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Reliability analysis of a system using semi Markov process

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Abstract

This study present we describe reliability analysis of a system using semi Markov process. The system consists one main unit A and two associate units B and C. System will be operable when main unit and at least one associate unit is in operative mode and the system will be in semi operable state state when the main unit is failed but both associate are in operative mode. As soon as job arrives, all the unit works with load. It is assumed that only one job is taken for processing at a time. There is a single server who visits the system immediately when preventive maintenance and repair required. The unit works as new after preventive maintenance and repair. The failures of the unit are distributed exponentially while the distribution of PM and repair time are taken is arbitrary semi-Markov and regenerating point technique are used in this model.

Keywords: Regenerative Point, MTSF, availability, busy period
2020 AMS Classification: 53A04, 53A05

1. Introduction

Many industrial systems under different operating conditions were analyzed in the past. The objective was to obtain the relevant indices reflecting the system's operational capabilities. Mostly, the semi-Markovian process was adopted for the system analysis. So many Researchers and Scientists are trying to improve the performance of industries using various reliability techniques. V.K.pathak ^[1] analyzed comparative study of reliability parameter of a system under different types of distribution functions. H. K. Saw, and V. K. Pathak ^[2] studied economic and performance evaluation of a stochastic model with hardware failure and human error, in this paper developed a stochastic Reliability model of single unit system in manufacturing plant. System has been investigated under the assumption that unit works in three different states: normal state, partial failed state and total failed state. Hemant Kumar Saw and H.L. Manker ^[3] develop a stochastic reliability model of one unit main system which two associate units. Each associate unit also has one sub units. Jyotishree Ghosh *et al.* ^[5] A two non indential unit parallel system with priority in repair. G. Taneja and R. Malhotra ^[6] studied cost-benefit analysis of a single unit system with scheduled maintenance and variation in demand. R. Bashir and J.P.S. Joorel ^[7] analyzed Reliability analysis of a three unit complex system working in seriesparallel configuration, S.M. Rizwan *et al.* ^[8] studied Reliability analysis of a three unit pumping system of a water pumping station is analyzed.

In view of above study in this paper an attempt has been made by authors by incorporating the concept of semi-up mode obtained the reliability parameters of working system using semi Markov process and regenerative point technique.

2. System Description about the model

The system consists of three units namely one main unit A and two associate units B & C. Here the main unit depends on associate unit B and C and the system is operable when the main unit and at least one associate unit is in operation and the system is semi operable when the main unit is failed and both associate are in operation. Associative units are essential in the operation of main unit A. As soon as a job arrives, all the units work with load.

It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on first come first served basis. Using regenerative point technique several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated. In the end the expected profit is also calculated.

3. Assumptions used in the model

1. The system consists of one main unit and two associate units.
2. The main unit A works with the help of associate unit B and C.
3. There is a single repairman which repairs the failed units on priority basis.
4. After random period of time the whole system goes to preventive maintenance.
5. All units work as new after repair.
6. The failure rates of all the units are taken to be exponential whereas the repair time distributions are arbitrary.
7. Switching devices are perfect and instantaneous.

Symbols and Notations

P_{ij} : Transition probabilities from S_i to S_j ,

μ_i : Mean Sojourn time at time t in regenerative state before transiting to other state,

E_0 : State of the system epoch $t=0$,

E : Set of regenerative states $S_0 - S_5$,

$q_{ij}(t)$: Probability density function of transition from S_i to S_j ,

$Q_{i,j}(t)$: Cumulative distribution function of transition time from S_i to S_j ,

$\pi_i(t)$: Cumulative distribution function of time to system failure when starting from state $E_0 = S_i \in E$,

$B_i(t)$: Repairman is Busy in the repair at time t / $E_0 = S_i \in E$

$r_1/r_2/r_3/r_4$: Constant Repair rate of Main unit A/Unit B/Unit C/ Unit A&B or A&C,

$g_1/g_2/g_3/g_4$: Probability density function of repair time of main unit F/Unit T/Unit U/Unit A&B or A&C,

$\bar{G}_1/\bar{G}_2/\bar{G}_3/\bar{G}_4$: Cumulative distribution function of repair time of main unit F/Unit T/Unit U/Unit A&B or A&C,

$a(t)$: Probability density function of preventive maintenance,

$b(t)$: Probability density function of preventive maintenance completion time,

$\bar{A}(t)$: Cumulative distribution functions of preventive maintenance,

$\bar{B}(t)$: Cumulative distribution functions of preventive maintenance completion time,

$\$$: Symbol for Laplace -Stieljes transforms.

