

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452

Maths 2024; 9(6): 08-16

© 2024 Stats & Maths

<https://www.mathsjournal.com>

Received: 13-08-2024

Accepted: 14-09-2024

Rajeev Kumar

Assistant Professor Mathematics,
Himachal Pradesh University
Department of Evening Studies,
The Mall, Shimla-1, Himachal
Pradesh, India

Aarti Manglesh

Assistant Professor,
Mathematics, ICDEOL,
Himachal Pradesh University,
Summer Hill, Shimla-5,
Himachal Pradesh, India

Ashish Kumar

Research Scholar, Department of
Mathematics and Statistics,
Himachal Pradesh University,
Summer Hill, Shimla-5,
Himachal Pradesh, India

Analysis of the effect of variable gravity on stationary convection in Rayleigh-Bénard convection for free-free, rigid-rigid and rigid-free boundary conditions

Rajeev Kumar, Aarti Manglesh and Ashish Kumar

DOI: <https://dx.doi.org/10.22271/math.2024.v9.i6a.1879>

Abstract

In this research article Rayleigh-Bénard convection has been studied considering variable gravity. We analysed gravitational field variation for six different cases, namely, (i) $f(z) = z^2 - 2z$, (ii) $f(z) = -z^2$, (iii) $f(z) = -z$, (iv) $f(z) = z$, (v) $f(z) = e^z$ and (vi) $f(z) = \log(1 + z)$. Galerkin type weighted residual method (GWRM) is used to derive mathematical expression for Rayleigh number (Stationary Convection), for all cases of gravitational field variation, considering free-free, rigid-rigid and rigid-free boundary conditions. It is observed that the system's stability is influenced by both the gravitational field variation and the type of boundary conditions. Decreasing gravity tends to stabilize the system, while increasing gravity tends to destabilize it. Additionally, rigid boundaries help stabilize the convection process, while free boundaries make the system more prone to instability.

Keywords: Rayleigh-Bénard convection, variable gravity, galerkin type weighted residual method, stationary convection

Introduction

The Rayleigh-Bénard problem, which describes the onset of natural convection in a fluid layer heated from below, is a fundamental problem in fluid dynamics, particularly in the study of heat transfer and stability. This problem involves the spontaneous conversion of potential energy (due to the thermal gradient) into kinetic energy (due to fluid motion), leading to convection when the temperature gradient exceeds a critical threshold. The stability of such convection is influenced by the interaction of buoyancy forces, viscosity, and thermal gradients.

The phenomenon was first experimentally demonstrated by Bénard ^[1], but it was Rayleigh ^[2] who laid down the theoretical framework for the onset of instability. Rayleigh formulated the Rayleigh number, a dimensionless number that characterizes the convection process. According to Rayleigh's criterion, convection becomes unstable when the Rayleigh number exceeds a critical value, which depends on the fluid properties (like viscosity and thermal conductivity), the temperature gradient, and the physical dimensions of the system. This threshold marks the transition from a stationary, stable state to a convective regime.

Subsequent studies, including those by Jeffreys ^[3], Pellew and Southwell ^[4], and Chandrasekhar ^[5], further refined the understanding of Rayleigh-Bénard convection, introducing the principle of the exchange of stabilities and exploring various effects such as rotation, magnetic fields, and the impact of different boundary conditions. For instance, the work by Gupta *et al.* ^[6] and Banerjee *et al.* ^[7] examined the influence of rotation and magnetic fields on the Rayleigh-Bénard problem, and their work provided upper bounds and sufficient conditions for the stability of the system under these conditions. Bodenschatz *et al.* ^[8] work provides a comprehensive summary of results on Rayleigh-Bénard convection, particularly focusing on compressed gases and gas mixtures. These systems are of interest because they exhibit different behaviors compared to conventional fluids, especially when the Prandtl number is near one or smaller.

Corresponding Author:**Rajeev Kumar**

Assistant Professor Mathematics,
Himachal Pradesh University
Department of Evening Studies,
The Mall, Shimla-1, Himachal
Pradesh, India

Tagare *et al.* [9] research investigates the linear and weakly nonlinear properties of Rayleigh-Bénard convection in rotating fluids and explores the effects of the Coriolis force on gravity-driven convection. Their work is particularly important in understanding how rotation influences convection, especially for idealized stress-free boundary conditions, which are a simplifying assumption often made in theoretical studies. All these researchers, in their study of Rayleigh-Bénard convection problem, consider Earth's gravitational field as constant.

A crucial simplification in many studies of Rayleigh-Bénard convection is the assumption of constant gravitational acceleration. However, this assumption may not always be appropriate, particularly in geophysical or astrophysical contexts, where gravity can vary with height. For example, in the Earth's atmosphere, ocean currents, or in planetary mantles, the gravitational field changes with altitude, and this can influence the convection process. Several studies have begun to explore the effect of variable gravity on the onset of instability in Rayleigh-Bénard convection. Gresho and Sani [10] were among the first to investigate the impact of time-dependent gravity on fluid stability. Later, researchers like Pradhan and Samal [11] did analytical investigation of the instability, sufficient condition for stability and a circle theorem for the growth rate of a layer of fluid under the action of a gravitational field varying with height, was derived. Straughan [12] showed that gravity variation could either stabilize or destabilize the convection depending on the direction and the magnitude of the change in gravity. Pradhan *et al.* [13] showed that increasing the vertical gravitational field accelerates the onset of convection. Rionero and Straughan [14] numerically investigated convection in a porous layer under a depth-dependent gravitational field, highlighting how gravity variations influence the thermal instability and convection patterns in porous media.

