 e^{q_2 \xi}}{p_3 e^{q_3 \xi} + p_4 e^{q_4 \xi}} \quad , \quad (6)$$

Where

$c_i, d_i, (i = 0, 1, \dots, N)$ are the unknown coefficients and $p_i, q_i, (i = 1, 2, 3, 4)$ are arbitrary constants.

Step 3: The positive integer N is obtained by applying the homogeneous balancing principle outlined in Equation (4).

Step 4: Substituting Equation (5) into Equation (4) lead to a polynomial with respect to $e^{q_i \xi} (i = 1, 2, 3, 4)$. Set $e^{q_i \xi} = 0 (i = 1, 2, 3, 4)$ lead to a set of equations, which can be solved by Maple.

Step 5: By substituting the parameter values obtained from the previous stage into Equation (5), the results of the wave described in Equation (4) can be derived in this manner.

3. Applications of exponential rational function method

First, suggest

$$u(x, y, t) = \psi(\xi) e^{i\mu}, \quad \xi = A_1 x + A_2 y - ct, \quad \mu = -k_1 x - k_2 y + wt \quad , \quad (7)$$

where A_1, A_2, c are arbitrary constants. The parameters k_1, k_2 mean frequency and the parameter w stands for wave number whilst. By using (7), Eq. (1) is transformed into

$$2bk_1 \psi^3 + (ak_1 k_2 + w) \psi - aA_1 A_2 \psi'' = 0, \quad c = -a(A_1 k_2 + A_2 k_1) \quad (8)$$

Utilizing the balancing principle as outlined in Equation (8) and assuming the solution to be represented by

$$\psi(\xi) = \frac{c_0 + c_1 \phi(\xi)}{d_0 + d_1 \phi(\xi)} \quad (9)$$

Family 1: If $p = [2 - i, 2 + i, i, -i]$ and $q = [i, -i, i, -i]$, then we obtain

$$\phi(\xi) = \frac{2 \cos \xi + \sin \xi}{-\sin \xi} \quad (10)$$

Case 1

$$c_0 = 2\sqrt{\frac{aA_1A_2}{bk_1}}d_1, \quad c_1 = 0, \quad d_0 = d_1, \quad d_1 = d_1, \quad w = a(2A_1A_2 - k_2k_1) \quad (11)$$

Solution of Equation (1) is

$$u_1(\xi) = 2\sqrt{\frac{aA_1A_2}{bk_1}} \frac{1}{1 - \frac{2\cos\xi + \sin\xi}{\sin\xi}} e^{i\mu} \quad (12)$$

Where

$$\xi = A_1x + A_2y + a(A_1k_2 + A_2k_1)t, \quad \mu = -k_1x - k_2y + a(2A_1A_2 - k_2k_1)t.$$

Case 2:

$$c_0 = \frac{\sqrt{4}}{4} \sqrt{\frac{aA_1A_2}{bk_1}}(d_0 - 5d_1), \quad c_1 = \frac{\sqrt{4}}{4} \sqrt{\frac{aA_1A_2}{bk_1}}(d_0 - d_1),$$

$$d_0 = d_0, \quad d_1 = d_1, \quad w = a(2A_1A_2 - k_2k_1) \quad (13)$$

Solution of Equation (1) is

$$u_2(\xi) = \frac{\sqrt{4}}{4} \sqrt{\frac{aA_1A_2}{bk_1}} \frac{-4d_1 \sin\xi - 2(d_0 - d_1) \cos\xi}{(d_0 - d_1) \sin\xi - 2d_1 \cos\xi} e^{i\mu} \quad (14)$$

Where

$$\xi = A_1x + A_2y + a(A_1k_2 + A_2k_1)t, \quad \mu = -k_1x - k_2y + a(2A_1A_2 - k_2k_1)t.$$

Case 3

$$c_0 = (-2 + \sqrt{3}) \sqrt{\frac{aA_1A_2}{bk_1}}d_1, \quad c_1 = \sqrt{3} \sqrt{\frac{aA_1A_2}{bk_1}}d_1,$$

$$d_0 = (1 + 2\sqrt{3})d_1, \quad d_1 = d_1, \quad w = a(2A_1A_2 - k_2k_1) \quad (15)$$

Solution of Equation (1) is

$$u_3(\xi) = \sqrt{\frac{aA_1A_2}{bk_1}} \frac{-2\sin\xi - 2\sqrt{3}\cos\xi}{2\sqrt{3}\sin\xi - 2\cos\xi} e^{i\mu} \quad (16)$$

