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An undergraduate student flow model: Semester system in university of Tabuk (KSA)

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Abstract

This paper focuses on modeling an undergraduate students flow at university of Tabuk-faculty of Science (KSA) with stochastic process model depending on Markov Chain. The proposed model built by a reducible discrete Markov chain with eight transient and three absorbing states. The probabilities of absorption (graduating, withdrawal and apologized) were obtained. Furthermore, the expected time student will spend when he is enrolled in a particular stage of the study program is estimated, the expected time student enrolled in the first semester can expect to spend before graduating is obtained and the probabilities of students' progression between successive semesters of the study program for each academic year is calculated. The model also enables the prediction of future probability of student repeat specific semester, withdraw, apologize or graduate.

Keywords: Markov chains; transition matrix; batches; stochastic process, tabuk

1. Introduction

A Markov chain is an important class of stochastic processes in which a future state of an experiment depends only on the present one, not on proceeding states (see Bharucha, 2012)^[8]. There are various statistical techniques used for prediction such as time series models, cohort, regression, ratio, Markov chain and simulation. Among these techniques, Markov chain seems to be the most suitable model for this study, because it is a method that not only can estimate promotion and repetition rates, but it can also estimate the number of dropouts, graduates and death rates in the matrix (Johnstone, 1974; Borden & Dalphin, 1998; Armacost & Wilson 2004)^[16, 9, 7]. Markov chain method can also measure detailed information on the students' progress such as the average time student spend in an education system whereby other techniques like regression and ratio are unable to measure this (Kinard & Krech, 1977; Healey & Brown, 1978; Grip & Young, 1999; Guo, 2002)^[17, 14, 12, 13].

The use of Markov chain to model and analyze the students flow in the higher education is not new. Reynolds & Porath (2008)^[19] studied absorbing Markov chain with four transient and one absorbing states to model the academic progress of students attending the University of Wisconsin-Eau Claire over a specific length of time. Al-Awadhi & Konsowa (2007)^[5] and Al-Awadhi & Konsowa (2010)^[6] have modelled student flow in a high educational institution at Kuwait University by a finite Markov chain with eight states and with five transient and two absorbing states. Brezavšek & Baggia (2015)^[10] and Brezavšek & Baggia (2017)^[11] have built a model of student flow at Slovenian universities by a reducible discrete Markov chain with five transient and two absorbing states. Shah and Burke (1999)^[20] used Markov chain to model the movement of undergraduates through the higher education system in Australia with 51 transient and two absorbing states. Rahel *et al.* (2013)^[18] have developed an enrolment projection model based on the Markov chain for postgraduate students at University Utara Malaysia classified students by age, field of study, students' status whether they have graduated, dropped out, or deferred from their programs. Hlavatý & Dömeová (2014)^[15] have used Markov chain to create a model of students' progress throughout the whole courses at the Czech university of life sciences (CULS) in Prague with four transient and four absorbing states. Very interesting and useful are the studies, which modelled the students' progression and their performance during higher education study using an absorbing Markov chain (see e.g., Adam, 2015;

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Adeleke *et al.*, 2014; Al- Awadhi & Ahmed, 2002; Auwalu *et al.*, 2013; Wailand & Authella, 1980)^[2, 3, 4, 1, 21].

This paper provides a model, which can be used for analyzing the undergraduate students flow at University of Tabuk-faculty of Science (KSA). The proposed model is built by a reducible discrete Markov chain with eight transient and three absorbing states. The eight transient states represent the eight stages (semesters) student should move around until graduate, while the three absorbing states are graduation, withdrawal from specific semester and apologized for the study program. The rest of the paper is outlined as follows. In Section 2, we describe states of the Markov chain. Mathematical model is demonstrated in Section 3. Numerical example is given in Section 4. Section 5 pertains to the results and section 6 provides the conclusions of the study.

2. Definition of states of the Markov Chain

The duration of bachelors' degree within University of Tabuk is four years divided into eight semesters. Therefore, to model the student academic progress we define the following states:

- S_1 : Student is enrolled into the first semester of the study program
- S_2 : Student is enrolled into the second semester of the study program
- S_3 : Student is enrolled into the third semester of the study program
- S_4 : Student is enrolled into the fourth semester of the study program
- S_5 : Student is enrolled into the fifth semester of the study program
- S_6 : Student is enrolled into the sixth semester of the study program
- S_7 : Student is enrolled into the seventh semester of the study program
- S_8 : Student is enrolled into the eighth semester of the study program
- G: Student has graduated and successfully finished the study program
- W: Student has withdrawn from specific semester
- A: Student has apologized for the study program

To develop the model, the following assumptions are considered:

- Student who is currently enrolled into the first, second, third, fourth, fifth, sixth or seventh semester of the study program can next semester either progress to a higher level or repeat a semester (remaining at the same level).
- Student who is currently enrolled into the eighth semester of the study program can be either remain into the current semester, or can graduate and finish the study program.
- Student who has withdrawn from specific semester will never join this semester unless takes the acceptance of the academic manager.
- Student who has apologized will leave this study program.

3. Mathematical model

The general form of the Markov chain model is given by

$$n(t) = Z(t)I + H(t) \quad (1)$$

Where $n(t)$ is the column vector whose i^{th} element represents the number of students in state i at time t . $Z(t)$ is the square matrix whose ij^{th} element represents the number of students moving from state i at time t to state j at time $t + 1$, I is the column vector of ones, and $H(t)$ is the number of students moving from transient state i at time t to absorbing state j at time $t + 1$.

Eq. (1) refers that the number of students at the beginning of semesters consists of those who will survive to the next semester and those who will leave the program in that semester.