Several studies [15, 16] have explored the effects of time-dependent gravity on thermal instability. Time-varying gravitational fields can occur in systems undergoing acceleration or deceleration, such as in rotating fluids or in systems under the influence of tidal forces or acceleration due to external influences. The impact of such time-dependent gravity is significant because it can modulate the strength of buoyancy forces that drive convection, leading to changes in the onset of instability and the nature of convection patterns. Suma and Indira [17] study examined the effect of throughflow (fluid movement due to pressure gradients) and variable gravity on thermal convection in porous media using the regular perturbation technique. Chand *et al.* [18] investigated the impact of variable gravity on thermal instability in a nanofluid in a porous medium. They showed that the gravity parameter (which characterizes the strength of gravity variation) has a significant effect on fluid velocity. For porous medium the effect of variable gravity for different kind of fluids was studied by different researchers [19-23]. The study by Han *et al.* [24] investigates the dynamic transition in Rayleigh-Bénard convection under the influence of internal heating and a varying gravity field. This work adds an important layer to our understanding of convection by considering both internal heating and variable gravity, which are common in many natural and engineered systems. Yadav [25] used Galerkin-weighted residual technique, to study the influence of varying downward gravity field and constant throughflow on the onset of Darcy-Brinkman convection in a porous medium layer. Shekar *et al.* [26] studied the effects of variable gravity on rotating convection in a sparsely packed porous layer. Using

the MATLAB-based bvp4c method, they showed that the gravity variation parameter and the Taylor number (which characterizes the effects of rotation on the convection) have an impact on the onset of convection. Specifically, they found that gravity variations and the Taylor number together could postpone the onset of convection, whereas the Darcy number (which relates to the permeability of the porous medium) could accelerate it. Surya and Gupta [27] investigated the instability of a liquid layer heated from below under the influence of a variable gravitational field. They considered thermally conducting permeable boundaries. Their results show that the onset of convection is significantly affected by the variation in gravity. Pant and Algehyne [28] numerically studied ferromagnetic convection influenced by variable gravity and viscosity. They hypothesized six different types of gravitational field variations to explore how gravity, in combination with magnetic effects, can influence convection patterns. Francis *et al.* [29] conducted a comparative study on the effect of gravity modulation on Rayleigh-Bénard convection with different boundary conditions. They found that gravity modulation has a stabilizing effect on the system, particularly when the boundaries are configured in certain ways. The effect of variable gravity field, using nonlinear stability analysis, on the onset of penetrative convection was analysed by Arone *et al.* [30] and conditions for the onset of steady convection were also obtained.

While much has been done to understand the role of gravity in Rayleigh-Bénard convection, the effect of varying gravity on convection with free-free, rigid-rigid and rigid-free boundary conditions; particularly when the gravity field varies in specific ways, such as the six distinct gravitational field variations hypothesized by Pant and Algehyne [28], has not yet been thoroughly studied. This gap in the literature motivates further investigation into how variable gravity might influence the onset of instability and the resulting convective patterns, especially in systems with free-free, rigid-rigid and rigid-free boundary conditions, where the fluid boundaries are not constrained.

1. Physical Problem and Governing Equations

Here, we study finitely heat conducting viscous Boussinesq fluid, which is confined statically, between two horizontal boundaries $z = 0$ and $z = d$, of infinite horizontal extension. At lower boundary $z = 0$ temperature is taken to be T_0 and $T_1 (< T_0)$ is temperature at upper boundary $z = d$ (see Figure 1). Variable gravity acting on entire system is assumed from Pant and Algehyne [28] of the form $\vec{g} = (0, 0, -g(1 + \delta f(z)))$ where $f(z)$ is changeable gravity function and δ is a gravity variation parameter ($\delta \in (0, 1)$). It is further assumed that the cross-diffusion effects of the stratifying agencies may be neglected.

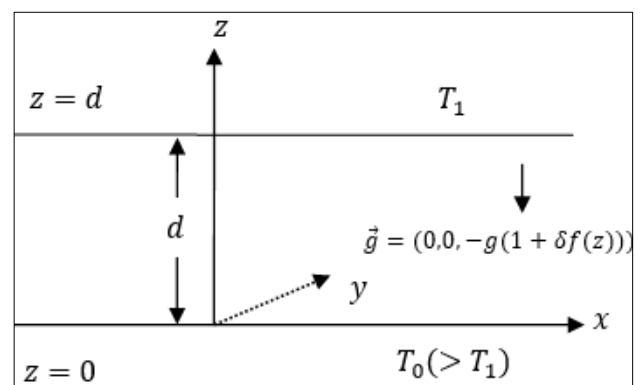


Fig 1: Geometrical configuration

Following the linear stability theory ^[5], making use of the initial state solutions, linearized perturbation equations and normal mode analysis, we get the following non-dimensional governing equations and boundary conditions

$$(D^2 - a^2) \left(D^2 - a^2 - \frac{p}{\sigma} \right) w = Ra^2(1 + \delta f(z))\theta, \quad (1)$$

$$(D^2 - a^2 - p)\theta = -w, \quad (2)$$

$$w = 0 = \theta = D^2 w \text{ at } z = 0 \text{ and } z = 1 \text{ (Lower and upper boundaries are free),} \quad (3)$$

$$w = 0 = \theta = Dw \text{ at } z = 0 \text{ and } z = 1 \text{ (Lower and upper boundaries are rigid),} \quad (4)$$

$$w = 0 = \theta = Dw \text{ at } z = 0 \text{ (Lower boundary is rigid),} \quad (5)$$

$$w = 0 = \theta = D^2 w \text{ at } z = 1 \text{ (Upper boundary is free),} \quad (6)$$

where $a, z, D = \frac{d}{dz}$, $\sigma > 0$, $R > 0$, $p = p_r + i p_i$, w and θ represents the wave number, vertical coordinate, differentiation with respect to z , Prandtl number, thermal Rayleigh number, complex growth rate (complex number) where p_r and p_i are real constants, vertical velocity and temperature, respectively.

