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Analytical expressions of a third grade fluid flow through porous medium using the Homotopy perturbation method

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Abstract

In this paper, we investigate the mathematical analyses of a reactive third grade fluid flow through porous medium. Using Homotopy perturbation method, we derive the approximate analytical expressions of the dimensionless velocity and dimensionless temperature. We also discuss the effect of various non-Newtonian fluid parameters on the velocity field, temperature field, skin friction and Nusselt number. We compare our analytical results graphically with previous work and report a satisfactory agreement.

Keywords: Third grade fluid; Non-Newtonian material; Frank-Kamenetskii parameter; Non-linear differential equations; Nusselt number; Homotopy perturbation method

1. Introduction

Nowadays the majority of the fluids dealt with by Engineers and Scientists, such as air, water and oils can be regarded as Newtonian under most conditions of interest. However, in many cases the assumption of Newtonian behavior is not valid and the rather more complex non-Newtonian response must be modeled. These include non-Newtonian fluid friction reduction, oil-pipeline friction reduction, surfactant applications to large-scale heating and cooling systems, scale-up, and flow tracers.

Such situations also arise in the chemical processing industry, plastics processing industry and mining industry. The simulation of non-Newtonian fluid flow phenomena is therefore of importance to industry. The classical Navier –Stokes equations cannot accurately describe the rheological behavior of these complex fluids about these applications. The third grade fluid model has been used to model the flow of non-Newtonian fluids ^[1-15] undergoing strongly exothermic chemical reaction due to its ability to explain the trim thickening/thinning properties of these fluids. The flow of a reactive fluid through a channel filled with saturated porous medium was examined by Makinde ^[7].

There are some methods to obtain approximate solutions of this kind of equations. One of them is Homotopy perturbation method. The method introduced by He in 1999 ^[16-18] has been developed and improved by himself. In this paper, the non linear differential equations are suitable to the presence of exponential temperature dependent internal heat generation occurring within the channel. So the approximate analytical solution will be obtained by Homotopy perturbation method.

2. Mathematical formulation of the problem

Consider the steady flow of a viscous incompressible third grade fluid between infinite parallel isothermal plates separated by distance $2a$. The channel is assumed saturated with porous materials and the geometry of the situation is shown in Figure 1. The channel walls are maintained at temperature T_0 .

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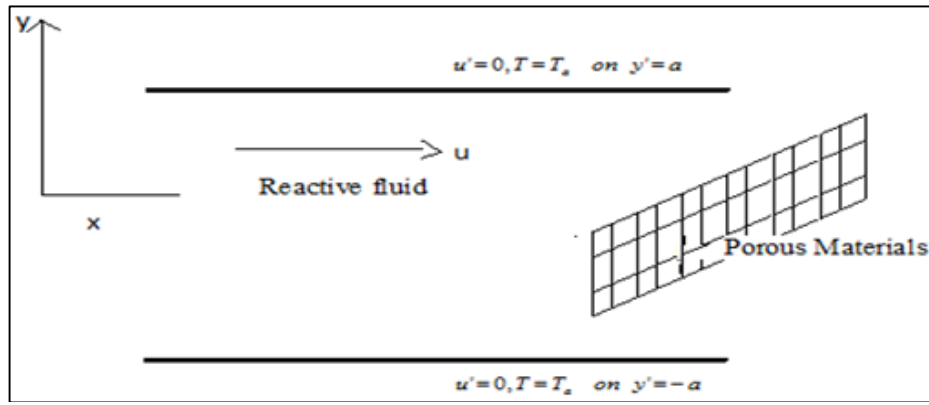


Fig 1: Flow Geometry

Disregarding the reactant consumption, the equations governing the hydrodynamically and thermally developed flow can be expressed as [1-10] follows:

$$-\frac{d\hat{p}}{dx} + \mu \frac{d^2 u'}{dy'^2} + 6\beta_3 \frac{d^2 u'}{dy'^2} \left(\frac{du'}{dy'} \right) - \frac{\mu}{K} \mu' = 0 \quad (1)$$

$$k \frac{d^2 T}{dy'^2} + \left(\frac{du'}{dy'} \right)^2 \left\{ \mu + 2\beta_3 \left(\frac{du'}{dy'} \right) \right\} + \frac{\mu}{K} u'^2 + QAC_0 e^{\frac{-E}{RT}} = 0 \quad (2)$$

Subject to the appropriate boundary conditions

$$u' = 0, T = T_a \quad \text{on } y' = a \quad (3)$$

$$u' = 0, T = T_a \quad \text{on } y' = -a \quad (4)$$

Here u' is the fluid velocity, P' is the fluid pressure, β_3 is the material coefficient, k is the thermal conductivity, μ is the dynamic viscosity, γ is the dimensionless third grade material parameter, T is the fluid temperature, U is the characteristic velocity, T_a is the wall temperature, Q is the heat of reaction, A is the rate constant, R is the universal gas constant, E is the activation energy, C_0 is the initial concentration of the reactant species, a is the channel width, y is the distance measured in the normal direction, additional term in the eqns. (1) and (2) is the third grade material effect defined in [1-9]. Now we introduce the following dimensionless parameters:

$$y = \frac{y'}{a}, \quad u = \frac{u'}{U}, \quad \gamma = \frac{\beta_3 U^2}{\mu a^2}, \quad \beta^2 = \frac{1}{Da}, \quad Da = \frac{K}{a^2}, \quad M = -\frac{a^2}{\mu U} \frac{dP}{dx}, \quad \lambda = \frac{QAEa^2 C_0 e^{\frac{-E}{RT_0}}}{kRT_a^2}, \quad \alpha = \frac{\mu U^2 e^{\frac{-E}{RT_0}}}{QAC_0 a^2}, \quad \varepsilon = \frac{RT_a}{E},$$

$$\theta = \frac{E(T - T_a)}{RT_a^2}$$

Then we obtain the following dimensionless eqns. (5) and (6) (with appropriate boundary conditions):

$$\frac{d^2 u}{dy^2} + 6\gamma \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right) - \beta^2 u = -M; \quad u(-1) = 0 = u(1) \quad (5)$$

$$\frac{d^2 \theta}{dy^2} + \lambda \left[e^{\frac{\theta}{1+\varepsilon\theta}} + \alpha \left(\frac{du}{dy} \right)^2 \left(1 + 2\gamma \left(\frac{du}{dy} \right)^2 \right) + \alpha \beta^2 u^2 \right] = 0; \quad \theta(-1) = 0 = \theta(1) \quad (6)$$

Here α is the viscous heating parameter, θ is the dimensionless temperature, u is the dimensionless velocity, M is the dimensionless pressure gradient, β^2 is the porous permeability parameter, Da is the Darcy parameter, K is the porous permeability, λ is the Frank-Kamenetskii parameter.

The physical quantities of this problem are the skin friction parameter C_f and Nusslet number Nu which are defined by

$$C_f = -\frac{du}{dy}(1) \quad (7)$$

$$Nu = -\frac{d\theta}{dy}(1) \quad (8)$$

3. Solution of the non-linear boundary value problem using the Homotopy perturbation method

Linear and non-linear phenomena are of fundamental importance in various fields of science and engineering. Most models of real-life problems are still very difficult to solve. Therefore, approximate analytical solutions such as Homotopy perturbation method (HPM) [16] were introduced. The HPM is the most effective and convenient method for both linear and non-linear equations. Perturbation method is based on assuming a small parameter. The majority of non-linear problems, especially those having strong non-linearity, have no small parameters at all and the solutions obtained by the perturbation methods are only approximate. In most cases, the solutions are valid only for small values of the small parameter. Generally, the perturbation solutions are uniformly valid as long as a scientific system parameter is small. However, we cannot rely fully on the approximations, because there is no criterion for the existence of a small parameter. Thus, it is essential to check the validity of the approximations numerically and/or experimentally. To overcome these difficulties, HPM have been proposed recently.

Recently, many authors have applied the HPM to solve the non-linear problem in physics and engineering sciences. A combination of Homotopy in topology and classic perturbation techniques is useful in solving some of the non-linear problem in physical sciences. The HPM is unique in its applicability, accuracy and efficiency. The HPM uses the imbedding parameters a small parameter, and only a few iterations are needed to search for an asymptotic solution.

The approximate analytical solutions of the eqns. (5) and (6) obtain by applying the Homotopy perturbation method is given by

$$u = \frac{M}{2}(1 - y^2) + \left(6\gamma M^3 - \frac{\beta^2 M}{2}\right) \frac{y^4}{12} + \frac{\beta^2 M}{4}(y^2 - 1) - \frac{1}{12} \left(6\gamma M^3 - \frac{\beta^2 M}{2}\right) \quad (9)$$

$$\theta = \frac{\lambda}{2}(1 - y^2) - \lambda \left[A \frac{y^2}{2} + By^4 + Cy^6 + Dy^8 + Ey^{10} + Fy^{12} + Gy^{14} \right] + H \quad (10)$$

The analytical expressions of the eqns. (7) and (8) obtained by using the eqns. (9) and (10) is given by

$$C_f = \left(1 - \frac{\beta^2}{2}\right) My - \left(6\gamma M^3 - \frac{\beta^2 M}{2}\right) \frac{y^3}{3} \quad (11)$$

$$Nu = \lambda [y + Ay + 4By^3 + 6Cy^5 + 8Dy^7 + 10Ey^9 + 12Fy^{11} + 14Gy^{13}] \quad (12)$$

Where

$$A = \frac{\lambda}{2} - \frac{\varepsilon \lambda^2}{4} + \alpha \beta^2 \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right)^2 + \frac{\alpha \beta^2}{144} \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right)^2 \\ + \frac{\alpha \beta^2}{6} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right) \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right) \quad (13)$$

$$B = \frac{-\lambda}{24} + \frac{\varepsilon \lambda^2}{24} + \frac{\alpha}{3} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right)^2 \\ + \alpha \beta^2 \left[\frac{-1}{6} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right)^2 - \frac{1}{72} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right) \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right) \right] \quad (14)$$

$$C = \frac{-\varepsilon\lambda^2}{120} + \frac{4\alpha}{90} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right) \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right) + \frac{16\alpha\gamma}{15} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right)^4$$

$$+ \frac{\alpha\beta^2}{15} \left[\frac{1}{2} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right)^2 - \frac{1}{144} \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right) \right]$$

$$\left[-\frac{1}{12} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right) \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right) \right] \quad (15)$$

$$D = \frac{\alpha}{504} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right) \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right)^2 + \frac{8\alpha\gamma}{21} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right)^3 \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right)$$

$$+ \frac{\alpha\beta^2}{336} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right) \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right) \quad (16)$$