Where

$$\xi = A_1x + A_2y + a(A_1k_2 + A_2k_1)t, \quad \mu = -k_1x - k_2y + a(2A_1A_2 - k_2k_1)t.$$

Case 4

$$c_0 = \frac{c_1d_0}{d_1}, \quad c_1 = c_1, \quad d_0 = d_1, \quad d_1 = d_1, \quad w = -\frac{ad_1^2k_2 + 2bc_1^2}{d_1^2} \quad (17)$$

Solution of Equation (1) is

$$u_4(\xi) = \frac{c_1}{d_1} e^{i\mu} \quad (18)$$

where

$$\xi = A_1 x + A_2 y + a(A_1 k_2 + A_2 k_1)t, \quad \mu = -k_1 x - k_2 y - \frac{ad_1^2 k_2 + 2bc_1^2}{d_1^2} t.$$

Family 2: Suppose $p = [2, 3, 1, 1]$, $q = [1, 0, 1, 0]$, we have

$$\varphi(\xi) = \frac{3e^\xi + 2}{e^\xi + 1} \quad (19)$$

Case 1:

$$c_0 = -\frac{\sqrt{4}}{4} \sqrt{\frac{aA_1 A_2}{bk_1}} (5d_0 + 12d_1), \quad c_1 = \frac{\sqrt{4}}{4} \sqrt{\frac{aA_1 A_2}{bk_1}} (2d_0 + 5d_1),$$

$$d_0 = d_0, \quad d_1 = d_1, \quad w = -\frac{a(A_1 A_2 + 2k_2 k_1)}{2} \quad (20)$$

Solution of Equation (1) is

$$u_1(\xi) = \frac{\sqrt{4}}{4} \sqrt{\frac{aA_1 A_2}{bk_1}} \frac{(d_0 + 3d_1)e^\xi - (d_0 + 2d_1)}{(d_0 + 3d_1)e^\xi + (d_0 + 2d_1)} e^{i\mu} \quad (21)$$

Where

$$\xi = A_1 x + A_2 y + a(A_1 k_2 + A_2 k_1)t, \quad \mu = -k_1 x - k_2 y - \frac{a(A_1 A_2 + 2k_2 k_1)}{2} t.$$

Case 2

$$c_0 = \frac{\sqrt{16}}{16} \sqrt{\frac{aA_1 A_2}{bk_1}} (5d_0 + 12d_1), \quad c_1 = 0, \quad d_0 = -\frac{5}{2}d_1, \quad d_1 = d_1, \quad w = -\frac{a(A_1 A_2 + 2k_2 k_1)}{2} \quad (22)$$

Solution of Equation (1) is

$$u_2(\xi) = \frac{16}{8} \sqrt{\frac{aA_1 A_2}{bk_1}} \frac{e^\xi + 1}{e^\xi - 1} e^{i\mu} \quad (23)$$

Where

$$\xi = A_1 x + A_2 y + a(A_1 k_2 + A_2 k_1)t, \quad \mu = -k_1 x - k_2 y - \frac{a(A_1 A_2 + 2k_2 k_1)}{2} t.$$

Case 3

$$c_0 = \frac{d_0 c_1}{d_1}, \quad c_1 = c_1, \quad d_0 = d_0, \quad d_1 = d_1, \quad w = -\frac{k_1(ak_2 d_1^2 + 2bc_1^2)}{d_1^2} \quad (24)$$

Solution of Equation (1) is

$$u_3(\xi) = \frac{c_1}{d_1} e^{i\mu} \quad (25)$$

Where

$$\xi = A_1 x + A_2 y + a(A_1 k_2 + A_2 k_1)t, \quad \mu = -k_1 x - k_2 y - \frac{k_1(ak_2 d_1^2 + 2bc_1^2)}{d_1^2} t$$