$\odot$: Symbol for Laplace Convolution.

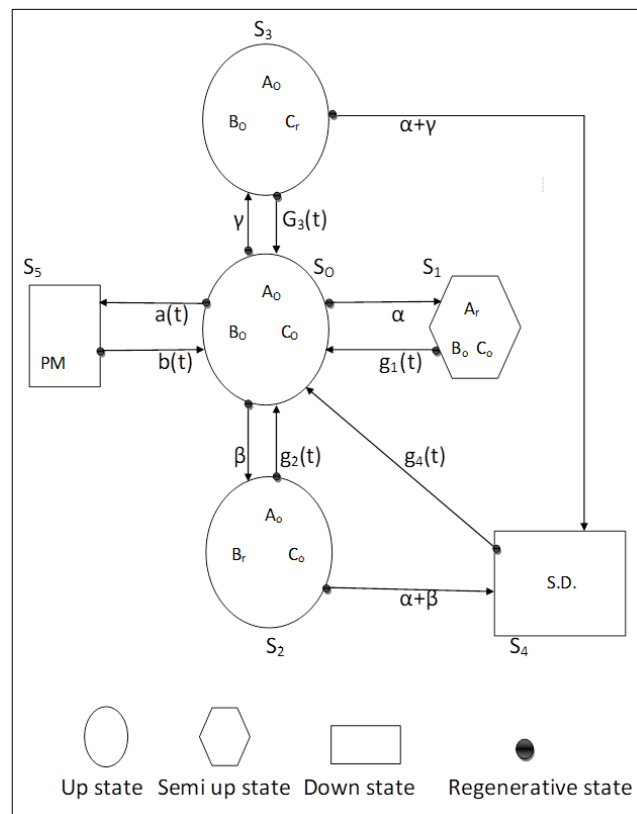


Fig 1: State transition diagram

Symbols used for states of the system

$A_0/A_g/A_r$: Main unit 'A' under operation/good and non-operative mode/ repair mode,

$B_0/B_g/B_r$: Associate Unit 'B' under operation/repair/ good and non-operative mode,

$C_0/C_g/C_r$: Associate Unit 'C' under operation/repair/good and non-operative mode,

P.M. : System under preventive maintenance.

Up states: $S_0 = (A_0, B_0, C_0); S_2 = (A_0, B_r, C_0); S_3 = (A_0, B_0, C_r)$

Semi Up states: $S_1 = (A_r, B_0, C_0)$

Down States: $S_4 = (S.D.); S_5 = (P.M)$

Transition probabilities

Simple probabilistic considerations yield the following non zero transition probabilities:

$$dQ_{01}(t) = \int_0^t \alpha e^{-(\alpha+\beta+\gamma)t} \bar{A}(t) dt$$

$$dQ_{02}(t) = \int_0^t \beta e^{-(\alpha+\beta+\gamma)t} \bar{A}(t) dt \quad (1)$$

$$dQ_{03}(t) = \int_0^t \gamma e^{-(\alpha+\beta+\gamma)t} \bar{A}(t) dt \quad (2)$$

$$dQ_{10}(t) = \int_0^t g_1(t) dt \quad (3)$$

$$dQ_{20}(t) = \int_0^t e^{-(\alpha+\beta)t} g_2(t) dt \quad (4)$$

$$dQ_{24}(t) = \int_0^t (\alpha + \beta) e^{-(\alpha+\beta)t} \bar{G}_2(t) dt \quad (5)$$

$$dQ_{34}(t) = \int_0^t (\alpha + \gamma) e^{-(\alpha+\gamma)t} g_3(t) dt \quad (6)$$

$$dQ_{30}(t) = \int_0^t e^{-(\alpha+\gamma)t} g_3(t) dt \quad (7)$$

$$dQ_{40}(t) = \int_0^t g_4(t) dt \quad (8)$$

$$dQ_{50}(t) = \int_0^t b(t) dt \quad (9)$$

$$dQ_{05}(t) = \int_0^t a(t) e^{-(\alpha+\beta+\gamma)t} dt \quad (10)$$

Where $x_1 = \alpha + \beta + \gamma$, Now letting $t \rightarrow \infty$, we get