Mathematical Analysis

We consider two categories of gravitational field variation, here, namely

- Decreasing gravitational field variation
- Increasing gravitational field variation

Table 1: Different cases for decreasing gravitational field variation.

Decreasing gravitational field variation		
Case (i)	$f(z) = z^2 - 2z$	Convex Decreasing
Case (ii)	$f(z) = -z^2$	Concave Decreasing
Case (iii)	$f(z) = -z$	Linear Decreasing

Table 2: Different cases for increasing gravitational field variation.

Increasing gravitational field variation		
Case (iv)	$f(z) = z$	Linear Increasing
Case (v)	$f(z) = e^z$	Convex Increasing
Case (vi)	$f(z) = \log(1 + z)$	Concave Increasing

We now solve the above equations (1)-(2) using Galerkin Method for different combinations of boundary conditions (3)-(6). We use Galerkin type weighted residual method to obtain Rayleigh numbers for free-free, rigid-rigid and rigid-free boundary conditions.

Stability Analysis when Lower and Upper boundaries are free

We consider the trial functions for w and θ , satisfying the boundary condition (3), as given below

$$w = w_0(z^4 - 2z^3 + z) \text{ and } \theta = \theta_0 z(z - 1). \quad (7)$$

Now, let us consider the case (iv) i.e. $f(z) = z$, for increasing gravitational field variation, and obtain the Rayleigh Number for free-free boundary condition.

Substituting equation (7) in equations (1)-(2), integrating each equation within the limits 0 to 1 and using Galerkin type weighted residual method, we get following systems of equations

$$\left[4032 + \left(2a^2 + \frac{p}{\sigma} \right) 408 + \left(a^4 + \frac{a^2 p}{\sigma} \right) \frac{124}{3} \right] w_0 + 17Ra^2(2 + \delta)\theta_0 = 0, \quad (8)$$

$$\text{and } 17w_0 + 14(10 + a^2 + p)\theta_0 = 0, \quad (9)$$

respectively.

For non-trivial solution of simultaneous system of equations (8) and (9), the determinant of coefficients of w_0 and θ_0 must vanish, mathematically

$$\begin{vmatrix} 4032 + \left(2a^2 + \frac{p}{\sigma} \right) 408 + \left(a^4 + \frac{a^2 p}{\sigma} \right) \frac{124}{3} & 17Ra^2(2 + \delta) \\ 17 & 14(10 + a^2 + p) \end{vmatrix} = 0.$$

This determinant on simplification, yields

$$R^{ff} = \frac{14(10+a^2+p) \left[12096 + \left(2a^2 + \frac{p}{\sigma} \right) 1224 + \left(a^4 + \frac{a^2 p}{\sigma} \right) 124 \right]}{289a^2(6+3\delta)},$$

when $f(z) = \log(1 + z)$. (10)

Following the same, above mentioned, steps, Rayleigh number for remaining five cases of gravitational field variation can be obtained and are given below

$$R^{ff} = \frac{14(10+a^2+p) \left[12096 + \left(2a^2 + \frac{p}{\sigma} \right) 1224 + \left(a^4 + \frac{a^2 p}{\sigma} \right) 124 \right]}{17a^2(102-73\delta)},$$

when $f(z) = z^2 - 2z$. (11)

$$R^{ff} = \frac{14(10+a^2+p) \left[12096 + \left(2a^2 + \frac{p}{\sigma} \right) 1224 + \left(a^4 + \frac{a^2 p}{\sigma} \right) 124 \right]}{17a^2(102-29\delta)},$$

when $f(z) = -z^2$. (12)

$$R^{ff} = \frac{14(10+a^2+p) \left[12096 + \left(2a^2 + \frac{p}{\sigma} \right) 1224 + \left(a^4 + \frac{a^2 p}{\sigma} \right) 124 \right]}{289a^2(6-3\delta)},$$

when $f(z) = -z$. (13)

$$R^{ff} = \frac{14(10+a^2+p) \left[6048 + \left(2a^2 + \frac{p}{\sigma} \right) 612 + \left(a^4 + \frac{a^2 p}{\sigma} \right) 62 \right]}{17a^2(51+1260\delta(1120-412e))},$$

when $f(z) = e^z$. (14)

$$R^{ff} = \frac{14(10+a^2+p) \left[6048 + \left(2a^2 + \frac{p}{\sigma} \right) 612 + \left(a^4 + \frac{a^2 p}{\sigma} \right) 62 \right]}{17a^2(51+20.286\delta)},$$

when $f(z) = \log(1 + z)$. (15)

Stationary Convection

Condition for stationary convection at the marginal state, is obtained by putting $p = 0$ in the equations (10)-(15), which on simplification yields Rayleigh Numbers (stationary convection) for six cases of gravitational field variation, given by

$$R_s^{ff} = \frac{14(10+a^2)(12096+2448a^2+124a^4)}{289a^2(6+3\delta)},$$

when $f(z) = z$. (16) (11)

$$R_s^{ff} = \frac{14(10+a^2)(12096+2448a^2+124a^4)}{17a^2(102-73\delta)},$$

$$\text{when } f(z) = z^2 - 2z. \quad (17)$$

$$R_s^{ff} = \frac{14(10+a^2)(12096+2448a^2+124a^4)}{17a^2(102-29\delta)},$$

$$\text{when } f(z) = -z^2. \quad (18)$$

$$R_s^{ff} = \frac{14(10+a^2)(12096+2448a^2+124a^4)}{289a^2(6-3\delta)},$$

$$\text{when } f(z) = -z. \quad (19)$$

$$R_s^{ff} = \frac{14(10+a^2)(6048+1224a^2+62a^4)}{17a^2(51+1260\delta(1120-412e))},$$

$$\text{when } f(z) = e^z. \quad (20)$$

$$R_s^{ff} = \frac{14(10+a^2)(6048+1224a^2+62a^4)}{17a^2(51+20.286\delta)},$$

$$\text{when } f(z) = \log(1+z). \quad (21)$$

In order to obtain the critical wave number (a_c), we differentiate R_s^{ff} with respect to square of wave number (a^2) and resultant is equated to zero, we get the value of a_c . For all six cases of gravitational field variation, when both boundaries are free, we get $a_c = 2.226972$.

