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Designing the Bayesian single sampling plan through Markov process

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Abstract

Pharmaceutical companies widely use sampling plans to test medications or other relevant components to make sure they are reliable and safe. A sampling plan is used to assess the quality of goods, keep an eye on the quality of the materials, methods, men power and confirm whether or not the yields are defect-free. If prior information about the manufacturing product is available, the Bayesian technique offers a better statistical approach to arriving at an appropriate conclusion. In an acceptance sampling plan under stochastic modelling, the inspected lot is rejected when the number of defective items falls above an upper control threshold value. However, the lot is accepted when it falls below a lower control threshold value, and if it falls within the thresholds, the process of inspecting the items continues, with the help of transition probability matrix. This plan is studied through the Gamma-Poisson model to safeguard both the producer and consumer by minimising the average sample number and total cost. Necessary tables and figures are constructed for the selection of optimal plan parameters, and suitable illustrations are provided that are applicable to the pharmaceutical industry.

Keywords: Bayesian single sampling plan, acceptance Sampling, gamma-poisson, stochastic process, transition probability matrix

1. Introduction

At the request of the Twentieth World Health Assembly (resolution WHA20.34), a team of experts created the first WHO draft language on good manufacturing practices (GMP) in 1967. It was later adopted when it was presented as "Draft requirements for good manufacturing practice in the manufacture and quality control of medicines and pharmaceutical specialties" to the Twenty-First World Health Assembly. The WHO Expert Committee on Specifications for Pharmaceutical Preparations considered the amended wording in 1968 and released it as an appendix to its twenty-second report. The content was later reprinted in 1971 in The International Pharmacopoeia Supplement, although with minor modifications. When the World Health Assembly adopted resolution WHA22.50 in 1969, recommending the first iteration of the WHO Certification Scheme on the quality of pharmaceutical items moving in international commerce, it also acknowledged the GMP text as an essential component of the Scheme. The book "Quality management in the medicines industry: philosophy and essential elements" describes the basic ideas of quality assurance (QA) and the main elements of GMP, which are shared by production and quality control management as well as top management. These include of persons, premises, materials, documentation, validation, self-inspection, hygiene, and equipment. "Good practices in production and quality control" offers instructions on what steps quality control and production staff should do independently to apply the main QA principles.

Commonly, WHO suggests three types of sampling plans: n , p , and r . The number of samples is the primary goal of the n -plan, the maximum number of defects in the plan is the focus of the p -plan, and the rejection number is the focus of the r -plan. This work makes use of p -plan. One of the pharmaceutical products is a capsule. Since the demand for capsule is growing daily, it would be wasteful to use 100% inspection; alternatively, sample inspection is safer for both producers and consumers. Inspection of raw materials, semi-finished items, or finished products is one aspect of quality assurance.

Acceptance sampling is the term used to describe the sort of inspection process used when the goal of the inspection is to determine whether a product should be accepted or rejected based on its adherence to a standard. In recent years, the industry has worked with suppliers to improve the performance of their processes, primarily using acceptance sampling for receiving or incoming inspections. A 100% inspection would be very expensive and time-consuming, the inspection mistake rate would be high enough, and there would be significant hazards of product responsibility. There are two main categories for acceptance sampling: (1) sample plans based on attributes and (2) sampling plans based on variables. There are various different kinds of sample plans that are used in lot-by-lot acceptance sampling for attributes, including sequential, continuous, chain, skip-lot, single, double, and multiple sampling plans. A popular and straightforward acceptance sampling technique is the single sampling plan (SSP) by attributes. It is identified by two factors, the acceptance number (c) and the sample size (n), which are established in accordance with predetermined standards. In this study focus on pharmaceutical industry.