$$\lim_{t \rightarrow \infty} Q_{ij}(t) = P_{ij}$$

$$P_{01} = \int_0^\infty \alpha e^{-(\alpha+\beta+\gamma)t} \bar{A}(t) dt = \frac{\alpha[1-a^*(x_1)]}{x_1} \quad (11)$$

$$dQ_{02}(t) = \int_0^\infty \beta e^{-(\alpha+\beta+\gamma)t} \bar{A}(t) dt = \frac{\beta[1-a^*(x_1)]}{x_1} \quad (12)$$

$$dQ_{03}(t) = \int_0^\infty \gamma e^{-(\alpha+\beta+\gamma)t} \bar{A}(t) dt = \frac{\gamma[1-a^*(x_1)]}{x_1} \quad (13)$$

$$dQ_{05}(t) = \int_0^\infty a(t) e^{-(\alpha+\beta+\gamma)t} dt = a^*(x_1) \quad (14)$$

$$dQ_{10}(t) = \int_0^\infty g_1(t) dt = 1 \quad (15)$$

$$dQ_{20}(t) = \int_0^\infty e^{-(\alpha+\beta)t} g_2(t) dt = g_2^*(\alpha + \beta) \quad (16)$$

$$dQ_{24}(t) = \int_0^\infty (\alpha + \beta) e^{-(\alpha+\beta)t} \bar{G}_2(t) dt = 1 - g_2^*(\alpha + \beta) \quad (17)$$

$$dQ_{30}(t) = \int_0^\infty e^{-(\alpha+\gamma)t} g_3(t) dt = g_3^*(\alpha + \gamma) \quad (18)$$

$$dQ_{34}(t) = \int_0^\infty (\alpha + \gamma) e^{-(\alpha+\gamma)t} \bar{G}_3(t) dt = 1 - g_3^*(\alpha + \gamma) \quad (19)$$

$$dQ_{40}(t) = \int_0^\infty g_4(t)dt = 1 \quad (20)$$

$$dQ_{50}(t) = \int_0^\infty b(t)dt = 1 \quad (21)$$

$$P_{10} = 1; P_{40} = 1; P_{50} = 1; \quad (22)$$

It is easy to see that $P_{01} + P_{02} + P_{03} + P_{05} = 1$; $P_{20} + P_{24} = 1$;

$$P_{30} + P_{34} = 1;$$

3.1 Mean sojourn times

The main sojourn times (μ_i) in the generative state "i" is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in the regenerative state 'i', then :

$$(\mu_i) = E(T) = P(T > t);$$

$$\mu_0 = \frac{[1 - a^*(x_1)]}{x_1}, \mu_1 = \int_0^\infty \overline{G}_3(t)dt$$

$$\mu_2 = \frac{[1 - g_2^*(\alpha + \beta)]}{\alpha + \beta}, \mu_3 = \frac{[1 - g_3^*(\alpha + \gamma)]}{\alpha + \gamma}$$

$$\mu_4 = \int_0^\infty \overline{G}_4(t)dt, \mu_5 = \int_0^\infty \overline{B}(t)dt$$

We note that the Laplace- Stieltjes transform of Q_{ij} is equal to Laplace- Stieltjes transform of q_{ij} i.e.

$$\overline{Q}_{ij}(s) = \int_0^\infty e^{-st} Q_{ij}(t)dt = L\{Q_{ij}\} = q_{ij}^*(s) \quad (23)$$

$$\overline{Q}_{01}(s) = \int_0^\infty \alpha e^{-(s+x_1)t} \overline{A}(t)dt = \frac{\alpha[1 - a^*(s+x_1)]}{s+x_1} \quad (24)$$

$$\overline{Q}_{02}(s) = \int_0^\infty \beta e^{-(s+x_1)t} \overline{A}(t)dt = \frac{\beta[1 - a^*(s+x_1)]}{s+x_1} \quad (25)$$

$$\overline{Q}_{03}(s) = \int_0^\infty \gamma e^{-(s+x_1)t} \overline{A}(t)dt = \frac{\gamma[1 - a^*(s+x_1)]}{s+x_1} \quad (26)$$

$$\overline{Q}_{05}(s) = \int_0^\infty e^{-(s+x_1)t} a(t)dt = a^*(s+x_1) \quad (27)$$

