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Wind speed prediction analysis using Rayleigh distribution

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Abstract

Two parameter Rayleigh distribution is considered for prediction analysis of quantiles at extreme upper and lower tail areas of the distribution. Four methods of estimation of the parameters namely, probability weighted moment (PWM), method of maximum likelihood (MLE), method of moment (MM), and ordinary least square method (OLS) are illustrated. The Rayleigh distribution is fitted to monthly averages and monthly maximum wind speed data in Oman by each of these four estimation methods. Theoretical quantiles at various probabilities were estimated by each of these method and were compared with empirical quantiles. The mean absolute error (MAE) and mean squared errors (MSE) of the fit due to each method were worked out. It is concluded that the two parameter Rayleigh distribution can best be fitted by probability weighted moment (PWM) method.

Keywords: Wind speed, Rayleigh distribution, probability weighted moment (PWM), maximum likelihood (ML), method of moment (MM), ordinary least square (OLS)

1. Introduction

Wind, that is moving air, possesses some kinetic energy due to its high speed. Wind is a result of the solar energy, as heating of land results in movement of air. Wind energy can be harnessed and used for generating electricity. The main advantage of wind energy is that harnessing it doesn't disrupt natural processes or harm the environment, unlike a lot of other energy sources. To generate electricity on a large scale, a number of windmills are set up over a large area, called a wind energy farm. Such areas need a wind speed of 15kmph^[1].

Many studies have found that wind speed is the most significant parameter of wind power. Thus, an accurate determination of the probability distribution of wind speed is an important parameter to measure before estimating the wind energy potential over a particular region.

Analysis of wind speed distribution is very important to evaluate the wind potential and the economic feasibility. There are several of distribution functions to describe a wind speed frequency curve but the Rayleigh distribution is the most broadly used and accepted in the specialized *research journal* [2, 3, 4]. Most of the studies on wind speed modeling consider various functional forms of the probability models using maximum likelihood method of estimation. Very little attempts were made to take into account more recent methods of estimation such as probability weighted moment (PWM) However this study focus on the methods of estimation of the quantiles rather than the choice of the model in particular in estimating the extreme quantiles.

2. Review of Literature

There are extensive studies on wind speed data modeling in energy literature e.g. [5, 6, 7] Some of these investigation related to the choice of a probability density function are reviewed below. Tanate Chaichana *et al.* [6] described some statistical probability distribution functions (pdf) which have been used to measure wind power and energy potential of Saint Martin's Island situated at 13 kilometers apart from the southern-most tip of Bangladesh. They compared some well-known statistical probability density functions (pdf) namely, Normal distribution, Weibull distribution, Gamma distribution, and Rayleigh distribution. Their performance test revealed that, Weibull probability distribution proved to be the best distribution function for the wind power estimation and analysis.

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Rayleigh distribution follows Weibull density in this research. Fatema ^[5] studied the use of Rayleigh, Inverse Weibull (IW), Burr Type III, Extreme Value (EV), Gamma, Inverse Gamma (IG), Marshall-Olkin extended Lindley (MOEL), generalized Extreme Value (GEV) and exponentiated Weibull (EW) distributions as alternatives to Weibull. Mahyoub H. Al Buhairi ^[7] investigated the wind energy modeling potential of Weibull and the Rayleigh models by fitting data from Yemen.

3. Two-parameter Rayleigh distribution

The two-parameter Rayleigh distribution is a continuous probability distribution which usually arises when a two dimensional vector has its two orthogonal components normally and independently distributed. The Euclidean norm of the vector will then have a Rayleigh distribution. The distribution may also arise in the case of random complex numbers whose real and imaginary components are normally and independently distributed. The modulus of these numbers will then be Rayleigh distributed. The Rayleigh variable X with parameter ε and parameter δ is characterized by the probability density function ^[4].

$$f(x) = \frac{(x-\varepsilon)}{\delta^2} e^{-\frac{(x-\varepsilon)^2}{2\delta^2}} \quad \text{for } \varepsilon < x < \infty \text{ and } \delta > 0$$

The corresponding cumulative distribution function (CDF) is given as follows:

$$F(x) = 1 - e^{-\frac{(x-\varepsilon)^2}{2\delta^2}}$$

The Mean and variance of this distribution are given as below:

$$E[x] = \delta \sqrt{\frac{\pi}{2}} + \varepsilon$$

$$\text{Var}(x) = \delta^2 \left[2 - \frac{\pi}{2} \right]$$

3. Estimation Methods

In this paper we investigate the estimation of the scale and threshold parameters using a modified maximum likelihood method (ML), the moment method (MM), the probability weighted moment method (PWM) and the ordinary least square method (OLS). These methods are illustrated as below.