$$E = \frac{24\alpha\gamma}{405} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right)^2 \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right)^2 + \frac{\alpha\beta^2}{12960} \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right)^2 \quad (17)$$

$$F = \frac{8\alpha\gamma}{891} \left(\frac{M}{2} \left(\frac{\beta^2}{2} - 1 \right) \right) \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right)^3 \quad (18)$$

$$G = \frac{\alpha\gamma}{7371} \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right)^4 \quad (19)$$

$$H = \lambda \left[\frac{A}{2} + B + C + D + E + F + G \right] \quad (20)$$

4. Result and Discussions

Figure 1 shows the flow geometry of a viscous incompressible third grade fluid through infinite parallel isothermal plates separated by a distance of $2a$. The fluid velocity profile with respect to increasing value of non-Newtonian parameter γ and porous permeability parameter β are shown in Fig. 2(a) and 2(b) respectively. A parabolic velocity profile is observed. However, a regular decrease in the magnitude of fluid velocity is noticed with an increase in value of non-Newtonian parameter γ . In the same way, level of fluid velocity is decreased with an increase in value of Porous permeability parameter β . The fluid temperature profiles have been carried out by assigning specific values to various thermophysical parameters controlling the flow system, as shown in Fig 3(a)-3(e). In Fig. 3(a), it can be noticed that slightly increasing the value of the activation energy parameter ε vaguely increases the fluid temperature profile. Shown in Fig. 3(b), the fluid temperature profile is decreased when the dimensionless third grade non-Newtonian material parameter γ is increased. In Fig.3(c), slightly increasing the porous permeability parameter β increases the fluid temperature profile. In Fig. 3(d), fluid temperature is increased when additional heat source of viscous parameter α within the channel is considerably increased. Vaguely decreasing the Frank –Kamenetskii parameter λ decreases the fluid temperature profile, as shown in Fig.3 (e). The skin friction decreases with increase in the values of non-Newtonian parameter γ and porous permeability parameter β , as shown respectively in Fig. 4(a) and 4(b).

The Nusselt number has been carried out by assigning specific values to various thermophysical parameters controlling the flow system, shown in Fig 5(a)-5(i). In Fig. 5(a)-5(c), elevation in the Nusselt number is observed with an increase in porous permeability parameter β . The elevation decreases when the parameters ε and γ are increased. But elevation increases when viscous parameter α is increased. Elevation of the Nusselt number is noticed with an increase in activation energy parameter ε , shown in Fig.5(d)-5(f). The elevation is increased when the parameters β and α are increased. But elevation is decreased when non-Newtonian parameter γ is increased. Fig. 5(g) through 5(i) plot effects of Nusselt number on the value of non-Newtonian material γ . From the Fig.5 (g), it is observed that rise in the value of activation energy parameter ε decreases the flow. From the result of the Fig.5 (h) & (i), it is observed that decreases in the values of porous permeability parameter β and viscous parameter

α enhance the flow in the fluid temperature. In the steady flow of a viscous incompressible third grade fluid, velocity profile, temperature profile, skin friction, and Nusselt number are observed about the effect of variation of parameters, In this flow field, the value of pressure gradient M is kept at 1 if it affects the flow.

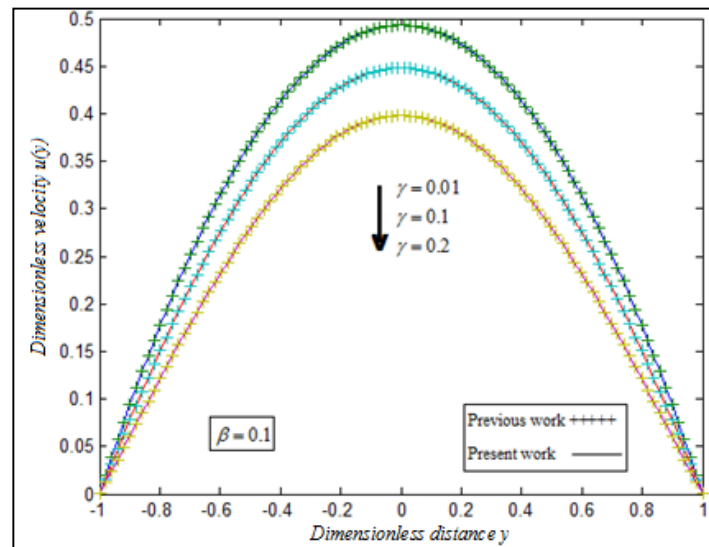


Fig 2(a): Dimensionless velocity $u(y)$ versus dimensionless distance y , The curves are plotted using the eqn. (9) for various values of γ and in some fixed values of the other dimensionless parameters.