$$\overline{Q}_{10}(s) = \int_0^\infty e^{-st} g_1(t)dt = g_1^*(s) \quad (28)$$

$$\overline{Q}_{20}(s) = \int_0^\infty e^{-(s+\alpha+\beta)t} g_2(t)dt = g_2^*(s+\alpha+\beta) \quad (29)$$

$$\overline{Q}_{24}(s) = \int_0^\infty (\alpha + \beta) e^{-(s+\alpha+\beta)t} \overline{G}_2(t)dt = \frac{(\alpha + \beta)[1 - g_2^*(s+\alpha+\beta)]}{s+\alpha+\beta} \quad (30)$$

$$\overline{Q}_{30}(s) = \int_0^\infty e^{-(s+\alpha+\gamma)t} g_3(t)dt = g_3^*(s+\alpha+\gamma) \quad (31)$$

$$\overline{Q}_{34}(s) = \int_0^\infty (\alpha + \gamma) e^{-(s+\alpha+\gamma)t} \overline{G}_3(t)dt = \frac{(\alpha + \gamma)[1 - g_3^*(s+\alpha+\gamma)]}{s+\alpha+\gamma} \quad (32)$$

$$\overline{Q}_{40}(s) = \int_0^\infty g_4(t)dt = g_4^*(s+x_1) \quad (33)$$

$$\overline{Q}_{50}(s) = \int_0^\infty e^{-st} b(t)dt = b^*(s) \quad (34)$$

We define m_{ij} as follows :- $m_{ij} = -\left[\frac{d\overline{Q}_{ij}(s)}{ds}\right]_{s=0} = -Q'_{ij}(0)$.

It can so that $m_{01} + m_{02} + m_{03} + m_{05} = \mu_0$;

$$m_{20} + m_{24} = \mu_2;$$

$$m_{30} + m_{34} = \mu_3;$$

Where $\alpha + \beta + \gamma = x_1$

3.2 Main time to system failure

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations.

$$\pi_0(t) = Q_{01}(t) + Q_{02}(t) \pi_2(t) + Q_{03}(t) \pi_3(t) + Q_{05}(t), \quad (35)$$

$$\pi_1(t) = Q_{10}(t) \pi_0(t), \quad (36)$$

$$\pi_2(t) = Q_{20}(t) \pi_0(t) + Q_{24}(t), \quad (37)$$

$$\phi_3(t) = Q_{30}(t) \pi_0(t) + Q_{34}(t). \quad (38)$$

Taking Laplace-Stieltjes Transforms (L.S.T.) on both side and solving we get, MTSF when the system starts from the state S_0 is given by

$$MTSF = E(t) = -\left[\frac{d\bar{\pi}_{ij}(s)}{ds}\right]_{s=0} = \frac{D'_1(0) - N'_1(0)}{D_1(0)}$$

$$MTSF = \frac{\mu_0 + \mu_1 P_{01} + \mu_2 P_{02} + \mu_3 P_{03}}{1 - P_{01}P_{10} - P_{02}P_{20} - P_{03}P_{30}} \quad (39)$$

3.3 Availability analysis of the system

Let $M_i(t)$ ($i = 0, 1, 2, 3$) denote the probability that system is initially in regenerative state $S_i \in E$ is up at time t without passing through any other regenerative state or returning to itself through one or more non regenerative states. i.e. either it continues to remain in regenerative S_i or a non regenerative state including itself. By probabilistic arguments, we have the following recursive relations

$$M_0(t) = e^{-(\alpha+\beta+\gamma)t} \bar{A}(t), M_1(t) = \bar{G}_1(t).$$

$$M_2(t) = e^{-(\alpha+\beta)t} \bar{G}_2(t), M_3(t) = e^{-(\alpha+\gamma)t} \bar{G}_3(t).$$

