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On record based transmuted inverse exponential distribution

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Abstract

Inverse Exponential distribution has been used to analyze lifetime datasets which has non monotone hazard function and has applications in reliability and biological study areas. The present study introduces new generalization of inverse exponential distribution using Record-Based transmutation. Record-Based transmuted inverse Exponential distribution is derived and its properties studied. The maximum likelihood estimation procedure is employed to estimate the parameters of the proposed distribution. Finally, an algorithm in R package is developed to carry out the simulation studies.

Keywords: Exponential distribution, inverse exponential, record-based transmuted inverse exponential, maximum likelihood estimation

1. Introduction

Exponential distribution is a popular lifetime distribution due to its simple structure and has constant hazard rate. Several generalizations of exponential distributions were introduced so far. For example new extended generalized exponential distribution (Dey *et al.* (2017) ^[7], slash exponential (Punathumparambath (2020) ^[12]), Inverse exponential distribution (Keller *et al.* 1982; Dey, 2007) ^[10, 8], exponentiated inverted exponential (Fatima and Ahamad, 2017) ^[9], generalized inverted exponential (Abouammoh and Alshingiti (2009) ^[1] and exponentiated generalized inverted exponential (Oguntunde, Adejumo and Balogun, 2014) ^[11]).

The cumulative distribution function (cdf) and probability density function (pdf) of inverse exponential (IE) distribution were respectively given by,

$$G(x) = e^{-\frac{\theta}{x}}, x \geq 0, \theta > 0. \quad (1.1)$$

$$g(x) = \frac{\theta}{x^2} e^{-\frac{\theta}{x}}, x \geq 0, \theta > 0. \quad (1.2)$$

Recently there is lot of interest in constructing new family of distributions which describes many real life phenomena. Shaw and Buckley (2007) proposed the transmuted-G (T-G) family of distributions. N. Balakrishnan and M. He (2021) ^[4], proposed a family of transmuted distributions based on the theory of records. The Cumulative distribution function (cdf) and probability density function (pdf) of record based transmuted distribution can be respectively given as follows.

$$F(x) = G(x) + p(1 - G(x)) \log(1 - G(x)), \quad (1.3)$$

$$f(x) = g(x)[1 - p - p \log(1 - G(x))] \quad (1.4)$$

Where, $0 < p < 1$, $G(x)$ and $g(x)$ respectively denote the cdf and pdf of the baseline distribution. Caner Tanış and Buğra Saraçoğlu (2020) ^[6] studied various properties and applications of the Record based transmuted Weibull (RTW) distribution.

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Arshad *et al.* (2021) ^[3] introduced Record-Based Transmuted Generalized Linear Exponential Distribution. In this paper we study new generalization of inverse exponential distribution using record based transmutation, referred as record based transmuted inverse exponential distribution.

2. Record based transmuted Inverse Exponential Distribution

In this section, we introduce the Record based transmuted Inverse exponential distribution (RTIE). The pdf and cdf of record based transmuted inverse Exponential distribution with parameters θ, p denoted by RTIE (θ, p) are respectively given by

$$f(x) = \theta x^{-2} e^{-\theta x^{-1}} [1 - p - p \log(1 - e^{-\theta x^{-1}})], \theta > 0, 0 < p < 1, \quad (2.1)$$

$$F(x) = e^{-\theta x^{-1}} + p(1 - e^{-\theta x^{-1}}) \log(1 - e^{-\theta x^{-1}}), \theta > 0, 0 < p < 1. \quad (2.2)$$

The plot of pdf and cdf of RTIE distribution for various values of the parameters are given below in figure1. As evident from the density function, our method, RTIE, has more flexibility due to its additional parameter and hence the study of statistical properties associated with it are of non-negligible significance.

