

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2024; 9(5): 34-39
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<https://www.mathsjournal.com>
Received: 06-07-2024
Accepted: 11-08-2024

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Application of central composite design to minimize product variability in iron sheet production process

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DOI: <https://doi.org/10.22271/math.2024.v9.i5a.1803>

Abstract

The CCD is a fundamental technique in RSM and is widely applied in scientific research to accurately estimate factors in a 2nd degree model. This paper demonstrates the use of CCD in optimizing process variables to keep the production of iron sheets on target and minimize variability. Specifically, the study focused on three key factors influencing variability in iron sheet manufacturing: the pressure of the press rollers (X_1), the flow rate of the hydraulic fluid (X_2), and the power applied to the digital setting (X_3). By utilizing CCD, the study systematically optimized these factors and considered their interaction effects. The analysis highlighted the magnitudes of coefficients across the linear, interaction, and quadratic models, illustrating the varying degrees of influence each factor exerts on the response variable. By quantifying these effects, the study provided deeper insights into the relationships between factors, helping to identify key drivers of efficiency and variability. The R^2 values for the linear, interaction, and quadratic models were 0.98, 0.95, and 0.973, in that order, indicating that the quadratic model provides a more accurate description of the production process. Surface plots comparing the pressure of the press rollers with the power applied to the digital setting, and the flow rate of the hydraulic fluid with the power applied to the digital setting, suggested that increasing the hydraulic fluid flow enhances production efficiency. The minimum variability was found at $X_1 = 8.15$, $X_2 = 5.022$, and $X_3 = 0.4550$.

Keywords: Response surface methodology, central composite designs, second- order model

1. Introduction

A Central Composite Design (CCD) integrates a 2 square factorial design having four middle points and four axial runs, creating a robust approach for modeling second-order response surfaces. CCDs include three types of points, edge points, which correspond to the 2-level designs (± 1), star points at $\pm \alpha$ (where $|\alpha| \geq 1$) to give quadratic properties, and middle points. This structure enables comprehensive modeling of factor interactions and quadratic effects, making CCD widely used in various scientific and industrial applications.

A notable study by Martin Kasina (2020) utilized CCD to optimize the production of *Pleurotus ostreatus* (Oyster mushroom) spawns. By varying temperature levels, sterilization times, and culture media concentrations, the study aimed to minimize the number of days required for full mycelium development. The statistical elements provided by CCD were crucial for evaluating these factors during spawn propagation.

This paper utilized the CCD within RSM to minimize variability in the critical dimensions of iron sheets. By optimizing key process variables namely, the pressure of the press rollers (X_1), the flow of the hydraulic fluid (X_2), and the power applied to the digital setting (X_3). The study aimed to improve the production process. The generated contour plots and response surface maps were essential in precisely identifying the optimal conditions, leading to improved process efficiency and higher product quality.

2. Methods

2.1 Optimization Process

In the iron sheet production process, four variables were examined to evaluate their effects on the quality of the final product. These variables are: the pressure of the press rollers (x_1), the flow of the hydraulic fluid (x_2), the spacing between the anode and cathode wheel encoder gap

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(x_3), and the power applied to the digital setting (x_4). The production process was represented by the function:

$$f(x_1, x_2, x_3, x_4) + \varepsilon \quad (2.1)$$

The objective was to identify the optimal settings for these variables to minimize variability in the iron sheets.

2.2 Rotatability Conditions

In a Central Composite Design (CCD) experiment, which involves k independent variables, the design matrix is composed of three distinct types of investigational runs. These experimental runs are combined to generate a design with N treatment combinations, typically denoted as L^N representing different levels of the variables. The factorial points explore the main effects and interactions, axial points extend the design for capturing curvature, and center points provide an estimate of experimental error and stability. A CCD is classified as rotatable of order d when the response surface model meets specific rotational symmetry criteria (Box & Hunter in 1957). The response surface for a CCD experiment is represented by a polynomial equation that accounts for linear, interaction, and quadratic effects, facilitating accurate predictions and optimizations across the design space.