Guenther ^[18] proposed an iterative process to determine the parameters of single sampling plan by attributes for two specific points on the OC curve, $(p_1, 1 - \alpha)$ and (p_2, β) , where p_1 is the acceptable quality level (AQL), p_2 is the limiting quality level (LQL), α is the producer's risk, and β is the consumer's risk. Tables of unity values for the development of single sampling plan by attributes based on the traditional Poisson distribution were provided by Cameron ^[2]. Hald ^[7] and Schilling ^[15] spoke about how to determine the plan parameters based on the unity values. The theory behind traditional sample plans by attributes is based on the basic assumption that the nonconforming process fraction or lot is a constant, implying that the manufacturing process is steady. Conventional sampling strategies cannot be used in these situations since the judgement about the submitted lots must take into account the variances between lots. As an alternative, sample strategies based on the Bayesian technique may be used, which makes use of information about process variance to decide how to dispose of the lot. Choosing a prior distribution for the lot fraction nonconforming has been discussed by Vijayaraghavan *et al.* ^[17] selecting a prior distribution for a metric quality characteristic of the production process has been studied by Loganathan *et al.* ^[10] When sampling from a Poisson distribution, the gamma distribution is a conjugate prior, according to Bayesian theory. The gamma distribution is the conjugate prior to the average number of nonconformities per unit. When sample units are randomly selected from a process, the number of nonconformities in the sample is distributed based on the Poisson equation. Under these circumstances, the operating characteristic (OC) function of SSP by characteristics labelled as gamma-Poisson SSP has been mathematically expressed by Hald ^[7]. Optimization of Bayesian repetitive group sampling plan for quality determination in Pharmaceutical products and related materials studied by Kaviyarasu and Siva kumar ^[8]. Bayesian single sampling plan using gamma prior were provided by Suresh K.K and Latha ^[17]. A novel control strategy was presented by Fallahnezhad and Nasab ^[11] addressing the acceptance sampling problem. Using the Binomial distribution, Mohammad Mirabi and Fallahnezhad ^[12] analyze the acceptance sampling on the single and double stage through the Markov chain. Fallahnezhad ^[13] provided a minimal angle method-based Markov chain acceptance sampling plan. In this work gives tables to determine the c_1

(lower threshold), c_2 (lower threshold), n (minimum sample size) and minimum cost of the Bayesian Acceptance Sampling Plan under the circumstances of using the Gamma-Poisson distribution.

2. Transition Probability Matrix for Acceptance Sampling

The Markov chain is one of the branches of stochastic processes it uses an optimum value of threshold to accept or reject the lot, which initiates the idea of employing a Markovian approach to model the acceptance sampling problem. Let $P = \{p_{ij}\}$ represent the transition probability matrix for the finite state space S with discrete-time. A transition probability matrix depicts the single-step process of moving from one state to another. It includes all of the information required to understand how the process moves across the states.

2.1 Assumptions

- The entries of matrixes are all non-negative.
- In the matrix each row equals one.

$$p_{11} + p_{12} + p_{13} + \dots + p_{1n} = 1$$

The three states of the problem are designed as follows:

- Accept the lot when the batch is in good state.
- Reject the lot when the batch is in bad state.
- Continue the inspection when it is in between the upper and lower thresholds.

Every phase is viewed as a state of the transition matrix P , which comprises the probabilities of accepting the batch, rejecting the batch, and continuing the process. The matrix P is an absorbing Markov chain where the states are known to be transient (continue inspection) and absorbing (reject or accept the lot). Finding the optimal threshold values for the sample plan is the aim of this Markovian technique.

Wilson and Burgess ^[4] specified the block form for the transition probability matrix is arranged as follows

$$P = \begin{bmatrix} P & 0 \\ R & Q \end{bmatrix} \quad (2.1)$$

Here

P = the probability p_{ij} of remaining in a state is represented by an identity matrix.

0 = the null matrix shows the probabilities of failing to reach the absorbing state.

R = the matrix encoding the column vector with entry $1 - \sum q_{ij}$ containing probabilities of transitioning from non-absorbent to absorbing state (from inspection to accept or reject the lot).

Q = the square matrix's q_{ij} components describe the odds of transitioning from one non-absorbing condition to another.

Using the fundamental matrix N_{ij} , the expected number of times an inspection continues in the long run m_{11} may be determined.

$$N_{ij} = \frac{1}{1 - Q} = m_{11} \quad (2.2)$$

$E(n)$ denotes the average number of units inspected.

$E(n) = m_{11} \times \text{sample size per inspection.}$ (2.3)

The long run absorption probability matrix ζ may be computed as follows:

$$\zeta = N_{ij} X R \quad (2.4)$$

ζ_{12} = The probability of the lot being accepted & ζ_{13} = The probability of the lot being rejected.