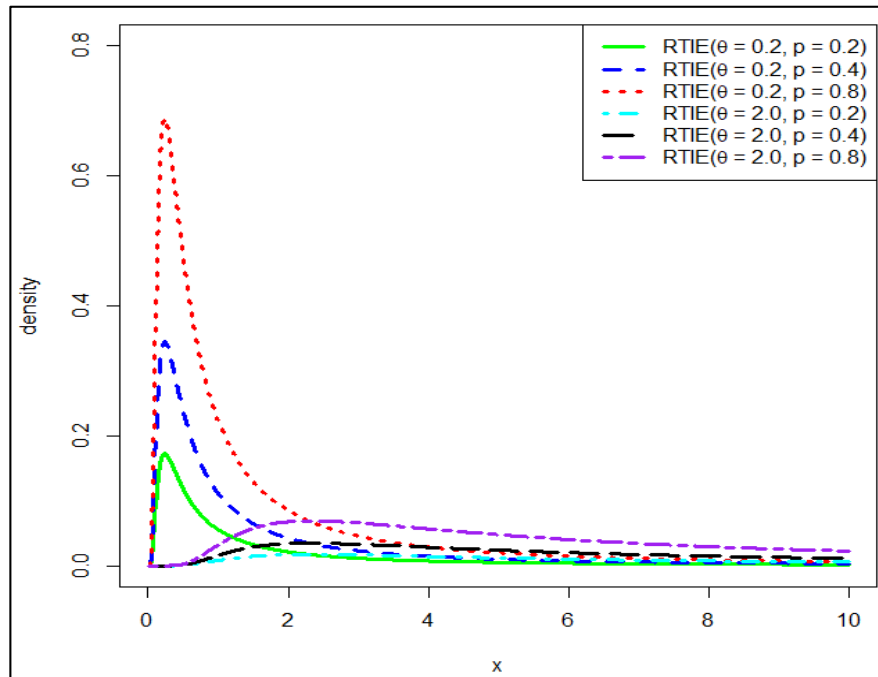


Fig 1: Plot of pdf of RTIE for various values of parameters

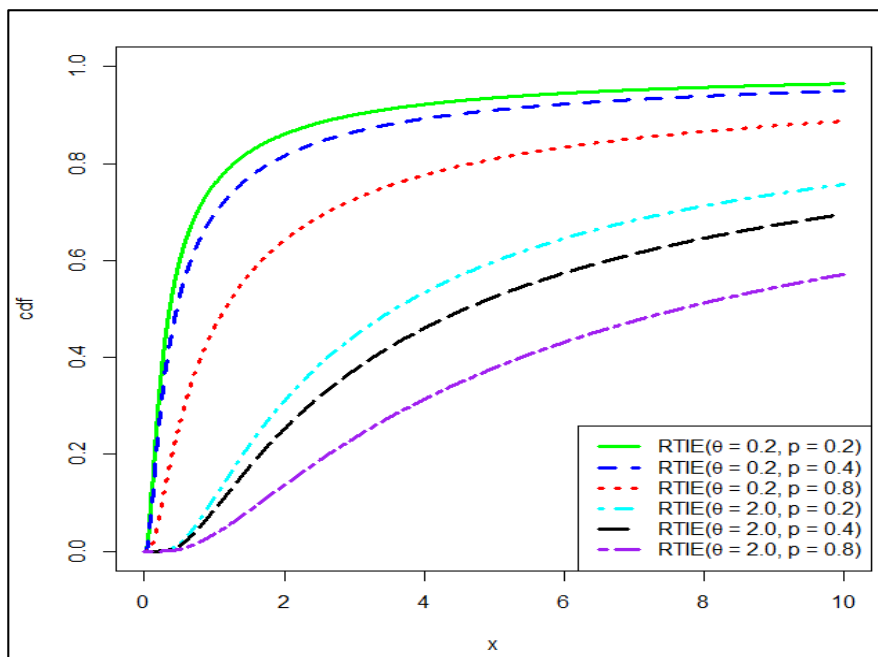


Fig 2: Plot of cdf of RTIE for various values of parameters.