$$Y_\mu = \beta_{0\mu} + \sum_{i=1}^n \beta_{i\mu} x_{i\mu} + \sum_{i=1}^n \beta_{ii\mu} x_{i\mu}^2 + \sum_{i=1}^{n-1} \sum_{j=2}^n \beta_{ij\mu} x_{i\mu} x_{j\mu} + \sum_{i=1}^{n-2} \sum_{j=2}^{n-2} \sum_{k=3}^n \beta_{ijk\mu} x_{i\mu} x_{j\mu} x_{k\mu} + \dots + \varepsilon_\mu$$

Here, x_i represents the i^{th} factor at the j^{th} treatment level. This model is used to identify the optimal operating conditions that bring critical product characteristics as close to the target value as possible while minimizing variability.

2.3 The First-Order Model

The initial step in analyzing the relationship between variables involves using a first-order model. This model, which takes the form of a linear polynomial, helps determine both the direction and the type of connection between the dependent variable (also called the response variable) and the independent variables. The model's simplicity allows for a straightforward interpretation of how changes in the independent variables affect the response variable.

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon \quad (2.3)$$

Where x_i are the design variables and β_i are the regression coefficients. Here, x_i denotes the design variables and β_i are the regression coefficients. This model provides insights into how changes in the independent variables affect the response and helps guide the experimenter along the path of steepest ascent or descent, facilitating the identification of optimal conditions for the response.

2.4 The Second-Order Model

This methodology is particularly effective when the response surface exhibits curvature, making the second-order model crucial for achieving more precise approximations. To determine the optimal operating conditions for iron sheet production, a second-order response surface model was applied. This model was developed using a Central Composite Design (CCD), which incorporates a 2^2 factorial

design along with four center points and four axial runs. The CCD is widely used for constructing second-order response surface models.

According to Box and Hunter's 1957 work, a key feature of an effective second-order response surface design is rotatability. This property maximizes the information gained from the data and reduces uncertainty in predictions. In a rotatable design, the variance of predictions stays the same for all points that are equally distant from the design space's center. This consistency enhances the overall prediction accuracy. By using a rotatable Central Composite Design (CCD), the model can effectively capture and represent the complex relationships and interactions between different factors. This approach leads to more reliable estimates, which are crucial for optimizing processes. The mathematical representation of this second-order response surface model takes a specific form, which builds upon the simpler first-order model by including additional terms to account for curvature and interactions.

$$y_\mu = \beta_{0\mu} + \sum_{i=1}^n \beta_{i\mu} x_{i\mu} + \sum_{i=1}^n \beta_{ii\mu} x_{i\mu}^2 + \sum_{i=1}^{n-1} \sum_{j=2}^n \beta_{ij\mu} x_{i\mu} x_{j\mu} + \varepsilon_\mu \quad (2.4)$$

In the context of the experimental design, y_μ denotes the response variable observed for the μ^{th} combination of factors, where $\mu = 1, 2, \dots, n$; Each factor in the experiment is represented by $x_{i\mu}$ which indicates the i^{th} factor during the μ^{th} , with i ranging from ($i = 1, 2, \dots, k$) in the run μ ($\mu = 1, 2, \dots, k$). The model coefficients to be estimated include β_i 's, which are the regression coefficients associated with each factor. Specifically β_0 represents the constant term or intercept of the model, β_i denotes the linear regression coefficients for each factor, β_{ii} represents the quadratic regression coefficients for each factor squared, and β_{ij} is the interaction coefficient for the interaction between the $(i, j)^{th}$ factors. The term $\varepsilon_{\mu/s}$ represents the random error associated with the μ^{th} observation. These errors are assumed to be uncorrelated, with each having a mean of zero and a constant variance denoted by σ^2 .

2.5 Contour Plots

Contour plots were created from the empirical polynomial model to nearly approximate the actual reply surface in the factor region. These plots, which overlay experimental responses, were employed to pinpoint the optimal "window" of operability. They provided a visual representation of factor interactions and their effects on the response.