Recursive relations giving point wise availability $A_i(t)$ given as follows:

$$A_0(t) = M_0(t) + \sum_{i=1,2,3,5} q_{0i}(t) \odot A_i(t); \quad (40)$$

$$A_1(t) = M_1(t) + q_{10}(t) \odot A_0(t); \quad (41)$$

$$A_2(t) = M_2(t) + \sum_{i=0,4} q_{2i}(t) \odot A_i(t); \quad (42)$$

$$A_3(t) = M_3(t) + \sum_{i=0,4} q_{3i}(t) \odot A_i(t); \quad (43)$$

$$A_4(t) = q_{40}(t) \odot A_0(t); \quad (44)$$

$$A_5(t) = q_{50}(t) \odot A_0(t); \quad (45)$$

Taking Laplace stieltjes transformation (L.S.T.) of above equations and solving the steady state availability is given by

$$A_0^*(\infty) = \lim_{s \rightarrow 0} A_0^*(s) = \frac{N_2(0)}{D'_2(0)} = \frac{\mu_0 L_0 + \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} \quad (46)$$

4. Busy period analysis

- Let $W_i(t)$ ($i = 1, 2, 3$) denote the probability that the repairman is busy initially with repair in regenerative state S_i and remain busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states. By probabilistic arguments we have

$$W_1(t) = \bar{G}_1(t), W_2(t) = \bar{G}_2(t), W_3(t) = \bar{G}_3(t).$$

Developing similar recursive relations as in availability, we have

$$B_0(t) = \sum_{i=1,2,3,5} q_{0i}(t) \odot B_i(t); \quad (47)$$

$$B_1(t) = W_1(t) + q_{10}(t) \odot B_0(t); \quad (48)$$

$$B_2(t) = W_2(t) + \sum_{i=0,4} q_{2i}(t) \odot B_i(t); \quad (49)$$

$$B_3(t) = W_3(t) + \sum_{i=0,4} q_{3i}(t) \odot B_i(t); \quad (50)$$

$$B_4(t) = q_{40}(t) \odot B_0(t); \quad (51)$$

$$B_5(t) = q_{50}(t) \odot B_0(t); \quad (52)$$

Taking Laplace stieltjes transformation (L.S.T.) of above equations and solving the steady state busy period of repairman due to repair is given by

$$B_0^{1*}(\infty) = \lim_{s \rightarrow 0} s B_0^{1*}(s) = \frac{N_2(0)}{D'_{12}(0)} = \frac{\sum_{i=1,2,3} \mu_i L_i}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} \quad (53)$$

• **Busy period of the Repairman in preventive maintenance in time (0, t].**

By probabilistic arguments we have $W_5(t) = \bar{B}(t)$

Similarly developing similar recursive relations as in 9(1), we have

$$B_0(t) = \sum_{i=1,2,3,5} q_{0i}(t) \odot B_i(t); \quad (54)$$

$$B_1(t) = q_{10}(t) \odot B_0(t); \quad (55)$$

$$B_2(t) = \sum_{i=0,4} q_{2i}(t) \odot B_i(t); \quad (56)$$

$$B_3(t) = \sum_{i=0,4} q_{3i}(t) \odot B_i(t); \quad (57)$$

$$B_4(t) = q_{40}(t) \odot B_0(t); \quad (58)$$

$$B_5(t) = W_5(t) + q_{50}(t) \odot B_0(t); \quad (59)$$

Taking Laplace stieltjes transformation (L.S.T.) of above equations and solving the steady state busy period of the repairman in preventive maintenance is given by

$$B_0^{2*}(\infty) = \lim_{s \rightarrow 0} s B_0^{2*}(s) = \frac{N_4(0)}{D'_{12}(0)} = \frac{\mu_5 L_5}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} \quad (60)$$

• **Busy period of the Repairman in Shut Down repair in time (0, t].**

By probabilistic arguments we have $W_4(t) = \bar{G}_4(t)$

Similarly developing similar recursive relations as in 9(2), we have

$$B_0(t) = \sum_{i=1,2,3,5} q_{0i}(t) \odot B_i(t); \quad (61)$$

$$B_1(t) = q_{10}(t) \odot B_0(t); \quad (62)$$

$$B_2(t) = \sum_{i=0,4} q_{2i}(t) \odot B_i(t); \quad (63)$$

$$B_3(t) = \sum_{i=0,4} q_{3i}(t) \odot B_i(t); \quad (64)$$

$$B_4(t) = W_4(t) + q_{40}(t) \odot B_0(t); \quad (65)$$

$$B_5(t) = q_{50}(t) \odot B_0(t); \quad (66)$$

Taking Laplace stieltjes transformation (L.S.T.) of above equations and solving the steady state busy period of the repairman in shut down repair is given by

$$B_0^{3*}(\infty) = \lim_{s \rightarrow 0} s B_0^{3*}(s) = \frac{N_5(0)}{D'_{12}(0)} = \frac{\mu_4 L_4}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} \quad (67)$$