2.1 Reliability measures of Record based Inverse Exponential distribution

In this section we discuss the various reliability measures of RTIE (θ, p). The survival (reliability) function (sf) of RTIE is defined as,

$$S(x) = 1 - e^{-\theta x^{-1}} - p(1 - e^{-\theta x^{-1}}) \log(1 - e^{-\theta x^{-1}}), \theta > 0, 0 < p < 1, \quad (3.1)$$

The corresponding hazard function (hf) is as follows

$$h(x) = \frac{\theta x^{-2} e^{-\theta x^{-1}} [1 - p - p \log(1 - e^{-\theta x^{-1}})]}{1 - e^{-\theta x^{-1}} - p(1 - e^{-\theta x^{-1}}) \log(1 - e^{-\theta x^{-1}})}, \theta > 0, 0 < p < 1. \quad (3.2)$$

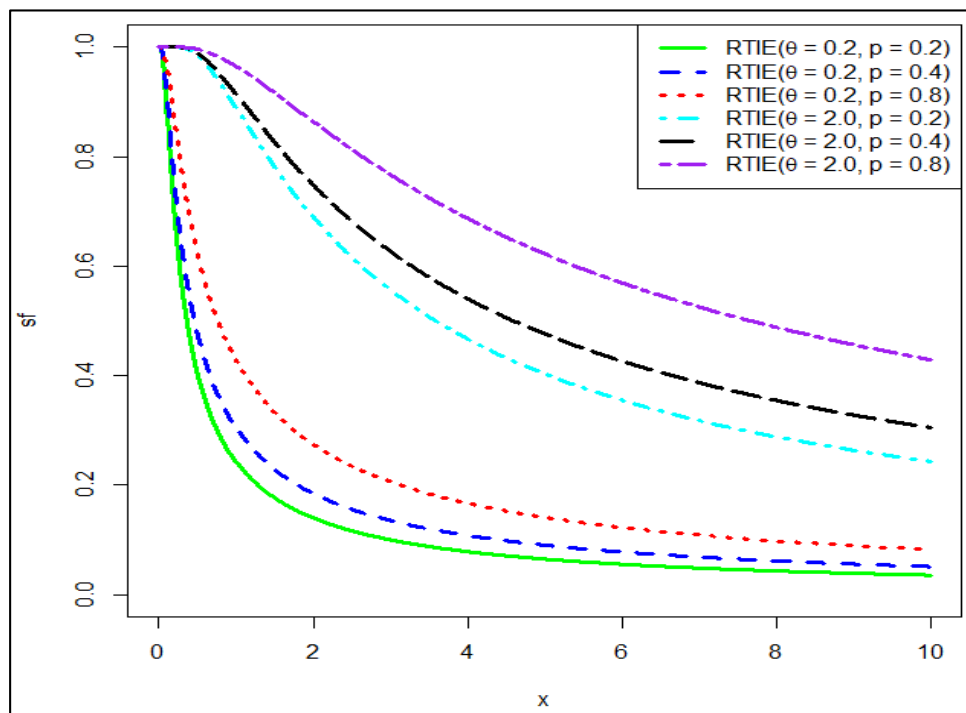


Fig 3: Plot of survival function (sf) of RTIE for various values of parameters.