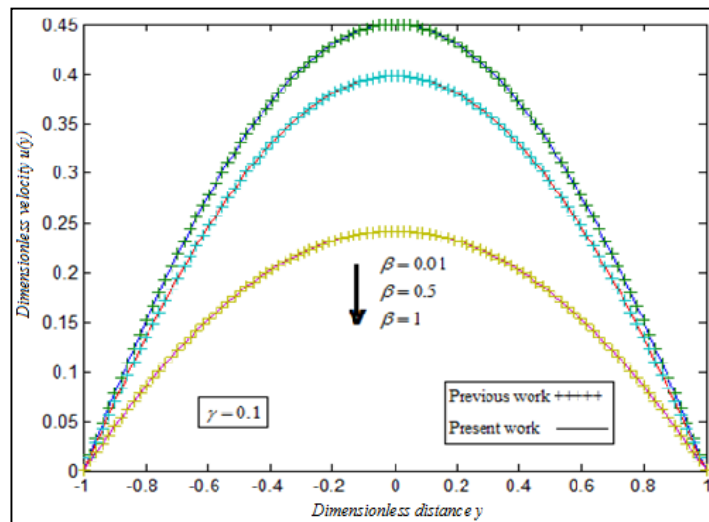


Fig 2(b): Dimensionless velocity $u(y)$ versus dimensionless distance y , The curves are plotted using the eqn. (9) for various values of β and in some fixed values of the other dimensionless parameters.

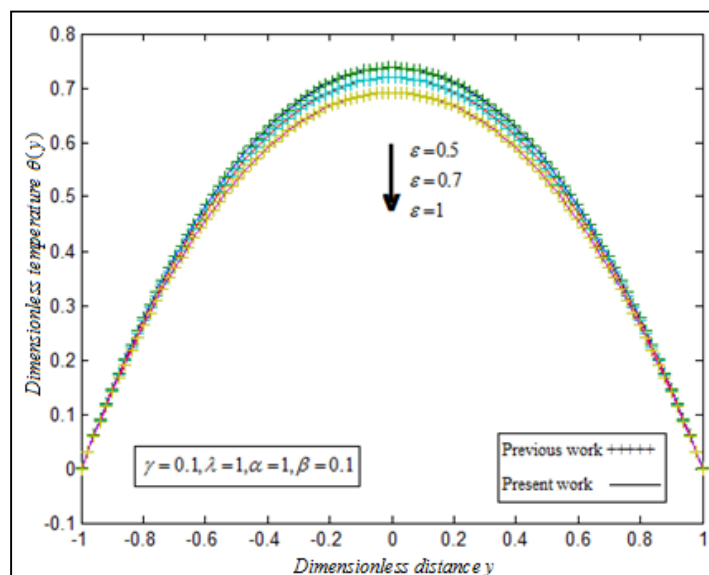


Fig 3(a): Dimensionless temperature $\theta(y)$ versus dimensionless distance y , The curves are plotted using the eqn. (10) for various values of ε and in some fixed values of the other dimensionless parameters

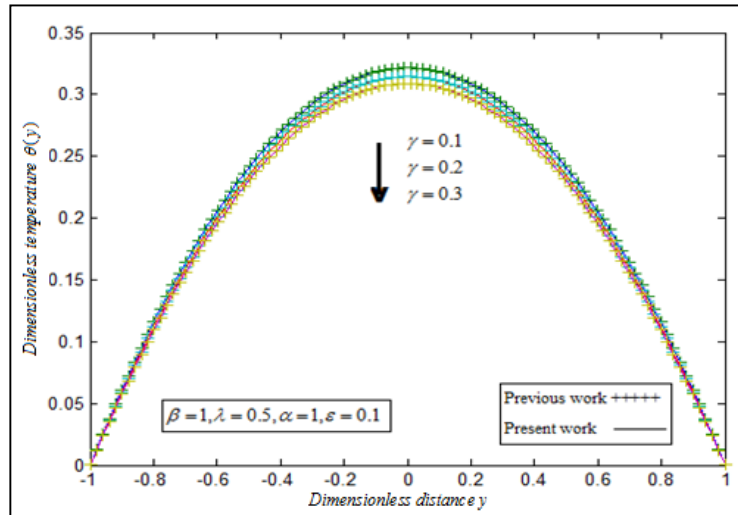


Fig 3(b): Dimensionless temperature $\theta(y)$ versus dimensionless distance y , The curves are plotted using the eqn.(10) for various values of γ and in some fixed values of the other dimensionless parameters.

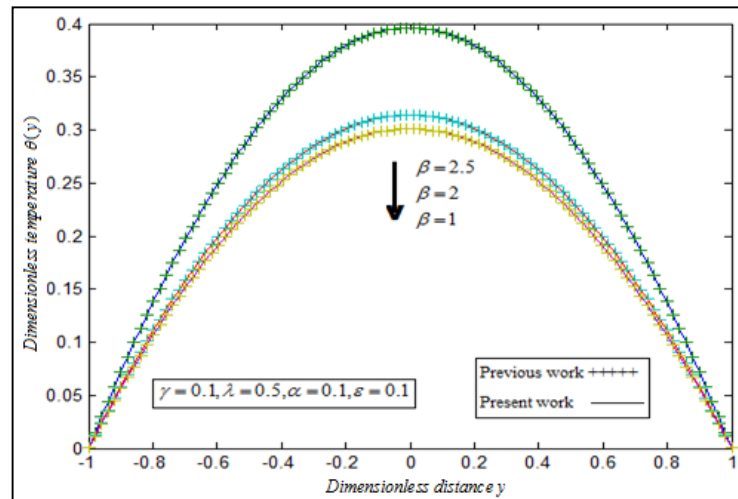


Fig 3(c): Dimensionless temperature $\theta(y)$ versus dimensionless distance y , The curves are plotted using the eqn. (10) for various values of β and in some fixed values of the other dimensionless parameters