The general form of the probability transition matrix of an absorbing Markov chain with r absorbing and t transient states is

$$P = \begin{bmatrix} Q & R \\ 0 & I \end{bmatrix} \quad (2)$$

Where

Q is a square matrix expressing transitions between the transient states.

R is a matrix expressing transitions from the transient states to the absorbing states.

0 is a zero matrix

I is an identity matrix

Base on the formulation in Shah & Burke (1999)^[20] and Rahel *et al.* (2013)^[18], the transition matrix (Q) and the absorbing matrix (R) are used to calculate the estimated average time student spend in the system and estimate the probability a student completing a course as follows, The matrix of transition probabilities given as

$$Q(t) = \hat{n}^{-1}(t)Z(t) \quad (3)$$

where $Q(t)$ is square matrix whose ij^{th} element represents the probability of student moving from state i at time t to state j at time $t + 1$ and $\hat{n}(t)$ is a diagonal matrix whose elements are the elements of $n(t)$.

The matrix of absorption probabilities is given by:

$$R(t) = \hat{n}^{-1}(t)H(t) \quad (4)$$

Where $R(t)$ is matrix whose ij^{th} element represents the probability of a student in state i at time t departing into an absorbing state j at time $t + 1$.

The fundamental matrix N of an absorbing Markov chain plays an important role in the assessment of the student completion attributes, and it defined as

$$N = (I - Q)^{-1} \quad (5)$$

Where N is a square matrix whose ij^{th} element represents the average time (in 6 month (semester)) that a student who commenced in state i , remains in state j before departing the program and I is the identity matrix.

The sum of the entries in the diagonal of N represents the expected time student enrolled in the first semester can expect to spend before graduating. That is,

$$E_G = \text{trace}(N) \quad (6)$$

The average time a student spends in the program is calculated by,

$$\mu = NI \quad (7)$$

Where μ is the mean time a student remains in the study program given that he commenced in state i .

The probability of a student moving into an absorbing state j , given that he commenced in transient state i , is given by the ij^{th} element of the matrix

$$B = NR \quad (8)$$

Formula below can be used to predict the future enrolment of the student

$$p^{(n)} = p^{(0)} P^n \quad (9)$$

Where, $p^{(0)}$ represents the initial probability distribution, P^n is the transition matrix P after n academic years and $p^{(n)}$ is the probability distribution after n academic years. The state transition diagram for the students' progression is illustrated in Figure 1.

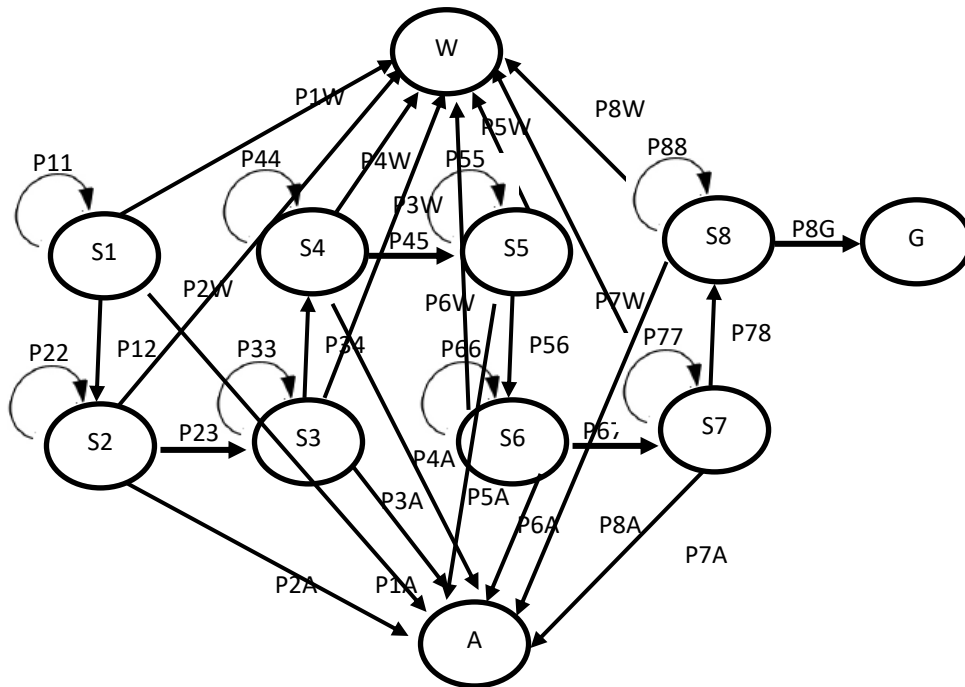


Fig 1: The state transition diagram for the students' progression

The probability transition matrix describing the progression of students from the first semester towards graduation is:

$$P = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & G & W & A \end{matrix} \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \\ G \\ W \\ A \end{matrix} & \begin{bmatrix} p_{11} & p_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{1W} & p_{1A} \\ 0 & p_{22} & p_{23} & 0 & 0 & 0 & 0 & 0 & 0 & p_{2W} & p_{2A} \\ 0 & 0 & p_{33} & p_{34} & 0 & 0 & 0 & 0 & 0 & p_{3W} & p_{3A} \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 & 0 & 0 & 0 & p_{4W} & p_{4A} \\ 0 & 0 & 0 & 0 & p_{55} & p_{56} & 0 & 0 & 0 & p_{5W} & p_{5A} \\ 0 & 0 & 0 & 0 & 0 & p_{66} & p_{67} & 0 & 0 & p_{6W} & p_{6A} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{77} & p_{78} & 0 & p_{7W} & p_{7A} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{88} & p_{8G} & p_{8W} & p_{8A} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix} \quad (10)$$