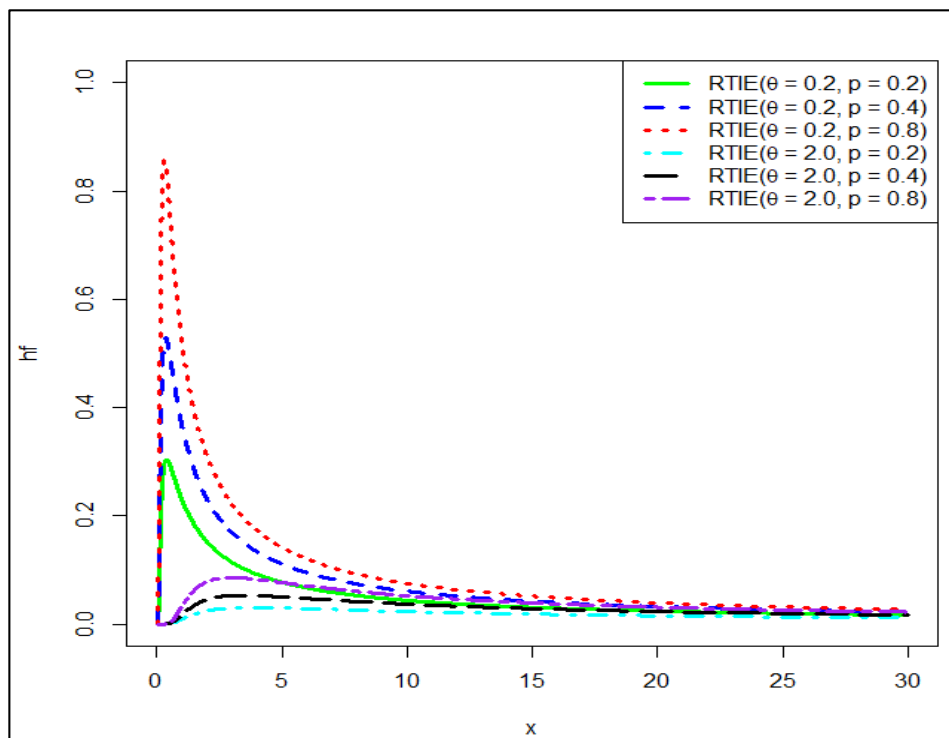


Fig 4: Plot of hf of RTIE for various values of parameters.

From figure 4 we can see that the hazard function of RTIE can be used to model datasets which has decreasing and increasing failure rates, that is DFR and IFR.

The reverse hazard function (rhf) is given as follows,

$$r(x) = \frac{\theta x^{-2} e^{-\theta x^{-1}} [1-p-p \log(1-e^{-\theta x^{-1}})]}{e^{-\theta x^{-1}} + p(1-e^{-\theta x^{-1}}) \log(1-e^{-\theta x^{-1}})}, \theta > 0, 0 < p < 1 \quad (3.3)$$

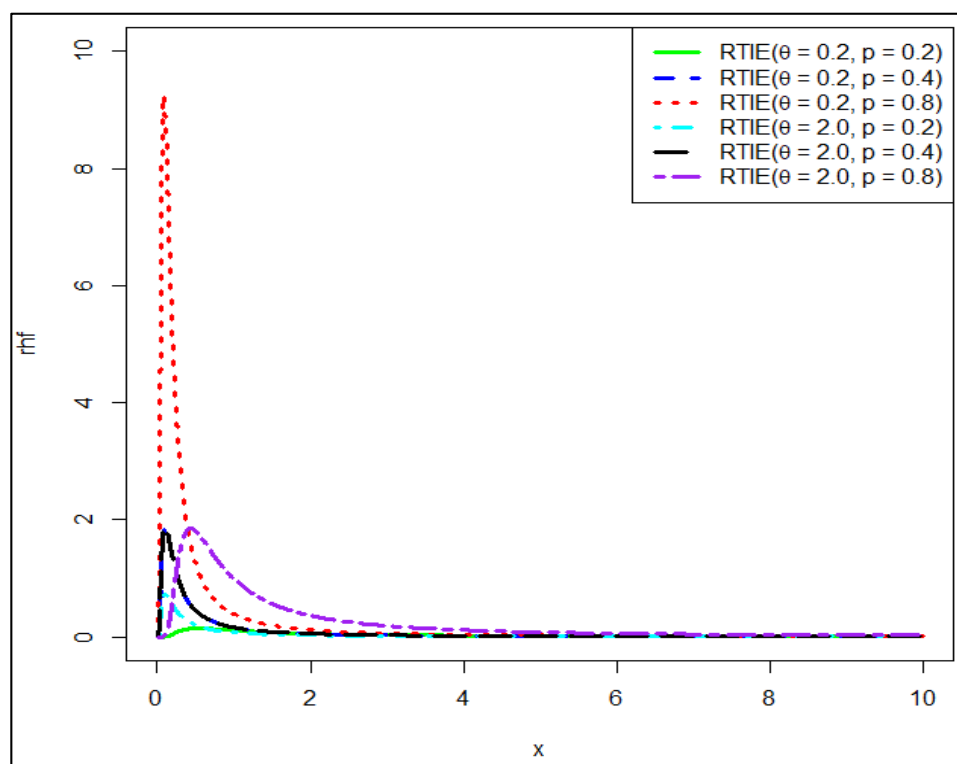


Fig 5: Plot of rhf of RTIE for various values of parameters the odd function (of) is given as follows,

$$o(x) = \frac{e^{-\theta x^{-1}} + p(1-e^{-\theta x^{-1}}) \log(1-e^{-\theta x^{-1}})}{1-e^{-\theta x^{-1}} - p(1-e^{-\theta x^{-1}}) \log(1-e^{-\theta x^{-1}})}, \theta > 0, 0 < p < 1 \quad (3.4)$$