The Poisson distribution with parameter $np = \lambda$, where λ is the average number of nonconformities per unit, is used to distribute the random variable, when it is taken at random from a production process that generates output in a continuous stream. The Poisson distribution is a suitable model for the number of nonconforming units in the sample, as noted by Hald [7] for $p < 0.10$ and by Schilling [16] when n is big and P is small such that $np < 5$.

$$P_{(n,p)}(d) = \frac{e^{-np}(np)^d}{d!} \text{ for } d = 0, 1, \quad (2.5)$$

A random variable, say L , must be introduced to represent the randomly varying distribution parameter when the distribution parameter λ is not constant but rather varies at random from lot to lot. This random variable is distributed according to a gamma distribution, which is the Poisson distribution's conjugate distribution.

The prior distribution of L 's density function can be found as follows:

$$f_{(a,m)}(\lambda) = \frac{e^{-a\lambda} a^m \lambda^{m-1}}{\Gamma(m)} \text{ for } 0 \leq \lambda < \infty, a, m > 0 \quad (2.6)$$

Where $a = \frac{m}{\bar{\lambda}}$, with $E[L] = \bar{\lambda}$ and m is the shape

When production is not stable, L is independently distributed, and according to Hald [7] the Gamma-Poisson probability distribution can be determined by the following

$$p = \sum_{d=0}^n \frac{(m+d-1)!}{d!(m-1)!} \left(\frac{np}{np+m} \right)^d \left(\frac{m}{np+m} \right)^m \quad (2.7)$$

Where,

N = number of observations

P = proportion defectives

M = shape parameter for gamma prior

D = number of defectives

2.2 Total Cost

The expected total cost associated with the inspection policy can be expressed using the equation (2.8) used to calculate the total cost. The acceptance cost is rejection cost is calculated

$$p(d_1 < d < r_2) = p_{11} = \sum_{d=0}^n \frac{(m+d-1)!}{d!(m-1)!} \left(\frac{np}{np+m} \right)^d \left(\frac{m}{np+m} \right)^m - p_{12} \quad (3.4)$$

$$p(d > r_2) = p_{13} = \sum_{d=0}^n \frac{(m+d-1)!}{d!(m-1)!} \left(\frac{np}{np+m} \right)^d \left(\frac{m}{np+m} \right)^m = 1 - p_{12} \quad (3.5)$$

$$p(d \leq r_1) = p_{12} = \sum_{d=0}^{r_1} \frac{(m+d-1)!}{d!(m-1)!} \left(\frac{np}{np+m} \right)^d \left(\frac{m}{np+m} \right)^m ; x = 0, \dots, r_1 \quad (3.6)$$

Where,

p_{11} = The probability of continuing inspection.

p_{12} = The probability of accepting the lot.

p_{13} = The probability of rejecting the lot.

by multiplying the rejection cost by the probability of rejecting the batch at stage 2. Multiplied by the probability of accepting the batch at stage 1. The expected inspection cost is calculated by multiplying the cost of inspecting one item in the batch by the sample size and the expected number of transitions sample size multiplying by the expected number of transitions.

The average total cost for single sampling strategy and the lot acceptance policy is given as

$$E(TC) = \text{Acceptance cost} + \text{Rejection cost} + \text{Inspection cost.} \quad (2.8)$$

3. Procedure for Bayesian stochastic single sampling plan

The steps involved in a single sampling plan are as follows:

Step 1: Select a random sample of n items and inspect each of items. Set the

lower threshold value c_1 and upper threshold value c_2 under binomial conditions.

Step 2: The lot will be accepted if the observed number of defects is less than c_1 .

Step 3: The lot will be rejected if the observed number of defects is greater than c_2 .

Step 4: The inspection is continued for n more items if the number of defects found in the first n items falls between c_1 and c_2 .

The transition probability matrix is

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (3.1)$$

The basic matrix is revealed by rearranging the matrix as

$$N_{ij} = \left[\frac{1}{1-p_{11}} \right] = m_{11} \quad (3.2)$$

The long-run absorption probability is given by

$$\zeta = \left[\frac{p_{12}}{1-p_{11}} \quad \frac{p_{13}}{1-p_{11}} \right] \quad (3.3)$$

The matrix's elements are each ζ_{12} and ζ_{13} , correspondingly.

The probability of arriving each states are obtained using the Gamma-Poisson distribution as

4. Different m Values for Different Threshold Values

This analysis is carried out to showcase the implementation of the proposed strategy in an acceptance sample design. Consider the following problem: $AQL = 0.05$, $LQL = 0.1$.