The Markov chain (10) is reducible. It consists of three closed sets of absorbing states $C_1 = \{G\}$, $C_2 = \{W\}$ and $C_3 = \{A\}$ and

of eight transient states $T = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$. The matrices Q, and R can be obtained from (10) as follows:

$$Q = \begin{matrix} & \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 \end{matrix} \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \end{matrix} & \begin{bmatrix} p_{11} & p_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_{22} & p_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & p_{56} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} & p_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{77} & p_{78} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{88} \end{bmatrix} \end{matrix}$$

$$R = \begin{matrix} & \begin{matrix} G & W & A \end{matrix} \\ \begin{matrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \end{matrix} & \begin{bmatrix} 0 & p_{1W} & p_{1A} \\ 0 & p_{2W} & p_{2A} \\ 0 & p_{3W} & p_{3A} \\ 0 & p_{4W} & p_{4A} \\ 0 & p_{5W} & p_{5A} \\ 0 & p_{6W} & p_{6A} \\ 0 & p_{7W} & p_{7A} \\ p_{8G} & p_{8W} & p_{8A} \end{bmatrix} \end{matrix}$$

4. Numerical example

To apply the model, data were obtained from the students' intake records at University of Tabuk-Faculty of Science (KSA). In our analysis, only the full time bachelor degree

students were included. The frequency data during five consecutive academic years from 1428/29 to 1432/33 Hijri calender are presented in Table 1.

Table 1: Students' progression through academic years from 1428/29 to 1432/33 Hijri calendar

Year 1428/1429												
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	G	W	A	Total
S_1	2	234	0	0	0	0	0	0	0	0	1	237
S_2	0	1	231	0	0	0	0	0	0	0	3	235
S_3	0	0	0	228	0	0	0	0	0	1	3	232
S_4	0	0	0	20	195	0	0	0	0	5	8	228
S_5	0	0	0	0	18	165	0	0	0	7	5	195
S_6	0	0	0	0	0	9	144	0	0	3	9	165
S_7	0	0	0	0	0	0	10	130	0	1	3	144
S_8	0	0	0	0	0	0	0	5	123	0	3	131
Year 1429/1430												
S_1	1	393	0	0	0	0	0	0	0	12	2	408
S_2	0	0	356	0	0	0	0	0	0	34	3	393
S_3	0	0	0	321	0	0	0	0	0	31	4	356
S_4	0	0	0	55	233	0	0	0	0	32	2	322
S_5	0	0	0	0	28	192	0	0	0	12	1	233
S_6	0	0	0	0	0	12	173	0	0	6	1	192
S_7	0	0	0	0	0	0	9	161	0	3	3	176
S_8	0	0	0	0	0	0	0	1	159	0	1	161
Year 1430/1431												
S_1	0	381	0	0	0	0	0	0	0	34	7	422
S_2	0	1	317	0	0	0	0	0	0	59	4	381
S_3	0	0	24	247	0	0	0	0	0	40	6	317
S_4	0	0	0	22	184	0	0	0	0	37	4	247
S_5	0	0	0	0	22	149	0	0	0	10	3	184
S_6	0	0	0	0	0	4	137	0	0	6	2	149
S_7	0	0	0	0	0	0	2	132	0	2	1	137
S_8	0	0	0	0	0	0	0	5	126	3	1	135
Year 1431/1432												
S_1	1	296	0	0	0	0	0	0	0	2	1	300
S_2	0	1	292	0	0	0	0	0	0	2	1	296
S_3	0	0	8	255	0	0	0	0	0	26	3	292
S_4	0	0	0	5	226	0	0	0	0	20	4	255
S_5	0	0	0	0	11	211	0	0	0	2	2	226
S_6	0	0	0	0	0	14	186	0	0	7	4	211
S_7	0	0	0	0	0	0	3	177	0	5	1	186
S_8	0	0	0	0	0	0	0	4	168	5	0	177
Year 1432/1433												
S_1	1	349	0	0	0	0	0	0	0	9	1	360
S_2	0	3	331	0	0	0	0	0	0	12	3	349
S_3	0	0	9	299	0	0	0	0	0	21	2	331
S_4	0	0	0	29	250	0	0	0	0	17	3	299
S_5	0	0	0	0	13	223	0	0	0	12	2	250
S_6	0	0	0	0	0	6	209	0	0	5	3	223
S_7	0	0	0	0	0	0	14	181	0	13	1	209
S_8	0	0	0	0	0	0	0	5	173	3	0	181

In Table 1 the last column labeled 'Total' shows the total number of students enrolled in each semester who either remain in the same semester, move to the next semester (or graduate), withdraw that semester or apologize of study program. For example, for the academic year 1428/29 the first row had total of 237 students of which 234 continue to the second semester 2 students remained and 1 student apologized.

Probability transition matrix

Data in Table 1 have used to estimate the transition probabilities for transition matrix for each academic year, that is, P_1, P_2, P_3, P_4 and P_5 . where P_1 denotes the transition probability matrix corresponds the first academic year

1428/29, while the other matrices P_2, P_3, P_4 and P_5 characterizes the second, third, fourth and fifth academic years respectively. The result in each entry of the transition matrix is obtained by dividing each value in each row to the corresponding row total. For example, for academic year 1428/29 the corresponding transition matrix is P_1 the first element in the first row 0.0084 is obtained by dividing 2 with the total value 237, other values in P_1 and transition matrices P_2, P_3, P_4 and P_5 follow the same definition. Moreover, the shaded rectangle area in each transition matrix represents the corresponding matrix Q while the dots rectangle explains matrix R .