Stability Analysis when Lower and Upper boundaries are rigid

We consider the trial functions for w and θ , satisfying the boundary condition (4), as given below

$$w = w_0(z^4 - 2z^3 + z^2) \text{ and } \theta = \theta_0 z(z-1). \quad (22)$$

Following the similar steps mentioned above, in section 3.1, we obtained Rayleigh Number for six different cases of gravitational field variation for rigid-rigid boundary condition and are given below

$$R_s^{rr} = \frac{14(10+a^2+p)[2016+(2a^2+\frac{p}{\sigma})48+(a^4+\frac{a^2p}{\sigma})4]}{3a^2(18-13\delta)},$$

$$\text{when } f(z) = z^2 - 2z. \quad (23)$$

$$R_s^{rr} = \frac{14(10+a^2+p)[2016+(2a^2+\frac{p}{\sigma})48+(a^4+\frac{a^2p}{\sigma})4]}{3a^2(18-5\delta)},$$

$$\text{when } f(z) = -z^2. \quad (24)$$

$$R_s^{rr} = \frac{14(10+a^2+p)[2016+(2a^2+\frac{p}{\sigma})48+(a^4+\frac{a^2p}{\sigma})4]}{3a^2(18-9\delta)},$$

$$\text{when } f(z) = -z. \quad (25)$$

$$R_s^{rr} = \frac{14(10+a^2+p)[2016+(2a^2+\frac{p}{\sigma})48+(a^4+\frac{a^2p}{\sigma})4]}{3a^2(18+9\delta)},$$

$$\text{when } f(z) = z. \quad (26)$$

$$R_s^{rr} = \frac{14(10+a^2+p)[2016+(2a^2+\frac{p}{\sigma})48+(a^4+\frac{a^2p}{\sigma})4]}{3a^2(18+2520\delta(1158-412e))},$$

$$\text{when } f(z) = e^z. \quad (27)$$

$$R_s^{rr} = \frac{14(10+a^2+p)[2016+(2a^2+\frac{p}{\sigma})48+(a^4+\frac{a^2p}{\sigma})4]}{3a^2(18+7.182\delta)},$$

$$\text{when } f(z) = \log(1+z). \quad (28)$$

1.1.1 Stationary Convection

For stationary convection, Rayleigh Number (R_s^{rr}) for different cases of gravitational field variation are obtained by putting $p = 0$ in equations (23)-(28), given by

$$R_s^{rr} = \frac{14(10+a^2)(2016+96a^2+4a^4)}{3a^2(18-13\delta)}, \quad (29)$$

$$\text{when } f(z) = z^2 - 2z.$$

$$R_s^{rr} = \frac{14(10+a^2)(2016+96a^2+4a^4)}{3a^2(18-5\delta)},$$

$$\text{when } f(z) = -z^2. \quad (30)$$

$$R_s^{rr} = \frac{14(10+a^2)(2016+96a^2+4a^4)}{3a^2(18-9\delta)},$$

$$\text{when } f(z) = -z. \quad (31)$$

$$R_s^{rr} = \frac{14(10+a^2)(2016+96a^2+4a^4)}{3a^2(18+9\delta)},$$

$$\text{when } f(z) = z. \quad (32)$$

$$R_s^{rr} = \frac{14(10+a^2)(2016+96a^2+4a^4)}{3a^2(18+2520\delta(1158-412e))},$$

$$\text{when } f(z) = e^z. \quad (33)$$

$$R_s^{rr} = \frac{14(10+a^2)(2016+96a^2+4a^4)}{3a^2(18+7.182\delta)},$$

$$\text{when } f(z) = \log(1+z). \quad (34)$$

In order to obtain the critical wave number (a_c), we differentiate R_s^{rr} with respect to square of wave number (a^2) and resultant is equated to zero, we get the value of a_c . For all six cases of gravitational field variation, when both boundaries are rigid, we get $a_c = 3.116524$.

Stability Analysis when Lower boundary is rigid and Upper boundary is free

We consider the trial functions for w and θ , satisfying the boundary condition (4), as given below

$$w = w_0(2z^4 - 5z^3 + 3z^2) \text{ and } \theta = \theta_0 z(z-1). \quad (35)$$

Following the similar steps mentioned above, in section 3.1, we obtained Rayleigh Number for six different cases of gravitational field variation for rigid-free boundary conditions and are given below

$$R_s^{rf} = \frac{14(10+a^2+p)[18144+(2a^2+\frac{p}{\sigma})864+(a^4+\frac{a^2p}{\sigma})76]}{13a^2(78-59\delta)},$$

$$\text{when } f(z) = z^2 - 2z. \quad (36)$$

$$R_s^{rf} = \frac{14(10+a^2+p)[18144+(2a^2+\frac{p}{\sigma})864+(a^4+\frac{a^2p}{\sigma})76]}{13a^2(78-25\delta)},$$

$$\text{when } f(z) = -z^2. \quad (37)$$

$$R^{rf} = \frac{14(10+a^2+p)\left[9072+\left(2a^2+\frac{p}{\sigma}\right)432+\left(a^4+\frac{a^2p}{\sigma}\right)38\right]}{13a^2(39-21\delta)},$$

when $f(z) = -z$.

(38)

$$R^{rf} = \frac{14(10+a^2+p)\left[9072+\left(2a^2+\frac{p}{\sigma}\right)432+\left(a^4+\frac{a^2p}{\sigma}\right)38\right]}{13a^2(39+21\delta)},$$

when $f(z) = z$.