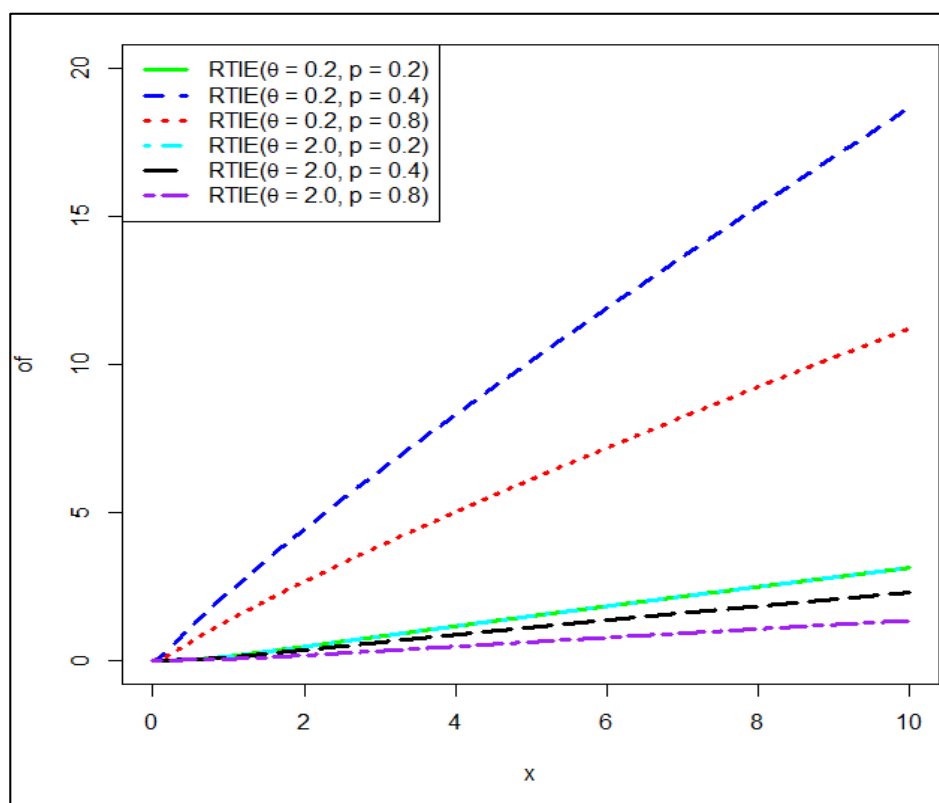


Fig 6: Plot of odd function of RTIE for various values of parameters the quantile function of Record based transmuted inverse exponential is given by

$$X_q = -\frac{1}{\theta} \left[\log \left(1 - \frac{q^{-1}}{qW\left((q-1)e^{-\frac{1}{p}}\right)} \right) \right]^{-1}, \theta > 0, 0 < p, q < 1, \quad (3.5)$$

Where, $W(x)$ is the Lambert W function which for real number takes the form, $W(x)e^{W(x)} = x$. Lambert W function is available in statistical softwares, such as R, Matlab, etc. Random sample generation from the RTIE distribution can be obtained using the quantile function (qf). The qf can be useful for goodness-of-fit, simulation purposes and computing measures of skewness and kurtosis.