$$P_1 = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & G & W & A \\ 0.0084 & 0.9873 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0042 \\ 0 & 0.0043 & 0.9830 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0128 \\ 0 & 0 & 0.0000 & 0.9828 & 0 & 0 & 0 & 0 & 0 & 0.0043 & 0.0129 \\ 0 & 0 & 0 & 0.0877 & 0.8553 & 0 & 0 & 0 & 0 & 0.0219 & 0.0351 \\ 0 & 0 & 0 & 0 & 0.0923 & 0.8462 & 0 & 0 & 0 & 0.0359 & 0.0255 \\ 0 & 0 & 0 & 0 & 0 & 0.0545 & 0.8727 & 0 & 0 & 0.0182 & 0.0546 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0694 & 0.9028 & 0 & 0.0069 & 0.0208 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0382 & 0.9389 & 0 & 0.0229 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & G & W & A \\ 0.0025 & 0.9632 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0049 \\ 0 & 0 & 0.9059 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0076 \\ 0 & 0 & 0 & 0.9017 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0112 \\ 0 & 0 & 0 & 0.1708 & 0.7236 & 0 & 0 & 0 & 0 & 0 & 0.0062 \\ 0 & 0 & 0 & 0 & 0.1202 & 0.8240 & 0 & 0 & 0 & 0 & 0.0046 \\ 0 & 0 & 0 & 0 & 0 & 0.0625 & 0.9010 & 0 & 0 & 0 & 0.0052 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0511 & 0.9148 & 0 & 0.0170 & 0.0170 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0062 & 0.9376 & 0 & 0.0062 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & G & W & A \\ 0 & 0.9028 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0106 \\ 0 & 0 & 0.8320 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0105 \\ 0 & 0 & 0 & 0.7792 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0189 \\ 0 & 0 & 0 & 0.0891 & 0.7449 & 0 & 0 & 0 & 0 & 0 & 0.0062 \\ 0 & 0 & 0 & 0 & 0.1196 & 0.8098 & 0 & 0 & 0 & 0 & 0.0163 \\ 0 & 0 & 0 & 0 & 0 & 0.0268 & 0.9195 & 0 & 0 & 0 & 0.0134 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0146 & 0.9635 & 0 & 0.0146 & 0.0073 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0370 & 0.9333 & 0.0222 & 0.0074 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_4 = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & G & W & A \\ 0.0033 & 0.9667 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0038 \\ 0 & 0.0034 & 0.9865 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0034 \\ 0 & 0 & 0.0274 & 0.8733 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0103 \\ 0 & 0 & 0 & 0.0196 & 0.8863 & 0 & 0 & 0 & 0 & 0 & 0.0157 \\ 0 & 0 & 0 & 0 & 0.0487 & 0.9336 & 0 & 0 & 0 & 0 & 0.0088 \\ 0 & 0 & 0 & 0 & 0 & 0.0664 & 0.8815 & 0 & 0 & 0 & 0.0190 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0161 & 0.9516 & 0 & 0.0269 & 0.0054 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0226 & 0.9492 & 0.0282 & 0.0054 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & G & W & A \\ 0.0028 & 0.9694 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0028 \\ 0 & 0.0086 & 0.9484 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0086 \\ 0 & 0 & 0.0272 & 0.9033 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0060 \\ 0 & 0 & 0 & 0.0970 & 0.8361 & 0 & 0 & 0 & 0 & 0 & 0.0100 \\ 0 & 0 & 0 & 0 & 0.0520 & 0.8920 & 0 & 0 & 0 & 0 & 0.0180 \\ 0 & 0 & 0 & 0 & 0 & 0.0269 & 0.9372 & 0 & 0 & 0 & 0.0135 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0670 & 0.8660 & 0 & 0.0622 & 0.0043 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0276 & 0.9558 & 0.0166 & 0.0043 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Using Q_1, Q_2, Q_3, Q_4 and Q_5 the corresponding fundamental matrices N_1, N_2, N_3, N_4 and N_5 of Eq. (5) are calculated:

$$N_1 = \begin{bmatrix} 1.0085 & 1 & 0.9830 & 1.0589 & 0.9978 & 0.8930 & 0.8375 & 0.7861 \\ 0 & 1.0043 & 0.9872 & 1.0634 & 1.0020 & 0.8968 & 0.8410 & 0.7894 \\ 0 & 0 & 1 & 1.0773 & 1.0150 & 0.9084 & 0.8520 & 0.7997 \\ 0 & 0 & 0 & 1.0962 & 1.0328 & 0.9244 & 0.8669 & 0.8137 \\ 0 & 0 & 0 & 0 & 1.1017 & 0.9860 & 0.9247 & 0.8679 \\ 0 & 0 & 0 & 0 & 0 & 1.0577 & 0.9920 & 0.9311 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0746 & 1.0086 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0397 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} 1.0025 & 0.9656 & 0.8747 & 0.9512 & 0.7823 & 0.6876 & 0.6529 & 0.6010 \\ 0 & 1 & 0.9059 & 0.9850 & 0.8101 & 0.7121 & 0.6762 & 0.6224 \\ 0 & 0 & 1 & 1.0874 & 0.8943 & 0.7861 & 0.7465 & 0.6871 \\ 0 & 0 & 0 & 1.2060 & 0.9919 & 0.8718 & 0.8279 & 0.7620 \\ 0 & 0 & 0 & 0 & 1.1366 & 0.9990 & 0.9487 & 0.8732 \\ 0 & 0 & 0 & 0 & 0 & 1.0667 & 1.0129 & 0.9324 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0539 & 0.9701 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0063 \end{bmatrix}$$