(39)

$$R^{rf} = \frac{14(10+a^2+p)\left[9072+\left(2a^2+\frac{p}{\sigma}\right)432+\left(a^4+\frac{a^2p}{\sigma}\right)38\right]}{13a^2(39+1260\delta(2490-916e))},$$

when $f(z) = e^z$.

(40)

$$R^{rf} = \frac{14(10+a^2+p)\left[9072+\left(2a^2+\frac{p}{\sigma}\right)432+\left(a^4+\frac{a^2p}{\sigma}\right)38\right]}{13a^2(39+16.5438\delta)},$$

when $f(z) = \log(1+z)$.

(41)

Stationary Convection

For stationary convection, Rayleigh Number (R_s^{rf}) for different cases of gravitational field variation are obtained by putting $p = 0$ in equations (36)-(41), given by

$$(R_s^{rf}) = \frac{14(10+a^2)(18144+1728a^2+76a^4)}{13a^2(78-59\delta)},$$

when $f(z) = z^2 - 2z$.

(42)

$$(R_s^{rf}) = \frac{14(10+a^2)(18144+1728a^2+76a^4)}{13a^2(78-25\delta)},$$

when $f(z) = -z^2$.

(43)

$$(R_s^{rf}) = \frac{14(10+a^2)(9072+864a^2+38a^4)}{13a^2(39-21\delta)},$$

when $f(z) = -z$.

(44)

$$(R_s^{rf}) = \frac{14(10+a^2)(9072+864a^2+38a^4)}{13a^2(39+21\delta)},$$

when $f(z) = z$.

(45)

$$(R_s^{rf}) = \frac{14(10+a^2)(9072+864a^2+38a^4)}{13a^2(39+1260\delta(2490-916e))},$$

when $f(z) = e^z$.

(46)

$$(R_s^{rf}) = \frac{14(10+a^2)(9072+864a^2+38a^4)}{13a^2(39+16.5438\delta)},$$

when $f(z) = \log(1+z)$.

(47)

In order to obtain the critical wave number (a_c), we differentiate R_s^{rf} with respect to square of wave number (a^2) and resultant is equated to zero, we get the value of a_c .

For all six cases of gravitational field variation, when lower boundary is rigid and upper boundary is rigid, we get $a_c = 2.66976781$.

Results and Discussion

In this paper we investigate the effect of variable gravity on the onset of stationary convection in Rayleigh Benard convection problem. Variation of Rayleigh number for six cases of gravitational field variation, for different combinations of boundaries are obtained, results are graphed and effects (stabilizing or destabilizing) are analysed analytically.

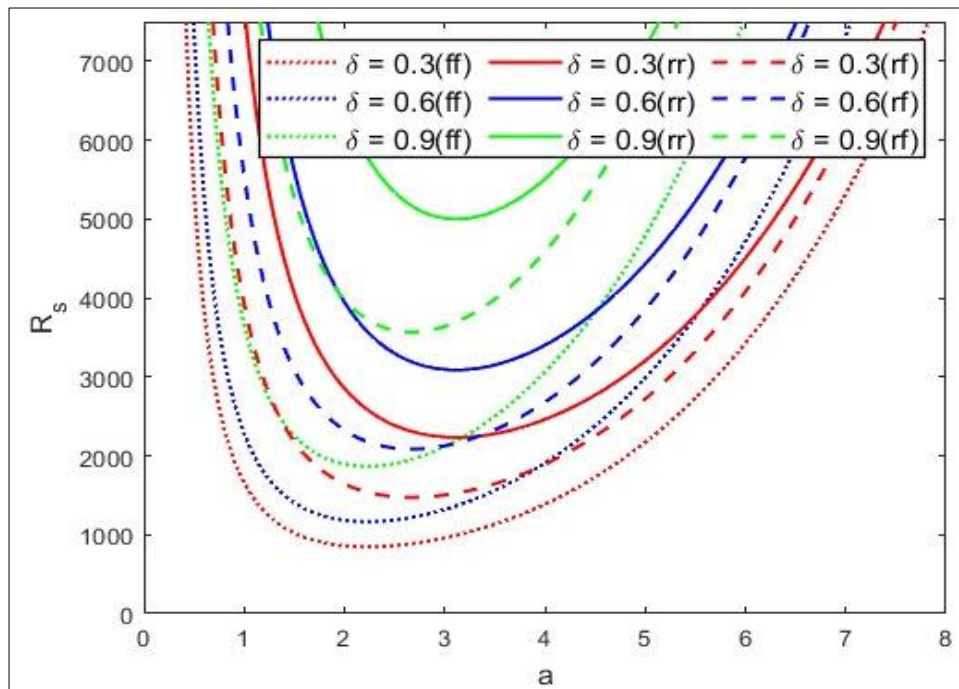


Fig 2: Graph between R_s and a for $f(z) = z^2 - 2z$, for $\delta = 0.3, 0.6, 0.9$, for free-free (ff), rigid-rigid (rr) and rigid-free (rf) boundary conditions.