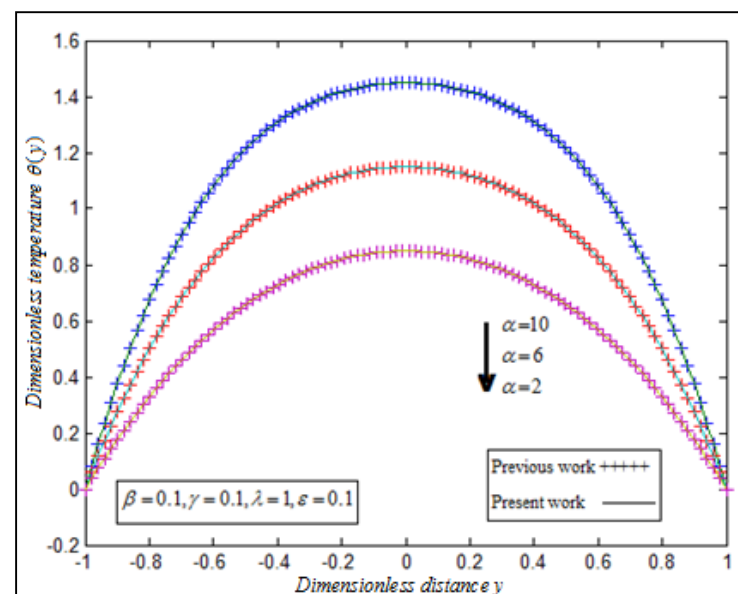


Fig 3(d): Dimensionless temperature $\theta(y)$ versus dimensionless distance y , The curves are plotted using the eqn.(10) for various values of α and in some fixed values of the other dimensionless parameters

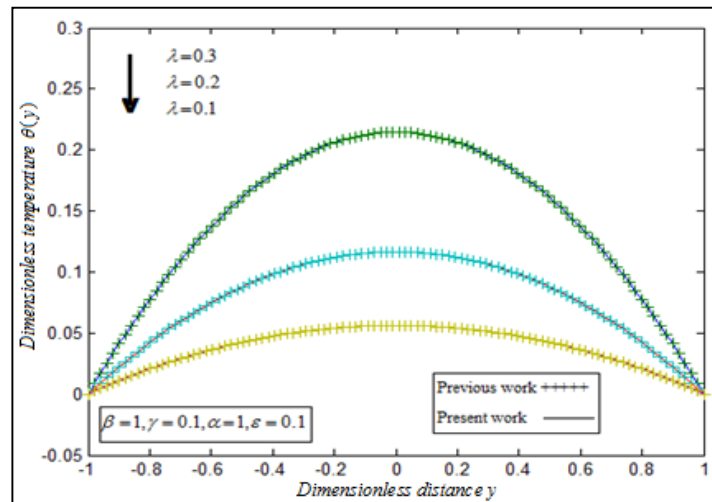


Fig 3(e): Dimensionless temperature $\theta(y)$ versus dimensionless distance y , The curves are plotted using the eqn.(10) for various values of λ and in some fixed values of the other dimensionless parameters

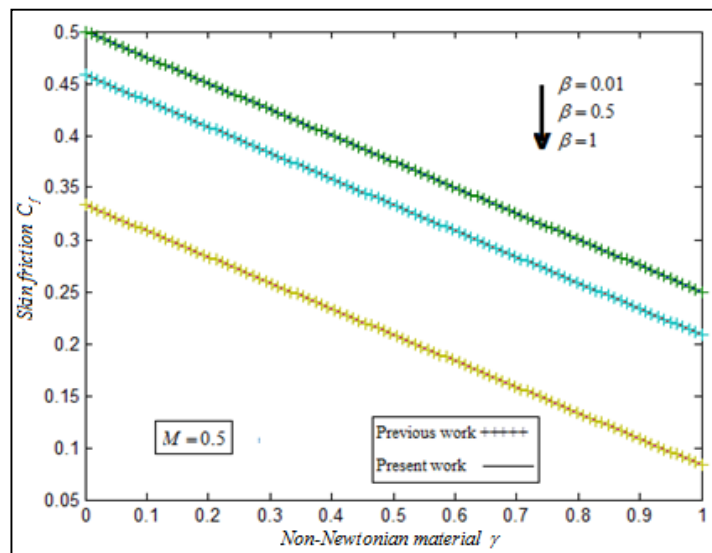


Fig 4(a): Skin friction C_f versus porous permeability parameter β . The curves are plotted using the eqn.(11) for various values of γ and in some fixed value of the other dimensionless parameter.

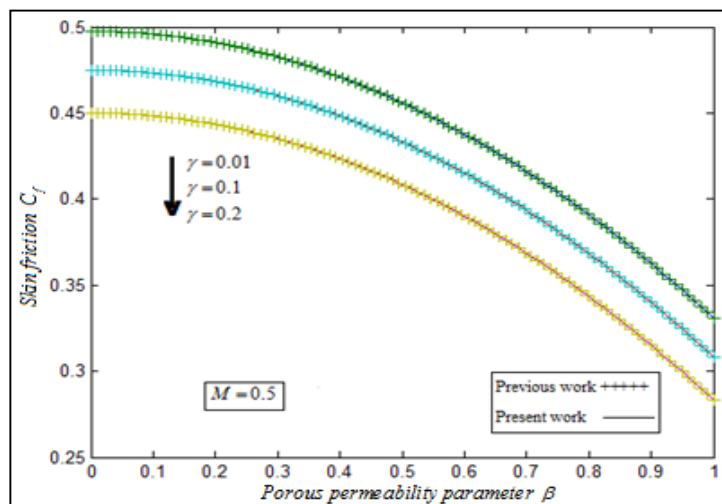


Fig 4(b): Skin friction C_f versus non-Newtonian material γ , The curves are plotted using the eqn. (11) for various values of β and in some fixed value of the other dimensionless parameter.