$$N_3 = \begin{bmatrix} 1 & 0.9052 & 0.8149 & 0.6970 & 0.5897 & 0.4907 & 0.4579 & 0.4581 \\ 0 & 1.0026 & 0.9025 & 0.7720 & 0.6532 & 0.5435 & 0.5072 & 0.5075 \\ 0 & 0 & 1.0819 & 0.9254 & 0.7830 & 0.6516 & 0.6080 & 0.6083 \\ 0 & 0 & 0 & 1.0978 & 0.9288 & 0.7729 & 0.7212 & 0.7216 \\ 0 & 0 & 0 & 0 & 1.1358 & 0.9451 & 0.8819 & 0.8824 \\ 0 & 0 & 0 & 0 & 0 & 1.0276 & 0.9588 & 0.9594 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0148 & 1.0154 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0385 \end{bmatrix}$$

$$N_4 = \begin{bmatrix} 1.0033 & 0.9933 & 1.0075 & 0.8974 & 0.8361 & 0.8360 & 0.7491 & 0.7293 \\ 0 & 1.0034 & 1.0177 & 0.9065 & 0.8445 & 0.8445 & 0.7567 & 0.7367 \\ 0 & 0 & 1.0282 & 0.9158 & 0.8532 & 0.8532 & 0.7644 & 0.7443 \\ 0 & 0 & 0 & 1.0200 & 0.9503 & 0.9502 & 0.8514 & 0.8289 \\ 0 & 0 & 0 & 0 & 1.0512 & 1.0511 & 0.9418 & 0.9169 \\ 0 & 0 & 0 & 0 & 0 & 1.0711 & 0.9596 & 0.9343 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0164 & 0.9896 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0231 \end{bmatrix}$$

$$N_5 = \begin{bmatrix} 1.0028 & 0.9806 & 0.9560 & 0.9563 & 0.8435 & 0.7732 & 0.7767 & 0.6917 \\ 0 & 1.0087 & 0.9834 & 0.9837 & 0.8676 & 0.7953 & 0.7989 & 0.7115 \\ 0 & 0 & 1.0280 & 1.0283 & 0.9069 & 0.8314 & 0.8351 & 0.7438 \\ 0 & 0 & 0 & 1.1074 & 0.9767 & 0.8953 & 0.8994 & 0.8010 \\ 0 & 0 & 0 & 0 & 1.0549 & 0.9669 & 0.9713 & 0.8651 \\ 0 & 0 & 0 & 0 & 0 & 1.0276 & 1.0323 & 0.9194 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0718 & 0.9546 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.0284 \end{bmatrix}$$

The elements of each fundamental matrix N_i represent the expected number of semesters student will spend when he is enrolled in a particular stage of the study program. For example, let we assume student is enrolled in the first semester of academic year 1428/29, it is expected that he will spend 1.0085 semesters (6 months and 1 day) for the first level, 1 semester (6 months) for the second level, 0.9830 semesters (5 months and 27 days) for the third level, 1.0589

semesters (6 months and 10 days) for the fourth level, 0.9978 semesters (5 months and 29 days) for the fifth level, 0.8930 semesters (5 months and 10 days) for the sixth level, 0.8375 semesters (5 months) for the seventh level, and 0.7861 semesters (4 months and 21 days) for the eighth level.

The mean time μ_i until absorption of Eq. (7) and the probability of absorption B_i of Eq.(8) for each academic year are respectively given as:

$$\begin{aligned} \mu_1 &= \begin{bmatrix} 7.5646 \\ 6.5841 \\ 5.6523 \\ 4.7340 \\ 3.8803 \\ 2.9807 \\ 2.0833 \\ 1.0397 \end{bmatrix} & B_1 &= \begin{bmatrix} 0.7380 & 0.0853 & 0.1766 \\ 0.7412 & 0.0857 & 0.1731 \\ 0.7508 & 0.0868 & 0.1624 \\ 0.7640 & 0.0839 & 0.1521 \\ 0.8149 & 0.0639 & 0.1212 \\ 0.8742 & 0.0261 & 0.0997 \\ 0.9471 & 0.0075 & 0.0455 \\ 0.9762 & 0.0000 & 0.0238 \end{bmatrix} & \mu_2 &= \begin{bmatrix} 6.5177 \\ 5.7118 \\ 5.2015 \\ 4.6596 \\ 3.9575 \\ 3.0119 \\ 2.0240 \\ 1.0063 \end{bmatrix} & B_2 &= \begin{bmatrix} 0.5936 & 0.3566 & 0.0498 \\ 0.6147 & 0.3388 & 0.0465 \\ 0.6786 & 0.2785 & 0.0429 \\ 0.7526 & 0.2123 & 0.0351 \\ 0.8624 & 0.1059 & 0.0317 \\ 0.9208 & 0.0506 & 0.0286 \\ 0.9580 & 0.0180 & 0.0240 \\ 0.9938 & 0.0000 & 0.0063 \end{bmatrix} \\ \mu_3 &= \begin{bmatrix} 5.4136 \\ 4.8885 \\ 4.6582 \\ 4.2423 \\ 3.8452 \\ 2.9458 \\ 2.0302 \\ 1.0385 \end{bmatrix} & B_3 &= \begin{bmatrix} 0.4276 & 0.4967 & 0.0757 \\ 0.4736 & 0.4609 & 0.0655 \\ 0.5677 & 0.3663 & 0.0659 \\ 0.6735 & 0.2726 & 0.0539 \\ 0.8286 & 0.1323 & 0.0442 \\ 0.8954 & 0.0767 & 0.0279 \\ 0.9477 & 0.0374 & 0.0149 \\ 0.9692 & 0.0231 & 0.0077 \end{bmatrix} & \mu_4 &= \begin{bmatrix} 7.0521 \\ 6.1101 \\ 5.1591 \\ 4.6008 \\ 3.9910 \\ 2.9650 \\ 2.0060 \\ 1.0231 \end{bmatrix} & B_4 &= \begin{bmatrix} 0.6922 & 0.2494 & 0.0584 \\ 0.6992 & 0.2451 & 0.0556 \\ 0.7064 & 0.2408 & 0.0528 \\ 0.7868 & 0.1662 & 0.0470 \\ 0.8703 & 0.0954 & 0.0343 \\ 0.8868 & 0.0877 & 0.0255 \\ 0.9393 & 0.0553 & 0.0055 \\ 0.9711 & 0.0289 & 0.0000 \end{bmatrix} \\ \mu_5 &= \begin{bmatrix} 6.9807 \\ 6.1492 \\ 5.3735 \\ 4.6798 \\ 3.8582 \\ 2.9793 \\ 2.0264 \\ 1.0284 \end{bmatrix} & B_5 &= \begin{bmatrix} 0.6611 & 0.2914 & 0.0475 \\ 0.6801 & 0.2740 & 0.0459 \\ 0.7109 & 0.2501 & 0.0390 \\ 0.7656 & 0.1991 & 0.0353 \\ 0.8268 & 0.1471 & 0.0261 \\ 0.8787 & 0.1025 & 0.0188 \\ 0.9124 & 0.0825 & 0.0051 \\ 0.9830 & 0.0170 & 0.0000 \end{bmatrix} \end{aligned}$$