3. Order Statistics

In this section, we derive the pdf and cdf of r^{th} order statistics from RTIE. Let $X_1, \dots, X_n$ is a random sample from RTIE distribution and $X_{1:n}, \dots, X_{n:n}$, denote the corresponding order statistics. Then the probability density function of the r^{th} order statistics is given by,

$$f_{r:n}(x) = \frac{f(x)}{B(r, n-r+1)} [F(x)]^{r-1} [1 - F(x)]^{n-r}, \quad (3.1)$$

$$f_{r:n}(x) = \frac{f(x)}{B(r, n-r+1)} \sum_{i=0}^{n-r} (-1)^i \binom{n-r}{i} [F(x)]^{r+i-1}. \quad (3.2)$$

Then the probability density function of the r^{th} order statistic of RTIE is given by,

$$f_{r:n}(x) = \frac{1}{B(r, n-r+1)} \theta x^{-2} e^{-\theta x^{-1}} [1 - p - p \log(1 - e^{-\theta x^{-1}})] \sum_{i=0}^{n-r} (-1)^i \binom{n-r}{i} [e^{-\theta x^{-1}} + p(1 - e^{-\theta x^{-1}}) \log(1 - e^{-\theta x^{-1}})]^{r+i-1} \quad (3.3)$$

Then the cumulative distribution function (cdf) of the r^{th} order statistic is given by

$$F_{r:n}(x) = \sum_{i=r}^n \binom{n}{i} [F(x)]^i [1 - F(x)]^{n-i} = \sum_{i=r}^n \sum_{j=0}^{n-i} \binom{n}{i} \binom{n-i}{j} [F(x)]^{i+j}. \quad (3.4)$$

Then the cdf of the r^{th} order statistic of RTIE is given by

$$F_{r:n}(x) = \sum_{i=r}^n \sum_{j=0}^{n-i} \binom{n}{i} \binom{n-i}{j} [e^{-\theta x^{-1}} + p(1 - e^{-\theta x^{-1}}) \log(1 - e^{-\theta x^{-1}})]^{i+j}. \quad (3.5)$$

We can derive the pdf's and cdf's of first and largest order statistics by substituting $r=1$ and $r=n$ respectively in equations (3.3) and (3.5).

4. Maximum Likelihood Estimation

In this section, we discuss the estimation of two unknown parameters $\Theta = (\theta, p)$ using the maximum likelihood estimation. Let $X = (X_1, \dots, X_n)$ be independent and identically distributed samples from an RTIE distribution.

The log-likelihood function takes the form,

$$\log L = n \log \theta - \sum_{i=1}^n \log x_i^{-2} - \theta \sum_{i=1}^n x_i^{-1} + \sum_{i=1}^n \log [1 - p - p \log(1 - e^{-\theta x_i^{-1}})] \quad (4.1)$$

The maximum likelihood estimation can be done using the optima function of the R statistical software, applying the BFGS algorithm in R (R core team 2020) [13]. Estimates of the standard errors were obtained by inverting the numerically differentiated information matrix at the maximum likelihood point.

5. Simulation

In this section, we illustrate the simulation studies. Now we generate $N = 1000$ sets of $n = 50, 100, 250$ samples from the RTIE distribution using algorithm developed in R package. Table 1 gives the estimated value of the parameters using MLE, and standard error (SE) of the estimates.

Table 1: Simulation study - Maximum likelihood estimates and standard error (SE) for various choices of parameters θ and p over 1000 replications of datasets of size $n = 50, 100, 250$.

Parameters	n	$\hat{\theta} (SE(\hat{\theta}))$	$\hat{p} (SE(\hat{p}))$
$\theta = 0.5$ $p = 0.5$	50	0.342 (0.054)	0.533 (0.097)
	100	0.514 (0.021)	0.521 (0.039)
	250	0.493 (0.009)	0.509 (0.017)
$\theta = 1.5$ $p = 0.8$	50	1.384 (0.047)	0.761 (0.054)
	100	1.487 (0.025)	0.813 (0.033)
	250	1.505 (0.010)	0.808 (0.021)

It is clear from Table 1 that the estimation algorithm works satisfactorily for various choices of parameters and the asymptotic standard errors of the maximum likelihood estimators decreases as sample size increases.

6. Conclusion

In this paper, we introduced a generalization of inverse exponential (IE) based on the record based transmutation introduced by Balakrishnan and He (2021) ^[4]. We studied various properties of the proposed distributions including reliability measures and order statistics. We discussed the estimation of parameters using maximum likelihood estimation and finally, we illustrated simulation studies using an algorithm developed in R package. Hazard function of RTIE has increasing and decreasing structure and hence can be used to model datasets which have to decrease and increasing failure rates that is DFR and IFR.

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