3. Results and Discussion

3.1 The Central Composite Design (CCD)

CCD form the cornerstone of RSM and are widely used in carrying out estimation in parameters in a full 2^{nd} degree model across various technical disciplines. In the manufacturing of iron sheets, three key factors influencing product variability were identified: the pressure of the press rollers (X_1), the flow of the hydraulic fluid (X_2), and the power applied to the digital setting (X_3). CCD enables systematic optimization of these factors while considering their interface effects. These models derived from CCD reveal how different factors contribute to the response. The analysis shows that the flow of the hydraulic fluid has the most significant impact on product variability.

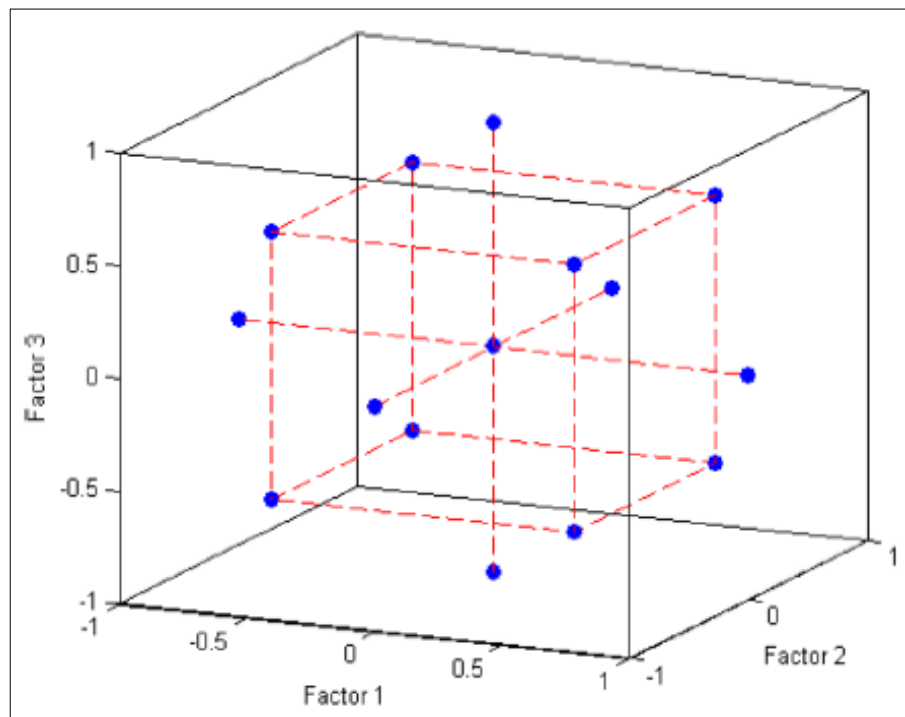


Fig 1: Central Composite Design for three factor two levels

The CCD can be rotatable by choosing the appropriate axial spacing. In a rotatable design, the standard deviation of the foreseen reply is consistent at all points that are equidistant from the center of the design. To achieve rotational symmetry, the axial distance α should be set according to the formula rotatability $\alpha = (F)^{\frac{1}{4}}$, where F shows the points in the factorial portion of the design. This specific axial distance

ensures that the design's predictive capability is uniform across the entire design space, enhancing the robustness of the model. By maintaining a constant standard deviation, the rotatable design allows for more reliable and consistent predictions, making it a valuable tool in experimental design for optimizing processes.