5. Results

Moving between different semesters estimation

Table 2 gives the probabilities of students' progression

between successive semesters of the study program for each academic year, which can be directly obtained from the probability transition matrices P_i $i = 1, 2, \dots, 5$.

Table 2: Probabilities of the students successfully moving between semesters until graduate for each academic year.

Academic year	Moving from 1 st to 2 nd .	Moving from 2 nd to 3 rd .	Moving from 3 rd to 4 th	Moving from 4 th to 5 th .	Moving from 5 th to 6 th .	Moving from 6 th to 7 th .	Moving from 7 th to 8 th .	Graduate
1428/29	0.9873	0.9830	0.9828	0.8553	0.8462	0.8727	0.9028	0.9389
1429/30	0.9632	0.9059	0.9017	0.7236	0.8240	0.9010	0.9148	0.9876
1430/31	0.9028	0.8320	0.7792	0.7449	0.8098	0.9195	0.9635	0.9333
1431/32	0.9867	0.9865	0.8733	0.8863	0.9336	0.8815	0.9516	0.9492
1432/33	0.9694	0.9484	0.9033	0.8361	0.8920	0.9372	0.8660	0.9558
Average	0.96188	0.93116	0.88806	0.80924	0.86112	0.90238	0.91974	0.95296

Row 1 in Table 2 concerning academic year 1428/29, indicates that, 98.73% of students moved successfully from first to second semester, 98.30% moved successfully from second to third, 98.28% moved from third to fourth, 85.53% moved from fourth to fifth, 84.62% from fifth to sixth, 87.27% from sixth to seventh, 90.28% from seventh to eighth

and 93.89% of students graduated. The last row represents the probabilities averages of the students moved successfully between semesters until graduate for the five academic years. Figure 1 shows probabilities of the students moved successfully between semesters until graduate for academic year 1428/29 and the averages of overall five academic years.

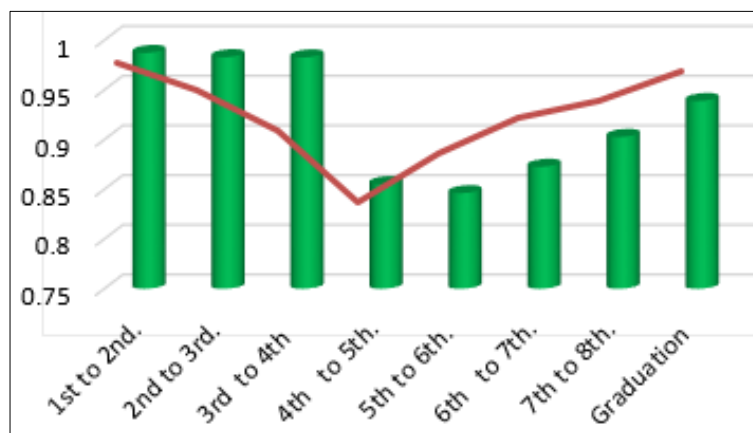


Fig 2: Probabilities moving between semesters until graduate for 1428/29 and the average of overall 5 academic years.

Expected time spend before graduation

Table 3 represents the expected time (semesters) student enrolled in the first semester can expect to spend before graduating for each academic year, which has been calculated according to Eq.(6)

Table 3: The expected time student enrolled in the first semester can expect to spend before graduating

Academic year	Expected time E_G
1428/29	8.3826
1429/30	8.4718
1430/31	8.3990
1431/32	8.2166
1432/33	8.3295
Expected time in last 5 academic years	8.3599

From Table 3 we note that, regarding the academic year 1429/30, the expected time student spend until graduate is 8.4718 semesters (4 years, 2 months and 24 days) greater than other academic years, while student of the academic year 1431/32 spend 8.2166 semesters (4 years, 1 month and 8 days) less than student of the rest academic years. Moreover, student of the two last academic years 1431/32 and 1432/33

spend time less than the average time of overall five academic years (see Figure 2).