The alternate values of m among the present ones provide various combinations.

In the case $N = 1500$, $cs = 1$, $p = 0.05$, $I = 2$, $rc = 100$, $n = 75$

Table 1: The feasible values of r_1 , r_2 , $E(n)$ and average cost among various values for $m = 1$

When $p_1 = 0.02$ Probability of accepting the lot	When $p_2 = 0.10$ Probability of rejecting the lot	r_1	r_2	$E(n)$	Average cost
0.65	0.85	0	2	93	287.77
0.75	0.84	0	3	104	303.18
0.83	0.73	1	3	91	274.56
0.89	0.71	1	4	99	291.02
0.91	0.63	2	4	89	267.68
0.95	0.59	2	5	99	288.34
0.95	0.53	3	5	90	267.71
0.97	0.51	3	6	93	272.99

Table 2: The feasible values of r_1 , r_2 , $E(n)$ and average cost among various values for $m = 3$

When $p_1 = 0.02$ Probability of accepting the lot	When $p_2 = 0.10$ Probability of rejecting the lot	r_1	r_2	$E(n)$	Average cost
0.59	0.98	0	2	79	257.28
0.75	0.97	0	3	95	288.28
0.86	0.91	1	3	89	275.68
0.93	0.90	1	4	100	296.87
0.95	0.82	2	4	91	277.65
0.98	0.80	2	5	102	299.73
0.98	0.72	3	5	92	276.38
0.99	0.69	3	6	103	297.29

Table 3: The feasible values of r_1 , r_2 , $E(n)$ and average cost among various values for $m = 5$

When $p_1 = 0.02$ Probability of accepting the lot	When $p_2 = 0.10$ Probability of rejecting the lot	r_1	r_2	$E(n)$	Average cost
0.57	0.99	0	2	82	263.84
0.75	0.99	0	3	90	278.99
0.87	0.95	1	3	87	271.77
0.95	0.95	1	4	97	292.40
0.96	0.88	2	4	90	277.91
0.99	0.87	2	5	103	302.38
0.99	0.78	3	5	93	280.56
0.99	0.75	3	6	106	306.20

Table 1 shows $m = 1$ for the probability of accepting and the probability of rejecting the lot. In this table, the minimum sample is 89 and the minimum total cost is 267.68, but this is not a feasible solution because the probability of accepting and the probability of rejecting the lot fail to meet the conditions. Table 2 shows $m = 3$ for the probability of accepting and the probability of rejecting the lot. In this table, the minimum sample is 79 and the minimum total cost is 257.28, but this is not a feasible solution because the

probability of accepting and the probability of rejecting the lot fail to meet the conditions. In table 3, $m = 5$ for the probability of accepting and probability of rejecting the lot. In this table, the minimum sample is 82 and the minimum total cost is 263.84, but this is not a feasible solution because the probability of accepting and probability of rejecting the lot fail to meet the conditions, but in this table, threshold values 1 and 4 do not fail to meet the conditions, so the feasible sample is 97 and the total cost is 292.40.