3.1 Method of Probability Weighted Moments

Using the distribution function of the Rayleigh distribution we obtain the probability weighted moment of order r and then replacing with their sample estimates we get:

$$\hat{\delta} = \frac{(r+1)\hat{\alpha}_r - (s+1)\hat{\alpha}_s}{\sqrt{\frac{\pi}{2(r+1)}} - \sqrt{\frac{\pi}{2(s+1)}}}$$

And

$$\hat{\varepsilon} = (r+1)\hat{\alpha}_r - \hat{\delta} \sqrt{\frac{\pi}{2(r+1)}} \quad \text{Where } \hat{\alpha}_r \text{ and likewise } \hat{\alpha}_s \text{ given by}$$

$$a_r = n^{-1} \sum_{i=1}^n \frac{c_i^{n-1}}{c_i^r} \quad \text{For } n > r \text{ and } c_j^k = 0 \text{ for } k < j \text{ are the unbiased estimators of } a_r \text{ and } a_s \text{ respectively.}$$

In our estimation we take $s=0$ and $r=1$, so we got this

$$\hat{\delta} = \frac{2\hat{\alpha}_1 - \hat{\alpha}_0}{\sqrt{\frac{\pi}{2(2)}} - \sqrt{\frac{\pi}{2}}}$$

$$\text{Where } \hat{\alpha}_1 = n^{-1} \sum_{i=1}^n \frac{n-i}{n-1} \text{ and } \hat{\alpha}_0 = \text{mean}(x)$$

$$\text{And } \hat{\varepsilon} = 2\hat{\alpha}_1 - \hat{\delta} \sqrt{\frac{\pi}{2(2)}}$$

3.2 Method of maximum likelihood

In the case of the Rayleigh situation, the maximization of the likelihood function with respect to δ and ε yields easily the ML parameter estimators

$$\hat{\varepsilon} = \min(x_1, x_2, \dots, x_n)$$

And

$$\hat{\delta} = \sqrt{\frac{\sum_{i=1}^n [x_i - \hat{\varepsilon}]^2}{2n}}$$

3.3 Method of moments

By equating theoretical moments with the sample estimates of mean and variance we get the following estimates of the parameters.

$$\hat{\delta} = \sqrt{\frac{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2}{2n^2(1 - \frac{\pi}{4})}}$$

And

$$\hat{\varepsilon} = \bar{x} - \hat{\delta} \sqrt{\frac{\pi}{2}}$$

3.4 The ordinary least square method (OLS)

By regression of ordered values of wind speed (y) on the estimated empirical distribution function the estimates are obtained as below.

$$\hat{\delta} = \frac{\sum x_i y_j - n \bar{x} \bar{y}}{\sum x_i^2 - n(\bar{x})^2}$$

$$\hat{\varepsilon} = \bar{y} - \hat{\delta} \bar{x}$$

4. Analysis and Results

We have considered data of 31 years taken monthly from 1980 to 2011 of the wind speed records for Muscat with 384 sample size with unit Knot (The knot is a unit of speed equal to one nautical mile (1.852 km) per hour, approximately 1.151 mph). This data consists of the mean and maximum of wind speed. The descriptive statistics is presented in Table #1 as below.

Table 1: Descriptive statistics

Wind Speed	Mean	Variance	St Dev	Min	Q1	Median	Q3	Max	Skewness	Kurtosis
Maximum	27.89	42.23	6.50	12	24	27	31	66	0.67	1.85
Mean	5.46	2.56	1.60	3	5	5	6	12	0.98	1.72

The table above shows that the mean of the maximum wind speed is 27.89 with variance 42.23 and the mean of average monthly wind speed is 5.46 with variance 2.56. The maximum wind speed has the highest volatility since it has the largest standard deviation comparing to the mean wind speed.

The estimates of quantiles by different method for maximum and mean wind speed are presented in Fig1 and Fig2 as below.