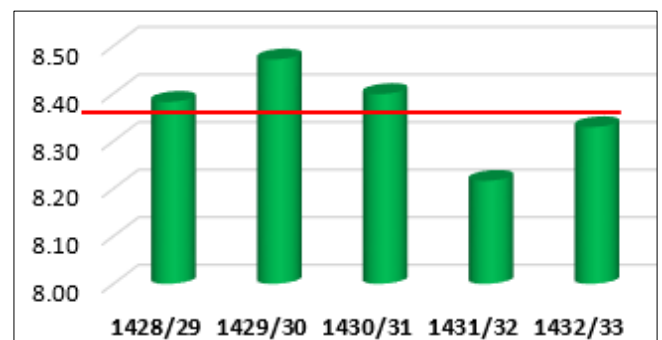


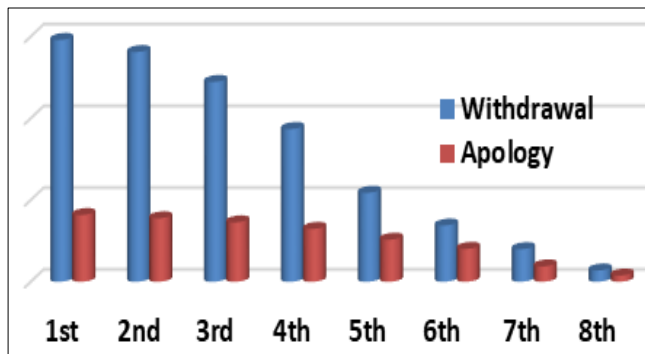
Fig 2: The expected time student enrolled in the first semester can expect to spend before graduating and average of overall 5 academic years

Probabilities of withdrawal and apology

Table 4 shows the probabilities of student withdraw from a particular semester and apologize of study program for each academic year. Note that the last two column give the averages of the five academic years.

Table 4: Probabilities of withdrawal and apology from a particular study stage

Year	1428/29		1429/30		1430/31		1431/32		1432/33		Average	
State	W	A	W	A	W	A	W	A	W	A	W	A
1 st	0.085	0.177	0.357	0.050	0.497	0.076	0.249	0.058	0.291	0.047	0.296	0.082
2 nd	0.086	0.173	0.339	0.047	0.461	0.066	0.245	0.056	0.274	0.046	0.281	0.078
3 rd	0.087	0.162	0.278	0.043	0.366	0.066	0.241	0.053	0.250	0.039	0.244	0.073
4 th	0.084	0.152	0.212	0.035	0.273	0.054	0.166	0.047	0.199	0.035	0.187	0.065
5 th	0.064	0.121	0.106	0.032	0.132	0.044	0.095	0.034	0.147	0.026	0.109	0.051
6 th	0.026	0.100	0.051	0.029	0.077	0.028	0.088	0.025	0.102	0.019	0.069	0.040
7 th	0.007	0.045	0.018	0.024	0.037	0.015	0.055	0.005	0.082	0.005	0.040	0.019
8 th	0.000	0.024	0.000	0.006	0.023	0.008	0.029	0.000	0.017	0.000	0.014	0.008

**Fig 3:** Probabilities averages of withdrawal and apology from a particular study stage

From Table 4 it can be seen that the probability of withdrawal and apology decrease as the students' progress to higher levels. This may be the result of the fact that they understand the system better as they pass from one level to another (see also Figure 3).

Predicting the future enrolment of students

To predict the future probability of student repeat specific semester, withdraw, apologize or graduate, let us assume that the initial state is in the last academic year 1432/33. So, form in Table 1 the total number of students who repeated each semester, graduated, withdrawn or apologized from study program of academic year 1432/33 are (1, 3, 9, 29, 13, 6, 14, 5, 173, 92, 15) which used to estimate the initial vector $P^{(0)}$,

$$P^{(0)} = \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 & S_7 & S_8 & G & W & A \end{bmatrix} = \begin{bmatrix} 0.0028 & 0.0083 & 0.0250 & 0.0806 & 0.0361 & 0.0167 & 0.0389 & 0.0139 & 0.4806 & 0.2556 & 0.0417 \end{bmatrix}$$

Using Eq.(9) we can predict the future enrolment of the students in next 4 academic years 1434/35, 1435/36, 1436/37

and 1437/38. The results are given in Table 5.

Table 5: Prediction of the future enrolment in the study program

	1434/35		1435/36		1436/37		1437/38	
	# of students	%	# of students	%	# of students	%	# of students	%
Repeats S_1	0	0.00%	0	0.00%	0	0.00%	0	0.00%
Repeats S_2	1	0.28%	0	0.00%	0	0.00%	0	0.00%
Repeats S_3	3	0.86%	1	0.29%	0	0.01%	0	0.00%
Repeats S_4	11	3.04%	4	1.07%	1	0.36%	0	0.04%
Repeats S_5	25	6.92%	10	2.90%	4	1.05%	1	0.36%
Repeats S_6	12	3.27%	23	6.26%	10	2.76%	4	1.01%
Repeats S_7	7	1.82%	11	3.18%	22	6.08%	11	2.99%
Repeats S_8	12	3.41%	6	1.67%	10	2.80%	19	5.35%
Graduate	178	49.38%	189	52.64%	195	54.24%	205	56.92%
Withdrawal	96	26.68%	99	27.50%	101	28.08%	103	28.64%
apologize	16	4.34%	16	4.49%	17	4.62%	17	4.70%
Total	360	100%	360	100%	360	100%	360	100%

From Table 5 we find out that

- 49.4%, 52.6%, 54.2% and 56.9% of students who join study program in the academic years 1434/35, 1435/36, 1436/37 and 1436/37 respectively will graduate, implies that the graduation is gradually increase (see Figure 4).
- 19.6%, 15.4%, 13.1% and 9.8% of students who join study program in the academic years 1434/35, 1435/36, 1436/37 and 1437/38 respectively will repeat one or more

semesters before graduate. Indicates that the probability of repeating semesters before graduation is gradually decrease (see Figure 5).

- 26.68%, 27.5%, 28.08% and 28.64% of students who join study program in the academic years 1434/35, 1435/36, 1436/37 and 1436/37 respectively will withdraw from one or more semesters before graduate, implies that the withdrawal is gradually increase.