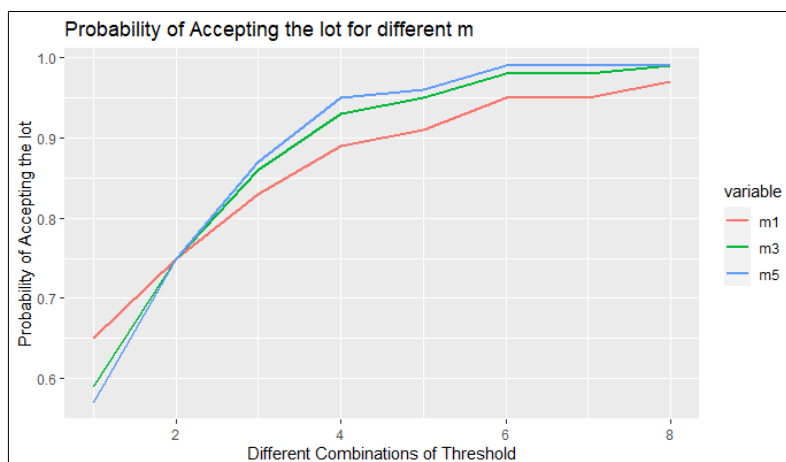


Fig 1: Probability of accepting lot for different m

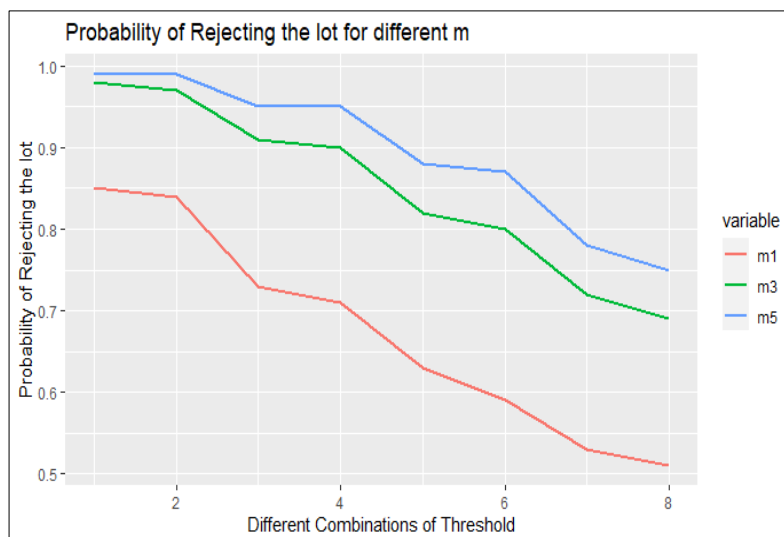


Fig 2: Probability of rejecting lot for different m

In figures 1 and 2, there are different combinations of threshold values for the probability of accepting and the probability of rejecting the lot. In these figures, the threshold

values are increasing, the probability of accepting is also increasing, and the probability of rejecting is decreasing.

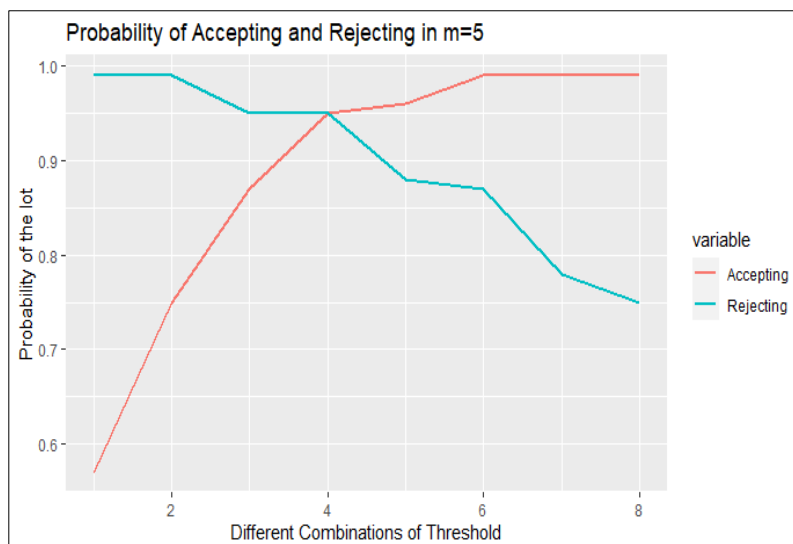


Fig 3: Probability of accepting and Probability of rejecting in m = 5

Figure 3 depicts the probability of accepting and rejecting a lot when $m = 5$. Both values of 0.95 for the combination of thresholds are 1 and 4.

5. Numerical illustration

The quality engineer should check the solubility of the capsule. In the lot, there are 1500 capsules, and the capsules

dissolve within three seconds. QE fixed the AQL = 0.05, LQL = 0.1, the cost of a defective item (cs) = 1, the proportion of the defective items in a batch is (p) = 0.05, the cost of inspecting an item in the batch (I) = 2, the cost of batch rejection (rc) = 100, and the sample size (n) = 75.