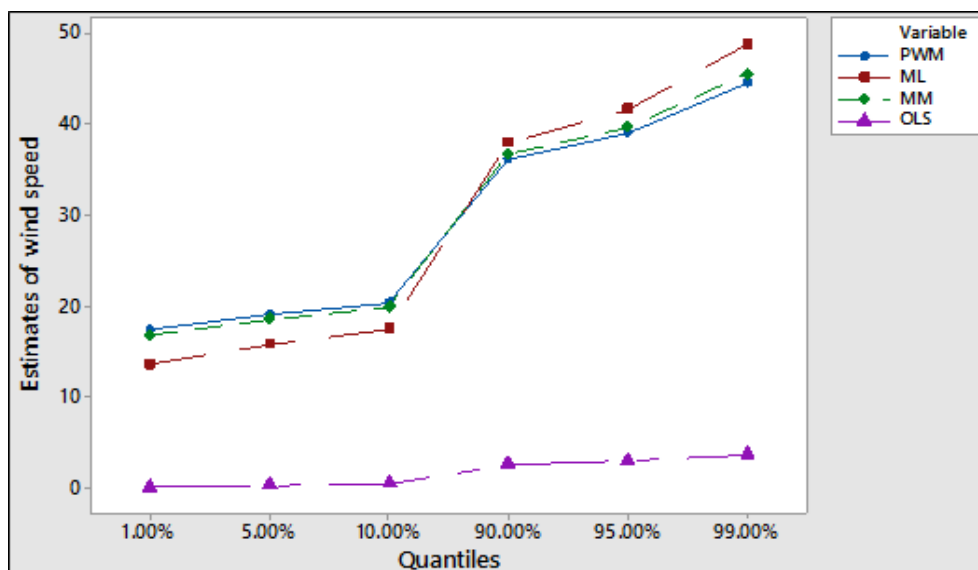


Fig 1: Estimated Quantile for maximum wind using different methods

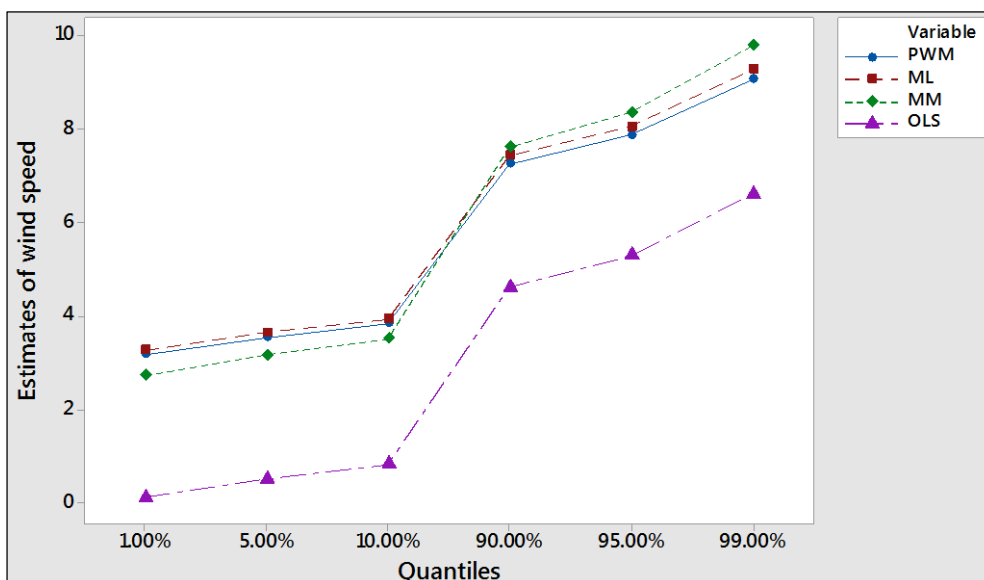


Fig 2: Estimated Quantiles for mean wind speed by different methods

From these Figures we can conclude that for the PWM for the maximum wind seed the probability of acceding the 99% quantile is $P[x_{.99} > 44.58] = 1 - .99 = 0.01$ and the probability of exceeding the 1% quantile is $[x_{.01} > 17.47] = 1 - .01 = 0.99$ and for mean speed the probability of exceeding the 99% quantile is $P[x_{.99} > 9.072] = 1 - .99 = 0.01$ while the probability of exceeding the 1% quantil is $[x_{.01} > 3.20] = 1 - .01 = 0.99$.

Tables 2 and 3 show the estimates of parameters of Rayleigh distribution for mean and maximum wind speed by different methods along with ranking with MAD and MSE as measures of accuracy. Clearly method pf PWM ranks one for fitting this distribution to both mean and maximum monthly wind speed.

5. Conclusion

Rayleigh distribution is widely used to model wind speed data. Numerous studies have been made to compare the Rayleigh distribution with other empirical distributions. Most of these studies have compared the performance of this distribution using method of maximum likelihood only. In this study we have used four methods of estimation namely, probability weighted moment (PWM), method of maximum likelihood (MLE), method of moment (MM), and ordinary least square method (OLS) to compare the estimate of the extreme quantiles rather than merely the estimates of the parameters. The ranking of the method of estimation of the quantiles was based on criterions of mean absolute error (MAE) and mean squared errors (MSE). It was concluded that probability weighted moment (PWM) performs best to predict the extreme quantiles of Rayleigh distribution. Therefor this distribution should be used with probability weighted moment (PWM) method of estimation rather than method of maximum likelihood.

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