Table 1: Response Surface Regression

(Intercept)	Estimate	Std. Error	t value	Pr(> t)
	55.1908	8.3493	6.6102	4.24e-08 ***
x_1	2.1391	5.4685	0.3912	0.007954 **
x_2	1.1948	5.4360	0.2198	0.0084695 ***
x_3	0.3791	6.6450	0.0570	0.000293 ***
x_4	1.5812	0.3763	4.2019	0.071452
x_1^2	0.1312	0.1215	1.0799	0.005276 **
x_2^2	0.1190	0.1283	0.9271	0.0000
x_3^2	0.4189	0.7802	0.5369	0.0006
x_4^2	0.0114	0.0041	2.7947	0.0027
x_1x_2	-1.654	5.158	0.279	0.0054
x_1x_3	1.586	7.833	0.265	0.0002
x_1x_4	1.342	3.124	0.124	0.0001
x_2x_3	-0.470	4.098	0.198	0.0046
x_2x_4	2.331	3.390	0.175	0.0028
x_3x_4	-0.345	2.346	0.146	0.0016
Residual standard error: 35 on 4 degrees of freedom				
Multiple R-squared: 0.98, Adjusted R-squared: 0.9499				
F-statistic: 32.6 on 6 and 4 DF, p-value: 0.002346				

Linear Mode

$$y = 55.1908 + 2.1391x_1 + 1.1948x_2 + 0.3791x_3 + 1.5812x_4$$

Interaction Model

$$y = 55.1908 + 2.1391x_1 + 1.1948x_2 + 0.3791x_3 + 1.5812x_4 - 1.654x_1x_2 + 1.586x_1x_3 + 1.342x_1x_4 - 0.47x_2x_3 + 2.331x_2x_4 - 0.345x_3x_4$$

Quadratic Model

$$y = 55.1908 + 2.1391x_1 + 1.1948x_2 + 0.3791x_3 + 1.5812x_4 - 1.654x_1x_2 + 1.586x_1x_3 + 1.342x_1x_4 - 0.47x_2x_3 + 2.331x_2x_4 - 0.345x_3x_4 + 0.1312x_1^2 + 0.1190x_2^2 + 0.4189x_3^2 + 0.0114x_4^2$$

Table 1 provides an overview of the predicted responses for various models. The R^2 values for the linear, interaction, and quadratic models were 0.98, 0.95, and 0.973, respectively. In comparison, the adjusted R^2 values for these models were 0.793, 0.748, and 0.949, respectively. These findings indicate that the quadratic model offers the most precise representation of the production process.

3.2 Statistical Analysis of product variability

The analysis of variance (ANOVA) results are shown in Table 2, which also shows that all three main components (press roller pressure, hydraulic fluid flow, and power delivered to the digital setting) have p-values of 0.000, indicating statistical significance. This suggests that these variables significantly affect the control signal's efficiency. For each of these parameters, the coefficient of determination (R^2) is 0.9624, 0.9713, and 0.9648. These figures show that the press roller pressure, hydraulic fluid flow, and power applied to the digital setting are highly connected with the

control signal efficiency, explaining 96.24%, 97.13%, and 96.48% of the response variability. A regression model is said to predict the outcome accurately if its R^2 value is near to 1, since 1 denotes a perfect prediction. Consequently, these strong R^2 values show that the link between the independent variables and the effectiveness of the control signals is consistently explained by the model. The appropriateness of the model was further supported by the fact that the p-value for the lack of fit test was not statistically significant for any of the three factors. When the observed and predicted values do not significantly differ from one another, the model is considered to be good. This is indicated by a non-significant F value of lack of fit. Therefore, additional evidence that the suggested regression model is appropriate for fitting the data comes from the high R^2 values and the non-significant F-value for lack of fit. The linkages between the roller press pressure, hydraulic fluid flow, power delivered to the digital setting, and control signal efficiency are captured by the model in an effective manner.

Table 2: Analysis Variance Model

	Sum of squares	Degree of freedom	Mean square	F	P
Linear Model					
Regression	2336.032	4	778.677	25.205	0.000
Residual	494.307	15	30.894		
Interaction Model					
Regression	2341.503	4	390.251	10.378	0.000
Residual	488.835	13	37.603		
Quadratic Model					
Regression	2755.109	9	306.123	40.692	0.000
Residual	75.229	10	7.523		
Total	2830	19			

Absolute differences concerning actual and predicted percentages for linear, interaction, and quadratic models were varied from 0.63-28.76%, 1.53-28.76% and 0.15-7.74% respectively. By comparing each of the four models deployed, these results reveal that the quadratic model gave the lowest level of error. To assess predictive accuracy the descriptive Root Mean Square Error (RMSE) was computed for the three models. The RMSE values were 5.39, 5.91, and 2.62 for the linear, interaction, and quadratic models, respectively. When converted to percentages, the RMSE values were 7.35% for the linear model, 8.06% for the interaction model, and 3.57% for the quadratic model. These findings suggest that the quadratic model is the most suitable for estimating the efficiency of the control signals. Notably, the interaction terms had a smaller influence, indicating less impact on variability.