• **Particular Cases:**

When all repair time distributions are n-phase Erlangian distributions i.e.

$$g_i(t) = \frac{n r_i (n r_i t)^{n-1} e^{-n r_i t}}{(n-1)!} \quad (68)$$

and survival function

$$\bar{G}_j(t) = \sum_{j=0}^{n-1} \frac{(nr_i t)^j e^{-nr_i t}}{j!} \quad (69)$$

And other distributions are negative exponential

$$a(t) = \theta e^{-\theta t}, b(t) = \eta e^{-\eta t}, \bar{A}(t) = e^{-\theta t}, \bar{B}(t) = e^{-\eta t}$$

$$\text{For } n=1 \quad g_i(t) = r_i e^{-r_i t}, \bar{G}_i(t) = e^{-r_i t}$$

$$\text{If } i=1,2,3,4 \quad g_1(t) = r_1 e^{-r_1 t}, g_2(t) = r_2 e^{-r_2 t}$$

$$g_3(t) = r_3 e^{-r_3 t}, g_4(t) = r_4 e^{-r_4 t}$$

$$\bar{G}_1(t) = e^{-r_1 t}, \bar{G}_2(t) = e^{-r_2 t}, \bar{G}_3(t) = e^{-r_3 t}, \bar{G}_4(t) = e^{-r_4 t},$$

$$\text{Also } P_{01} = \frac{\alpha}{x_1 + \theta}, P_{02} = \frac{\beta}{x_1 + \theta}, P_{01} = \frac{\gamma}{x_1 + \theta}, P_{05} = \frac{\theta}{x_1 + \theta},$$

$$P_{10} = 1, P_{20} = \frac{r_2}{\alpha + \beta + r_2}, P_{24} = \frac{\alpha + \beta}{\alpha + \beta + r_2}, P_{30} = \frac{r_3}{\alpha + \gamma + r_3}, P_{34} = \frac{\alpha + \gamma}{\alpha + \gamma + r_3},$$

$$\mu_0 = \frac{1}{x_1 + \theta}, \mu_1 = \frac{1}{x_1}, \mu_2 = \frac{r_2}{\alpha + \beta + r_2}, \mu_3 = \frac{r_3}{\alpha + \gamma + r_3}, \mu_4 = \frac{1}{r_4}, \mu_5 = \frac{1}{\eta},$$

$$\text{MTSF} = \frac{\mu_0 + \mu_1 P_{01} + \mu_2 P_{02} + \mu_3 P_{03}}{1 - P_{01} P_{10} - P_{02} P_{20} - P_{03} P_{30}}, A_0^*(\infty) = \frac{\mu_0 L_0 + \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} B_0^{1*}(\infty) = \frac{\sum_{i=1,2,3} \mu_i L_i}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} B_0^{2*}(\infty) = \frac{\mu_5 L_5}{\sum_{i=0,1,2,3,4,5} \mu_i L_i} B_0^{3*}(\infty) = \frac{\mu_4 L_4}{\sum_{i=0,1,2,3,4,5} \mu_i L_i}$$

$$\text{Where } L_0 = 1; L_1 = P_{01}; L_2 = P_{02}; L_3 = P_{03}; L_4 = P_{02} P_{24} + P_{03} P_{34}; L_5 = P_{05}$$

5. Profit analysis

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0,t]. Therefore, G(t) = Expected total revenue earned by the system in (0,t] - Expected repair cost of the failed units - Expected repair cost of the repairman in preventive maintenance - Expected repair cost of the Repairman in shut down

$$= C_1 \mu_{up}(t) - C_2 \mu_{b1}(t) - C_3 \mu_{b2}(t) - C_4 \mu_{b3}(t)$$

$$= C_1 A_0 - C_2 B_0^1 - C_3 B_0^2 - C_4 B_0^4$$

Where

$$\mu_{up}(t) = \int_0^t A_0(t) dt; \mu_{b1}(t) = \int_0^t B_0^1(t) dt;$$

$$\mu_{b2}(t) = \int_0^t B_0^2(t) dt; \mu_{b3}(t) = \int_0^t B_0^3(t) dt;$$

C_1 is the revenue per unit time and C_2, C_3, C_4 are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair respectively

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