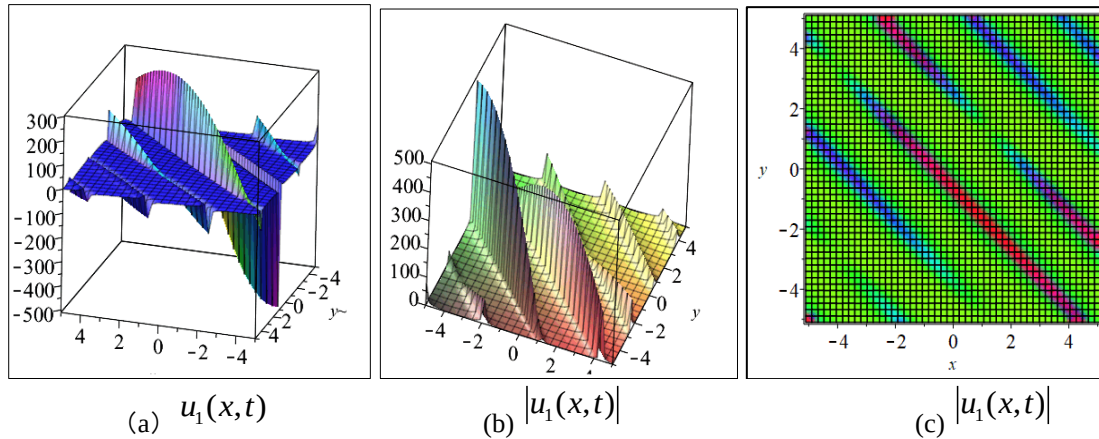


Fig 1: Traveling waves of solutions (12) with different shapes are plotted at $a = 2, b = 1, A_1 = A_2 = 1, d_0 = d_1 = 1, k_1 = k_2 = 1$

In figure 1, (a) provides the 3D-plot of $u_1(x, t)$, (b) and (c) give the 3D-plot and contour plot of absolute value of $|u_1(x, t)|$, respectively.

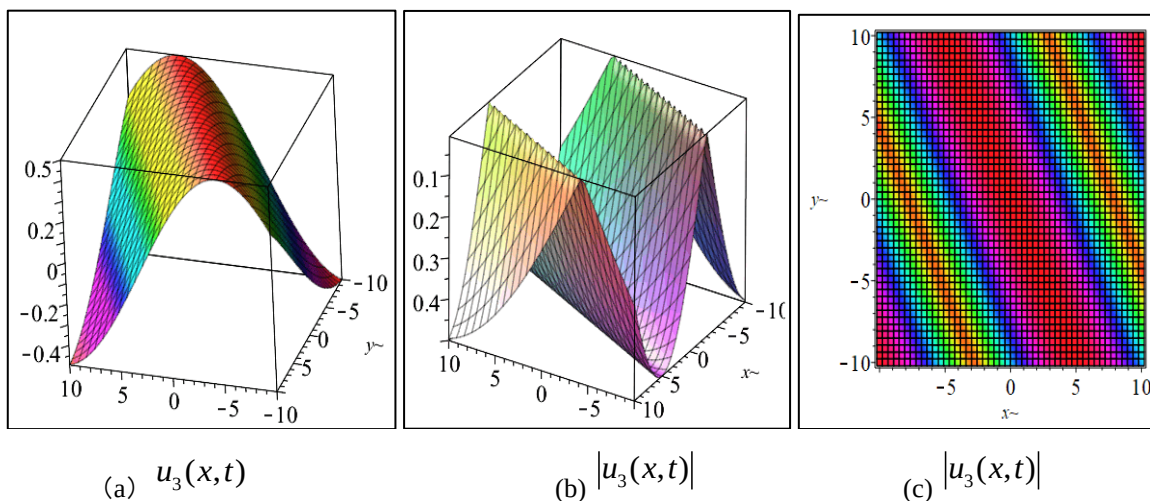


Fig 2: Traveling waves of solutions (24) with different shapes are plotted at $a = 2, b = 1, A_1 = A_2 = 1, c_1 = 0.5, d_0 = 0.3, d_1 = 1, k_1 = 0.2, k_2 = 1$

In figure 2, (a) provides the 3D-plot of $u_3(x, t)$, (b) and (c) give the 3D-plot and contour plot of absolute value of $|u_3(x, t)|$, respectively.

Graphical Representations

The results obtained from this study differ significantly from those reported by various researchers using previous techniques, primarily due to the expected outcomes outlined in equations (5) and (6) of the proposed method. Notably, the ordinary differential equation (ODE) presented in equation (6) yields several distinct types of solutions, which are derived by assigning specific values to the parameters involved.

Conclusions

In this paper, we derived the exact solution of the KMN equation and used the transform method as well as exponential rational function to separate the considered equation. We have obtained a bright periodic solution, as well as a dark wave soliton. The findings indicate that the application of this equation within the field of nonlinear optics offers a valuable approach for addressing

other nonlinear equations in mathematical physics. In future research, we can explore the possibility of integrating multiple functions to enhance the methods for correcting the equation and investigate the precise solutions of nonlinear compound equations.

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