Table 4: Dissolving time for the capsules in minutes

Time	0	1	1	0	2	3	1	0	1	1	0	0	2	1	0	0	1	1	1	2	0	1	3	2	0
A/R	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
Time	1	2	1	0	0	1	1	0	5	1	3	0	2	1	2	0	1	3	1	2	2	1	2	2	1
A/R	P	P	P	P	P	P	P	P	F	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
Time	0	1	0	0	1	1	1	3	0	1	0	2	3	1	0	2	1	1	1	2	0	2	2	1	2
A/R	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P

Using R software to generate the Gamma-Poisson data (mean = 0.405 < Standard deviation = 0.667) and take the sample with the help of random sampling, the samples are in Table 4. In this table, dissolving seconds are given from 0 to 3, which

passes the inspection; more than 3 seconds fail the inspection. In this condition, table 3 gives the feasible solution, where the lower threshold value is 1, the upper threshold value is 4, the

sample size is 97, and the total cost is 292.40. The above data has only one failed inspection, so the lot is accepted.

6. Conclusion

The goal of this paper is to try to find a better threshold value for the lot acceptance policy for a Bayesian single sampling plan using the Markovian matrix. In the traditional method, there is no change to continue the inspection, but this proposed model using TPM gives a change to continue the inspection. Furthermore, this technique makes improvements while limiting the hazards of acceptable and rejectable quality levels. The model appears to be efficient in determining the upper and lower threshold values for the single sampling plan. In the illustration provided with the Gamma-Poisson probability distribution, a single sample plan performs better in arriving at measurements. This benefits both the manufacturer and the consumer by allowing them to arrive at an ideal policy and avoid financial losses.

7. References

1. Calvin TW. How and when to perform acceptance sampling. Wisconsin: American Society for Quality Control; c1990.
2. Cameron JM. Tables for constructing and for computing the operating characteristics of single sampling plans. *Ind Qual Control*. 1952;9(1):37-39.
3. Dodge HF, Romig HG. Sampling inspecting tables, single and double sampling. New York: John Wiley and Sons; c1959.
4. Wilson EB, Burgess AR. Multiple sampling plans viewed as finite Markov chains. *Technometrics*. 1971;13:371-383.
5. Golub A. Designing single sampling inspection plans when the sample size is fixed. *J Am Stat Assoc*. 1953;48:278-288.
6. Gupta SK, Phung D, Venkatesh S. A nonparametric Bayesian Poisson gamma model for count data. In: *Proceedings of the 21st International Conference on Pattern Recognition (ICPR2012)*. IEEE; c2012, p. 1815-8.
7. Hald A. On the theory of single sampling inspection by attributes based on two quality levels. *Rev Int Stat Inst*. 1967;35:1-29.
8. Kaviyarasu V, Sivakumar P. Implication of production and monitoring techniques in Bayesian single sampling plan using Gamma-Zero Inflated Poisson distribution. *Int J Recent Technol Eng*. 2019;8(4):2277-2878.
9. Kaviyarasu V, Sivakumar P. Optimization of Bayesian single sampling plan for the zero inflated Poisson distribution involving risk minimization using tangent angle method. *Int J Sci Res Math Stat Sci*. 2021;8(3):1-11.
10. Loganathan A, *et al*. Selection of Bayesian single sampling plans by attributes with desired discrimination. *Econ Qual Control*. 2013;28(2):167-177.
11. Lord D, Moreno MLF. Effects of low sample mean values and small sample size on the estimation of the fixed dispersion parameter of Poisson-gamma models for modeling motor vehicle crashes: A Bayesian perspective. *Saf Sci*. 2008;46(5):751-770.
12. Fallahnezhad MS, Hosseini Nasab H. Designing a single stage acceptance sampling plan based on the control threshold policy. *Int J Ind Eng Prod Res*. 2011;22:143-150.
13. Fallah Nezhad MS. A new Markov chain-based acceptance sampling policy via the minimum angle method. *Iran J Oper Res*. 2012;3:104-111.
14. Peach P, Littauer SB. A note on sampling inspection. *Ann Math Stat*. 1946;17(1):81-4.
15. Schilling EG. Acceptance sampling. *Quality handbook*; c1999.
16. Suresh KK, Latha M. Bayesian single sampling plans for a gamma prior; c2001.
17. Vijayaraghavan R, Rajagopal K, Loganathan A. A procedure for selection of a gamma-Poisson single sampling plan by attributes. *J Appl Stat*. 2008;35(2):149-160.
18. Guenther WC. Use of the binomial, hypergeometric and Poisson tables to obtain sampling plans. *J Qual Technol*. 1969;1(2):105-109.
19. World Health Organization. Quality assurance of pharmaceuticals: A compendium of guidelines and related materials. Volume 2. Good manufacturing practices and inspection. World Health Organization; c2024.