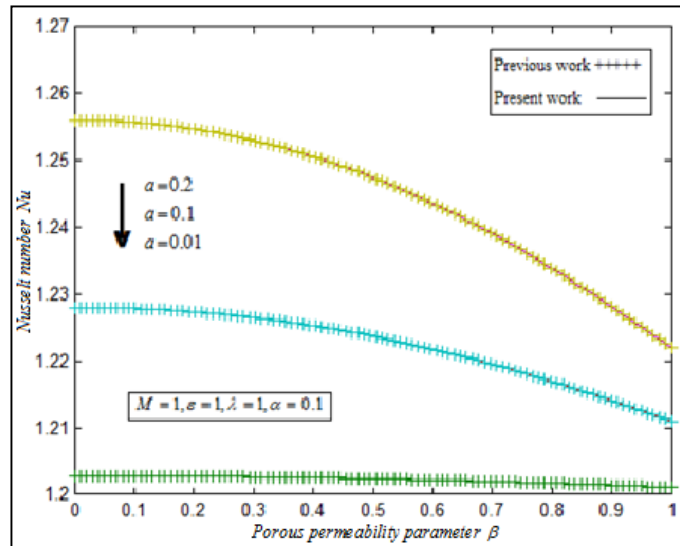


Fig 5(a): Nusselt number Nu versus porous permeability parameter β , The curves are plotted using the eqn.(12) for various values of α and in some fixed values of the other dimensionless parameters.

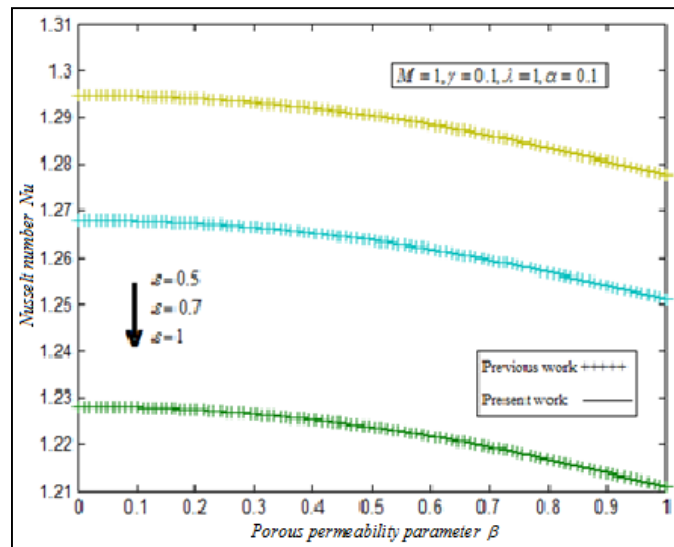


Fig 5(b): Nusselt number Nu versus porous permeability parameter β , The curves are plotted using the eqn. (12) for various values of ϵ and in some fixed values of the other dimensionless parameters.

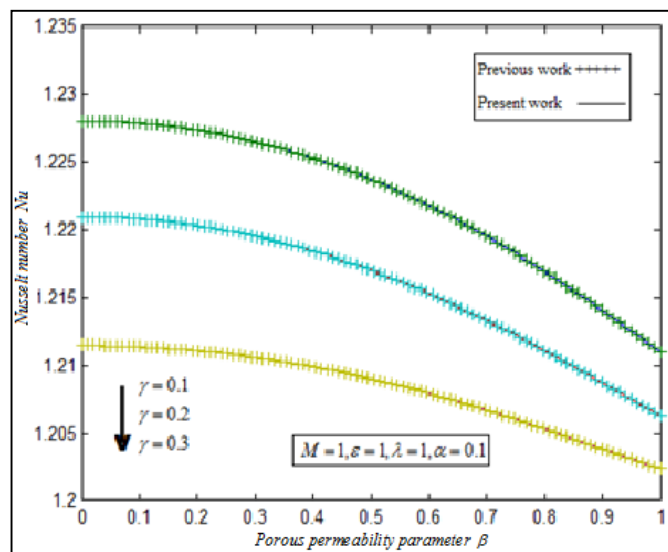


Fig 5(c): Nusselt number Nu versus porous permeability parameter β . The curves are plotted using the eqn. (12) for various values of γ and in some fixed values of the other dimensionless parameters.

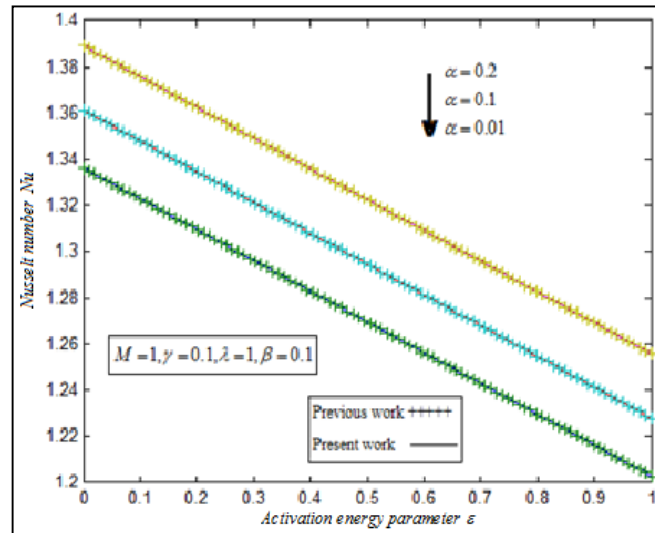


Fig. 5(d): Nusselt number Nu versus activation energy parameter ε , The curves are plotted using the eqn. (12) for various values of α and in some fixed values of the other dimensionless parameters.