Level of fitness was assessed through regression ANOVA model. By null hypothesis, the model will not have any capacity to predict the results. These results are summarized in Table 2 which shows that p-values for all linear, interaction as well as quadratic models are less than 0.05 and therefore these models provided good prediction of the efficiency of the control signals. Table 2 also contains the values of the predictable coefficients of the regression, t and p-values of the coefficients. All linear, interaction, and quadratic terms had p-

values below 0.05, except for the spacing between the anode and cathode wheel encoder. This suggests that the pressure applied to the roller press, hydraulic fluid flow, and power applied to the digital setting significantly impact the response. Interestingly, while the spacing between the anode and cathode wheel encoder showed a non-significant linear term, its quadratic term was significant. This suggests that the response is more influenced by the quadratic relationship rather than the linear one, indicating that the spacing between the anode and cathode wheel encoder has a quadratic effect on the response.

3.3 Graphical and numerical optimization

Optimization offers a visual representation of the range of operating conditions that ensure all response variables meet desired targets. This approach helps identify feasible regions where optimal conditions are achieved, providing a clear, intuitive overview for decision-making in complex, multivariate processes. To identify the optimal range for pressure of the press rollers, flow of the hydraulic fluid, and power applied to the digital setting, contour plots of these four responses are superimposed to create overlay plots. This optimal region highlights the coordinates for the potential optimal levels of these factors, focused on achieving an authorized goods in efficiency and overall quality.

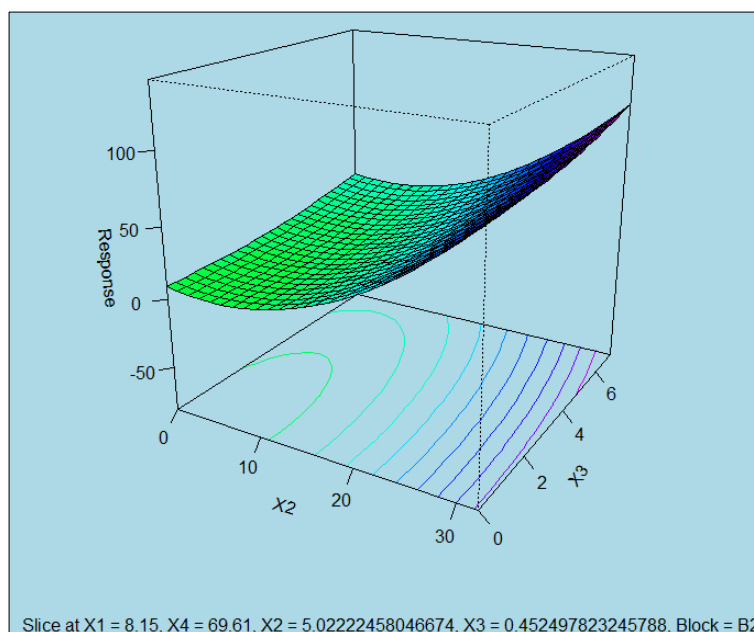


Fig 2: 3D Surface plot

Figure 2 presents 3D response surface plots showing the efficiency of control signals in relation to two factors. The surface plots illustrate the relationships between (pressure applied to the roller press versus power applied to the digital setting) and (flow of hydraulic fluid versus power applied to

the digital setting). The plots reveal that higher hydraulic fluid flow enhances the production process, leading to increased efficiency. The point of minimum variability is identified at $x_1=8.15$, $x_2=5.022$ and $x_3=0.4550$.