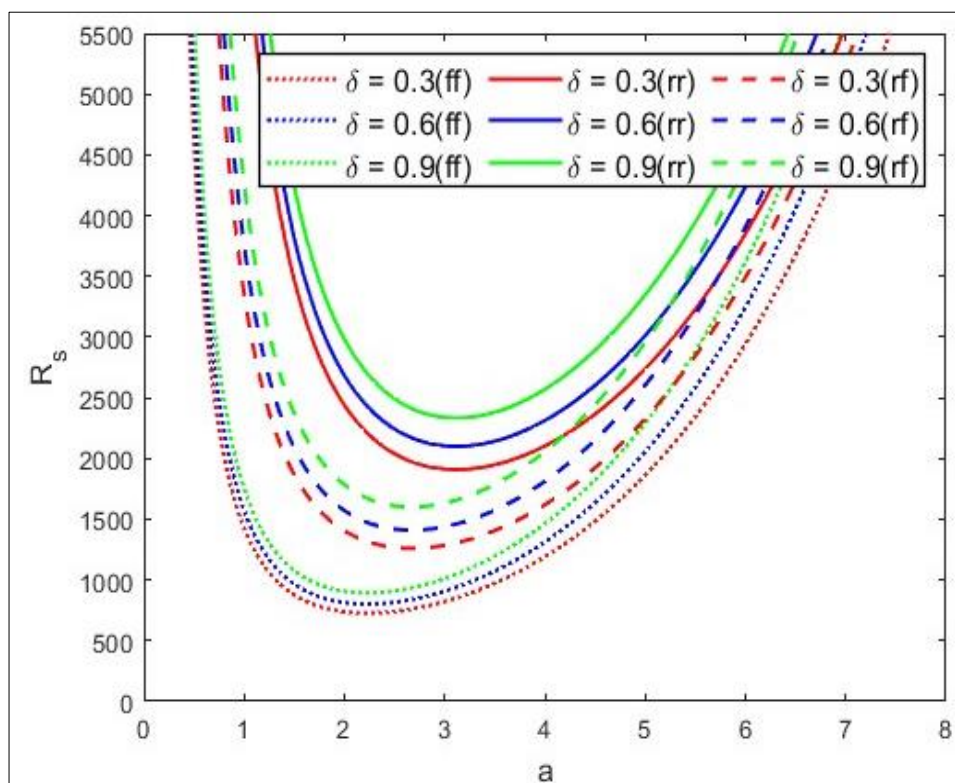


Fig 3: Graph between R_s and a for $f(z) = -z^2$, for $\delta = 0.3, 0.6, 0.9$, for free-free (ff), rigid-rigid (rr) and rigid-free (rf) boundary conditions.

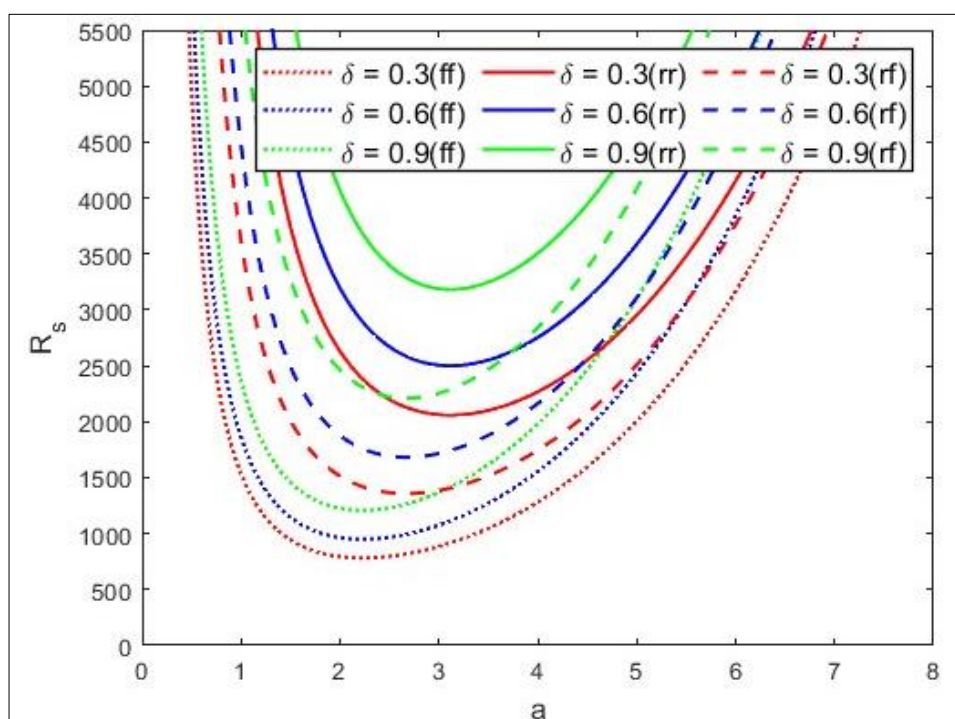


Fig 4: Graph between R_s and a for $f(z) = -z$, for $\delta = 0.3, 0.6, 0.9$, for free-free (ff), rigid-rigid (rr) and rigid-free (rf) boundary conditions.

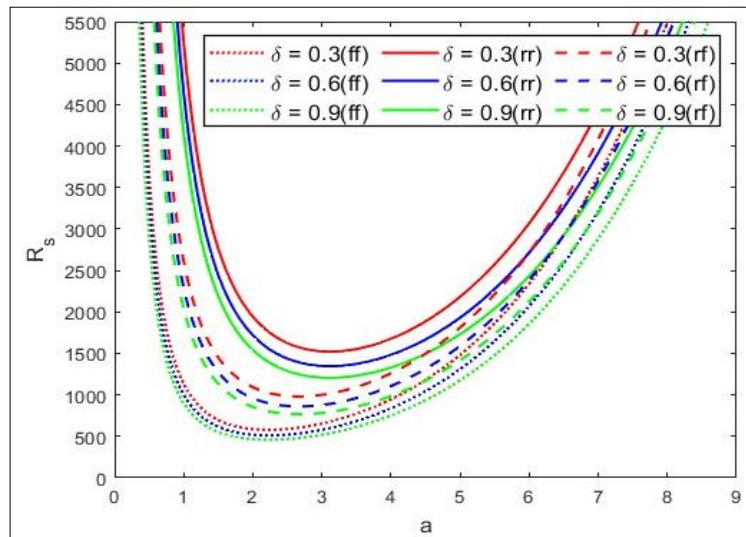


Fig 5: Graph between R_s and a for $f(z) = z$, for $\delta = 0.3, 0.6, 0.9$, for free-free (ff), rigid-rigid (rr) and rigid-free (rf) boundary conditions.