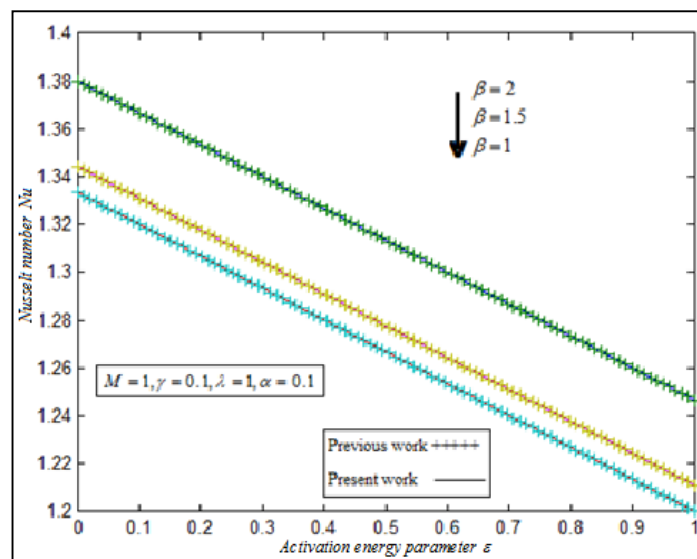


Fig 5(e): Nusselt number Nu versus activation energy parameter ε , The curves are plotted using the eqn. (12) for various values of β and in some fixed values of other the dimensionless parameters.

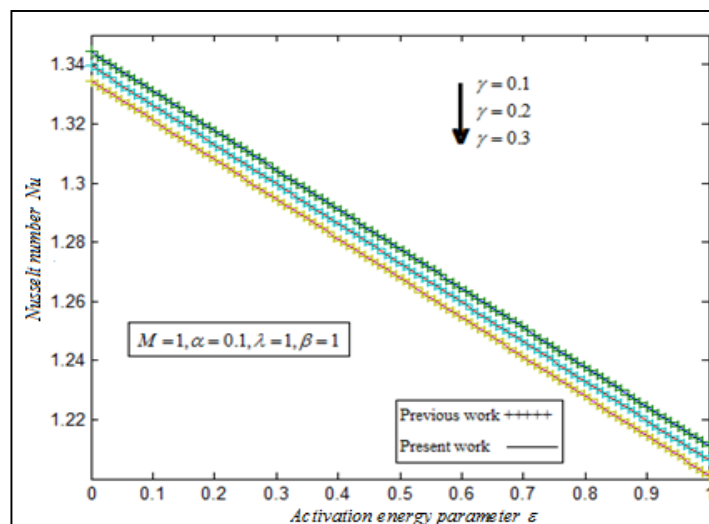


Fig 5(f): Nusselt number Nu versus activation energy parameter ε , The curves are plotted using the eqn. (12) for various values of γ and in some fixed values of the other dimensionless parameters.

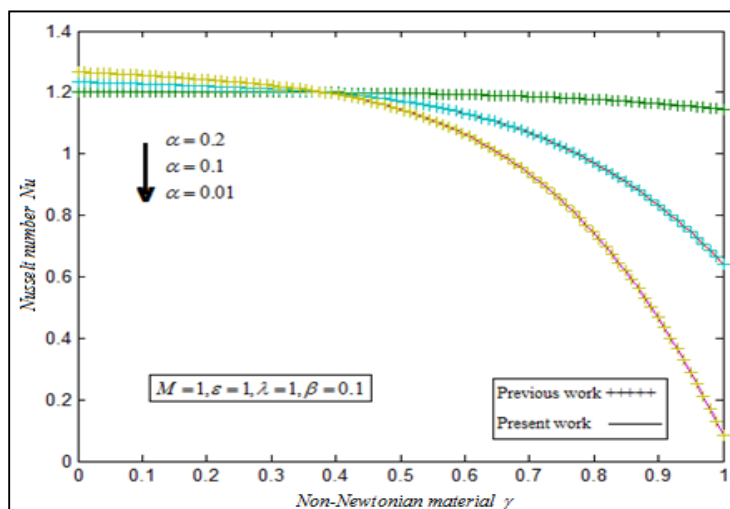


Fig 5(g): Nusselt number Nu versus non-Newtonian material γ , The curves are plotted using the eqn. (12) for various values of α and in some fixed values of the other dimensionless parameters.