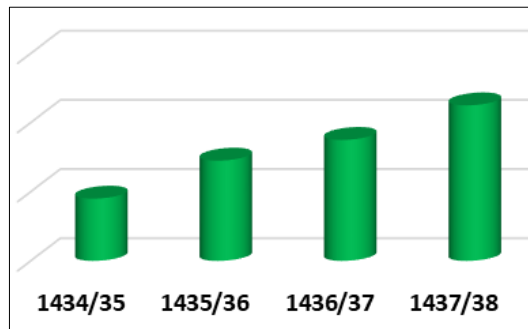


Fig 4: Prediction of graduation in 4 next academic years

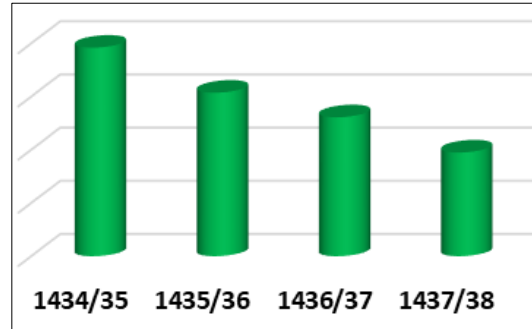


Fig 5: Prediction of Repeating in 4 next academic years

6. Conclusion

This study focuses on modeling an undergraduate students flow at university of Tabuk-faculty of Science (KSA) with stochastic process model depending on Markov Chain. The proposed model built by a reducible discrete Markov chain with eight transient and three absorbing states. The eight transient states represent an eight semesters student should move around until graduate, while the three absorbing states are graduation, withdrawal from specific semester and apologized for the study program. The probabilities of absorption (Graduating, withdrawal and apologized) were obtained. Furthermore, the model also provides estimates for the expected time student will spend when he is enrolled in a particular stage of the study program, the expected time student enrolled in the first semester can expect to spend before graduating and the probabilities of students' progression between successive semesters of the study program for each academic year. The model enables the prediction of future probability of student repeat specific semester, withdraw, apologize or graduate.

7. References

1. Auwalu A, Mohammed LB, Saliu A. Application of Finite Markov Chain to a Model of Schooling. *Journal of Education & Practice*. 2013; 4(17):1-9.
2. Adam RY. An Application of Markov Modeling to the Student Flow in Higher Education in Sudan. *International Journal of Science and Research (IJSR)*. 2015; 4(2):49-54.
3. Adeleke R, Oguntuase K, Ogunsakin R. Application of Markov Chain to the Assessment of Students' Admission and Academic Performance in Ekiti State University. *International Journal of Scientific & Technology Research*. 2014; 3(7):349-357.
4. Al-Awadhi S, Ahmed M. Logistic models and a Markovian analysis for student attrition. *Kuwait journal of science & engineering*. 2002; 29(2):25-40.
5. Al-Awadhi S, Konsowa M. An application of absorbing Markov analysis to the student flow in an academic institution. *Kuwait Journal of Science & Engineering*. 2007; 34(2A):77-89.
6. Al-Awadhi S, Konsowa M. Markov Chain Analysis and Student Academic Progress: An Empirical Comparative Study. *Journal of Modern Applied Statistical Methods*. 2010; 9(2):584-595.
7. Armacost R, nd Wilson A. Using Markov chain models to project University and program level enrolment, annual conference, 2004.
8. Bharucha-Reid AT. *Elements of the Theory of Markov Processes and their Applications*. Toronto: Courier Dover Publications, 2012.
9. Borden V, Dalphin J. Simulating the effect of student profile changes on retention and graduation rates: A Markov chain analysis, paper presented at the 38th annual forum of the association for institutional research, Minneapolis, 1998.
10. Brezavšček A, Baggia A. Analysis of students' flow in higher education study programmes using discrete homogeneous Markov. 13th International Symposium on Operational Research in Slovenia. Ljubljana: Slovenian Society Informatika, Section for Operational Research, 2015, pp.473-478.
11. Brezavšček A, Baggia A. Markov Analysis of Students' Performance and Academic Progress in Higher Education. *Organizacija*. 2017; 50(2):83-954.
12. Grip RS, Young JW. Predicting school enrollments using the modified regression technique, paper presented at the annual meeting of American Educational Research Association, Montreal, Canada, 1999.
13. Guo S. Three enrollment forecasting models: Issues in enrollment projection for community colleges, presented at the 40th RP Conference, California, 2002.
14. Healey MT, Brown DJ. Forecasting university enrollments by ratio smoothing, *Higher Education*. 1978; 7:417-429.
15. Hlavatý R, Dömeová L. Students Progress throughout Examination Process as a Markov Chain. *International Education Studies*. 2014; 7(12):20-29.
16. Johnstone JN. Mathematical models developed for use in educational planning: A review, *Review of educational research*. 1974; 44:177-201.
17. Kinard, Krech. Projected degree-credit enrollments through 1985 in South Carolina College and universities, 1977.
18. Rahela R, Haslinda I, Maznah M, Farah A. Projection Model of Postgraduate Student Flow. *Applied Mathematics & Information Sciences*. 2013; 7(2L):383-387.
19. Reynolds D, Porath J. Markov chains and student academic progress. Unpublished manuscript, Department of Mathematics, University of Wisconsin-Eau Claire, USA, 2008.
20. Shah C, Burke G. An undergraduate student flow model: Australian higher education, *Higher Education*. 1999; 37:359-375.
21. Wailand E, Authella M. Student Flow in a University Department: Results of a Markov Analysis. *Interfaces*. 1980; 10(2):52-59.