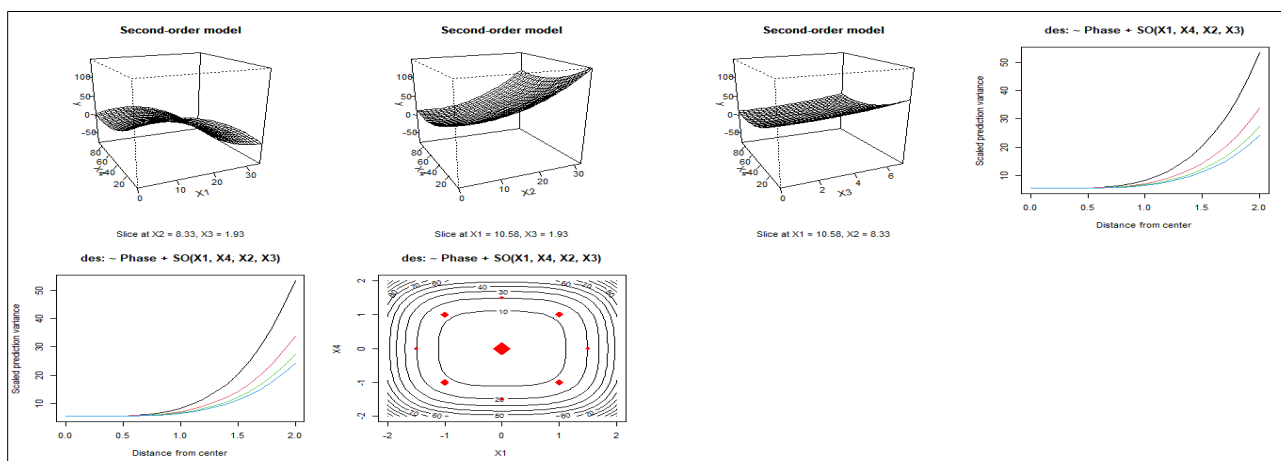


Fig 3: Optimal Levels of efficiency

Efficiency also improves with increased roller press pressure and power applied to the digital setting, though these factors have a less significant impact compared to the hydraulic fluid flow. This aligns with the coefficient values obtained from the quadratic model for each factor. The optimal hydraulic fluid flow for achieving maximum product efficiency is found to be 56.8, after which efficiency starts to decrease.

4. Conclusion and Recommendations

4.1 Conclusion

The quadratic model, incorporating the pressure applied to the roller press, hydraulic fluid flow, and power applied to the digital setting, provides the best fit for predicting the production efficiency of iron sheets and reducing product variability. Among these factors, hydraulic fluid flow has the most significant impact. The optimal conditions for achieving maximum efficiency (88.19%) are a hydraulic fluid flow of 4.56, roller press pressure of 56.8, and power applied to the digital setting at 42.9. The response obtained (86.54%) under

these modelled optimal conditions closely aligns with the predicted response.

Contour plots and response surface maps proved invaluable for quickly identifying optimal conditions and accurately characterizing complex surfaces. These visual tools enable process engineers to better interpret results and make informed decisions about necessary process adjustments. Leveraging these tools to determine optimal process conditions can lead to significant improvements in product quality and manufacturing efficiency.

4.2 Recommendations

Future research should focus on integrating real-time data analytics with multivariate control charts. The growing use of sensor data and Internet of Things (IoT) devices in manufacturing opens up opportunities to create more adaptive and responsive control systems. Exploring methods to efficiently process and analyze high-volume, high-velocity

data in real-time could drive significant advancements in process control and quality assurance.

In conclusion, this study has made notable contributions to the field of statistical process control, especially within the iron sheet manufacturing sector. The developed methodologies and insights offer a strong foundation for future research and practical applications. As manufacturing processes become increasingly complex, there is a growing need for advanced yet practical quality control techniques. The recommendations provided here are intended to guide future research efforts towards tackling the most critical challenges and opportunities in this essential field.

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