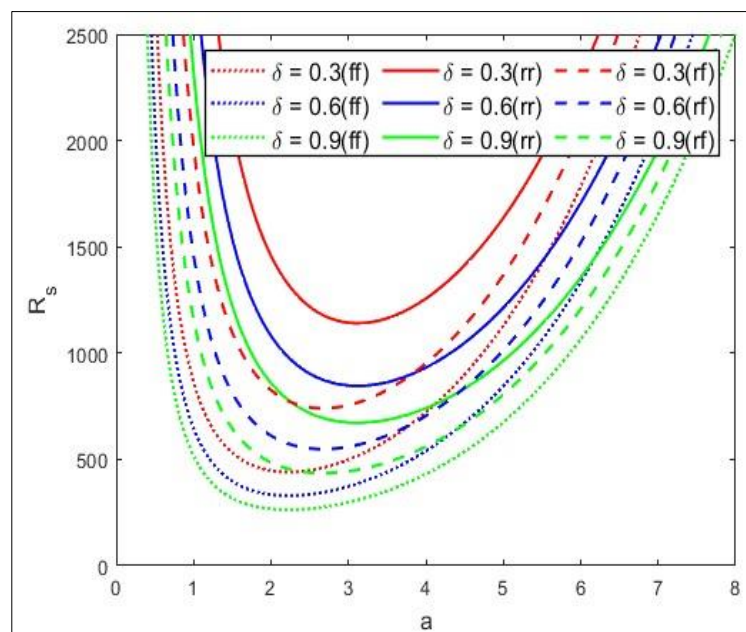


Fig 6: Graph between R_s and a for $f(z) = e^z$, for $\delta = 0.3, 0.6, 0.9$, for free-free (ff), rigid-rigid (rr) and rigid-free (rf) boundary conditions.

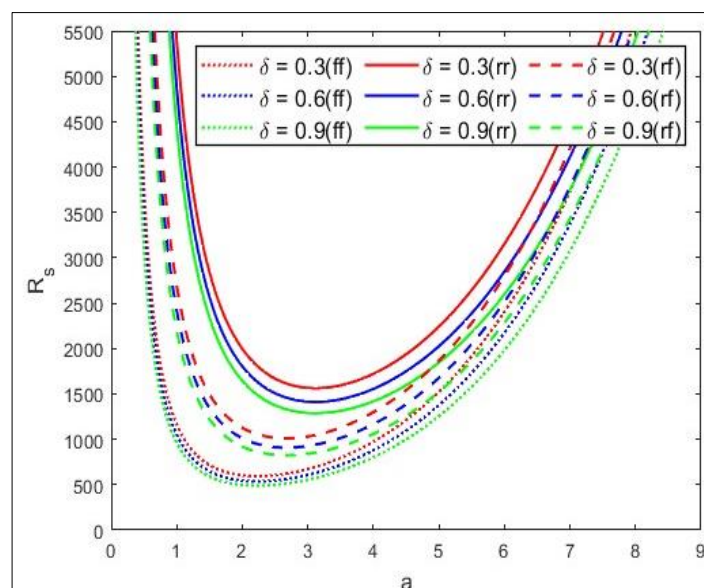


Fig 7: Graph between R_s and a for $f(z) = \log(1+z)$, for $\delta = 0.3, 0.6, 0.9$, for free-free (ff), rigid-rigid (rr) and rigid-free (rf) boundary conditions.

Figures 2-4 are the graphs between R_s and a , for the decreasing gravitational field variation $f(z) = z^2 - 2z$, $f(z) = -z^2$ and $f(z) = -z$ respectively, for different values of $\delta = 0.3, 0.6, 0.9$, for free-free, rigid-rigid and rigid-free boundary conditions. It is seen that as the value of δ increases, value of Rayleigh number increases, which clearly shows the stabilizing effect of δ on stationary convection for decreasing gravitational field variation. This happens because, an increase in the value of δ decreases the gravitational force for decreasing gravitational field variation. Hence, we can say that decreasing gravitational force postpone the onset of convection, resulting in enhancement of the stability of system.

Figures 5-7 are the graphs between R_s and a for the increasing gravitational field variation $f(z) = z$, $f(z) = e^z$ and $f(z) = \log(1+z)$ respectively. It is seen that as the value of δ increases the value of R_s decreases. Hence, in these cases, gravitational forces prepone the onset of convection by helping in early moments of fluid molecules, resulting in destabilizing the system. This is due to the reason that for increasing gravitational field variation, as the value of δ increases gravitational forces also increases.

Conclusion

One of the main findings of this study is that the stability of the convection system is highly dependent on both the type of gravitational field variation and the boundary conditions. Here's what we reveal:

- a) **Effect of Decreasing Gravitational Field (Stabilizing):** When the gravitational field decreases with height ($f(z) = z^2 - 2z$, $f(z) = -z^2$ and $f(z) = -z$), the system becomes more stable, particularly when rigid-rigid boundary conditions are applied. This suggests that decreasing gravity tends to suppress convection and stabilize the fluid motion.
- b) **Effect of Increasing Gravitational Field (Destabilizing):** Conversely, when gravity increases with height ($f(z) = z$, $f(z) = e^z$ and $f(z) = \log(1+z)$), the system becomes more unstable. This destabilizing effect is most pronounced under free-free boundary conditions, where the fluid is more susceptible to instabilities.
- c) **Comparing Boundary Conditions:**
 - **Rigid-Rigid Boundaries:** These boundary conditions generally lead to the most stable convection systems, especially when the gravitational field decreases with height. The rigidity of the boundaries seems to limit the fluid motion and prevent large convection cells from forming.
 - **Rigid-Free Boundaries:** These are more unstable than rigid-rigid, but less so than free-free boundaries. The rigidity of one surface helps control fluid motion, but the free surface still allows some convection.
 - **Free-Free Boundaries:** These are the least stable, particularly when the gravitational field increases. The absence of boundary constraints on both sides of the fluid allows for greater fluid motion, leading to more pronounced convection instabilities.

References

1. Bénard H. Les tourbillons cellulaires dans une nappe liquide. *Rev Gen Sci Pures Appl.* 1900;11:1216-1271, 1309-1328.
2. Rayleigh L. On convective currents in a horizontal layer of fluid when the higher temperature is on the underside. *Philos Mag.* 1916;32:529-546.
3. Jeffreys H. The stability of a layer of fluid when heated from below. *Philos Mag.* 1926;2:833-844.
4. Pellew A, Southwell RV. On maintained convective motion in a fluid heated from below. *Proc R Soc Lond A.* 1940;176:312-343.
5. Chandrasekhar S. *Hydrodynamic and Hydromagnetic Stability.* Oxford University Press; Dover Publications, Inc., New York: 1981.
6. Gupta JR, Sood SK, Bhardwaj UD. On Rayleigh-Bénard convection with rotation and magnetic field. *J Appl Math Phys.* 1984;35:252-256.
7. Banerjee MB, Gupta JR, Shandil RG, Sood SK, Banerjee B, Banerjee K. On the principle of the exchange of stabilities in the magnetohydrodynamic simple Bénard problem. *J Math Anal Appl.* 1985;108(1):216-222.
8. Bodenschatz E, Pesch W, Ahlers G. Recent developments in Rayleigh-Bénard convection. *Annu Rev Fluid Mech.* 2000;32:709-778.
9. Tagare SG, Benerji BA, Rameshwar Y. Rayleigh-Bénard convection in rotating fluids. *Int J Heat Mass Transfer.* 2008;51:1168-1178.
10. Gresho PM, Sani RL. The effects of gravity modulation on the stability of a heated fluid layer. *J Fluid Mech.* 1970;40:783-806.
11. Pradhan GK, Samal PC. Thermal stability of a fluid layer under variable body forces. *J Math Anal Appl.* 1987;122:487-495.
12. Straughan B. Convection in a variable gravity field. *J Math Anal Appl.* 1989;140:467-475.
13. Pradhan G, Samal P, Tripathy U. Thermal stability of a fluid layer in a variable gravitational field. *Indian J Pure Appl Math.* 1989;20:736-745.
14. Rionero S, Straughan B. Convection in a porous medium with internal heat source and variable gravity effects. *Int J Eng Sci.* 1990;28:497-503.
15. Clever R, Schubert G, Busse FH. Two-dimensional oscillatory convection in a gravitationally modulated fluid layer. *J Fluid Mech.* 1993;253:663-680.
16. Saunders BV, Murray BT, McFadden GB, Coriell SR, Wheeler AA. The effect of gravity modulation on thermosolutal convection in an infinite layer of fluid. *Phys Fluids A.* 1992;4(6):1176-1189.
17. Suma SP, Indira R. Effect of throughflow and variable gravity field on thermal convection in a porous layer. *Int J Eng Sci Technol.* 2011;3(10):7657-7668.
18. Chand R, Rana G, Kumar S. Variable gravity effects on thermal instability of nanofluid in anisotropic porous medium. *Int J Appl Mech Eng.* 2013;18:631-642.
19. Aghighi MS, Ammar A. Aspect ratio effects in Rayleigh-Bénard convection of Herschel-Bulkley fluids. *Eng Comput.* 2017;34(5):1658-1676.
20. Harfash A. Convection in a porous medium with variable gravity field and magnetic field effects. *Trans Porous Media.* 2014;103:361-379.
21. Mahabaleshwar U, Basavaraja D, Wang S, Lorenzini G, Lorenzini E. Convection in a porous medium with variable internal heat source and variable gravity. *Int J Heat Mass Transfer.* 2017;111:651-656.
22. Mahajan A, Sharma MK. Convection in a magnetic nanofluid saturating a porous medium under the influence

- of a variable gravity field. *Eng Sci Technol Int J*. 2018;21:439-450.
23. Pant S, Bhagat N, Kharayat H. Effect of suspended particles and magnetic field on thermal convection in ferromagnetic fluid with varying gravitational field in porous medium. *Int J Adv Res*. 2016;4:939-946.
24. Han D, Hernandez M, Wang Q. Dynamic bifurcation and transition in the Rayleigh-Bénard convection with internal heating and varying gravity. *Commun Math Sci*. 2019;17(1):175-192.
25. Yadav D. The onset of Darcy-Brinkman convection in a porous medium layer with vertical throughflow and variable gravity field effects. *Heat Transfer*. 2020;49:3161-3173.
26. Shekhar S, Ragoju R, Yadav D. The effect of variable gravity on rotating Rayleigh-Bénard convection in a sparsely packed porous layer. *Heat Transfer*. 2022;51(1):4187-4204.
27. Surya D, Gupta A. Thermal instability in a liquid layer with permeable boundaries under the influence of variable gravity. *Eur J Mech B Fluids*. 2022;91:219-225.
28. Pant S, Algehyne EA. Convection in a ferromagnetic fluid layer influenced by changeable gravity and viscosity. *Mathematics*. 2022;10:1737-1757.
29. Francis R, Narayana M, Siddeshwar PG. Gravity-modulated Rayleigh-Bénard convection in a Newtonian liquid bounded by rigid free boundaries: a comparative study with other boundary conditions. *J Eng Math*. 2023;5:139-170.
30. Arnone G, Capone F, Iovanna F, Massa G. Variable gravity effects on penetrative porous convection. *Int J Non-Linear Mech*. 2024;158:104579.