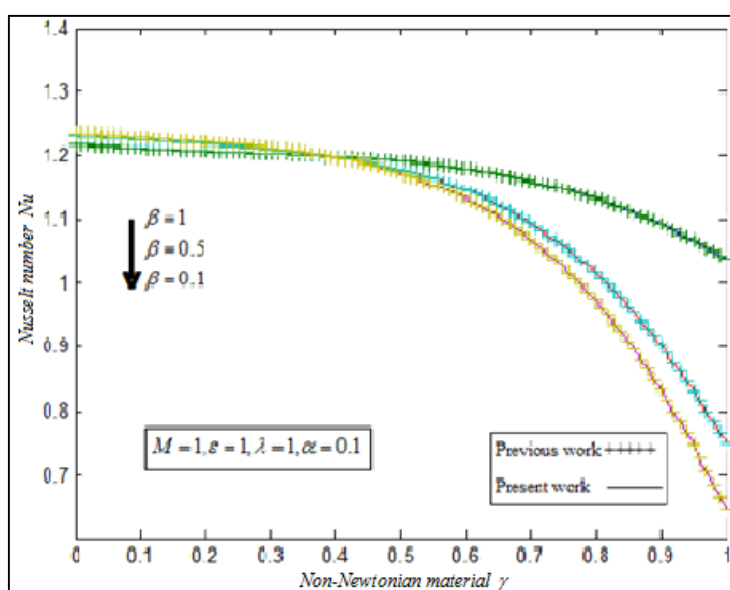


Fig 5(h): Nusselt number Nu versus non-Newtonian material γ using the eqn. (12) for various values of β and in some fixed values of the other dimensionless parameters.

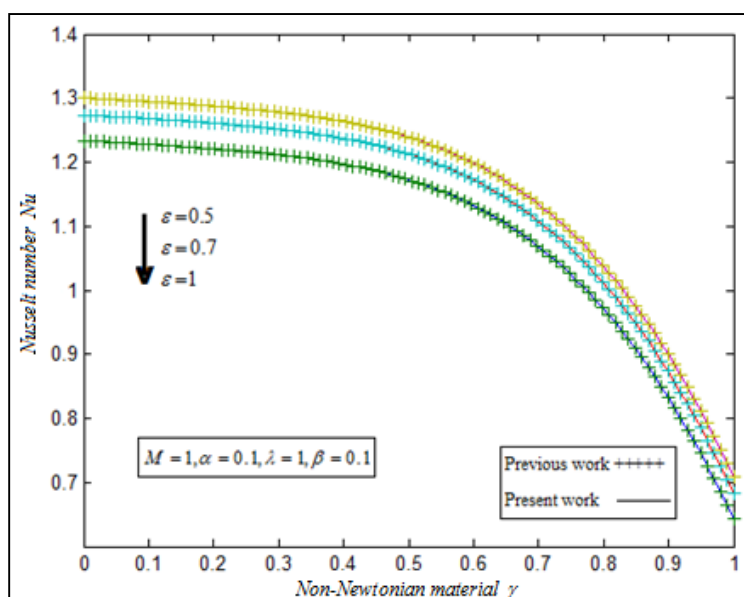


Fig 5(i): Nusselt number Nu versus non-Newtonian material γ . The curves are plotted using the eqn. (12) for various values of ϵ and in some fixed values of the other dimensionless parameters.

5. Conclusion

In this paper a mathematical analysis of the reaction of third grade fluid flow through the porous channel is done. The analytical expressions for the velocity and temperature profiles are obtained using Homotopy perturbation method. The analytical expressions of skin friction and Nusselt number are also obtained. If non-Newtonian parameter value is increased then the fluid flow decreases within the channel. The Frank-Kamenetskii parameter becomes large in the fluid temperature due to internal heat generation as a result of the exothermic chemical reactions. Our analytical results are compared with previous work and a satisfactory agreement is noted. This method can be easily extended to solve nonlinear boundary value problem in fluid mechanics, heat and mass transfer and other fields of engineering.

6. References

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Appendix A

Solution of non-linear differential equations (5) and (6) using the HPM

In this appendix, we indicate how the eqns.(5) and (6) in this paper are derived by HPM.

$$\frac{d^2 u}{dy^2} + 6\gamma \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right)^2 - \beta^2 u = -M \quad (\text{A.1})$$

$$\frac{d^2 \theta}{dy^2} + \lambda \left[e^{\frac{\theta}{1+\theta}} + \alpha \left(\frac{du}{dy} \right)^2 \left(1 + 2\gamma \left(\frac{du}{dy} \right)^2 \right) + \alpha \beta^2 u^2 \right] = 0 \quad (\text{A.2})$$

$$(1-p) \left[\frac{d^2 u}{dy^2} + M \right] + p \left[\frac{d^2 u}{dy^2} + 6\gamma \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right)^2 - \beta^2 u + M \right] = 0 \quad (\text{A.3})$$

We construct the Homotopy as follows

$$(1-p) \left[\frac{d^2 \theta}{dy^2} + \lambda \right] + p \left[\frac{d^2 \theta}{dy^2} + \lambda \left[e^{\frac{\theta}{1+\theta}} + \alpha \left(\frac{du}{dy} \right)^2 \left(1 + 2\gamma \left(\frac{du}{dy} \right)^2 \right) + \alpha \beta^2 u^2 \right] \right] = 0 \quad (\text{A.4})$$

The analytical solution of the eqns. (A.1) and (A.2) is

$$u = u_0 + pu_1 + p^2 u_2 + \dots \dots \dots \quad (\text{A.5}) \quad \theta = \theta_0 + p\theta_1 + p^2 \theta_2 + \dots \dots \dots \quad (\text{A.6})$$

Substituting the eqns. (A.5) and (A.6) into the eqns. (A.3) and (A.4) respectively we get

$$(1-p) \left[\frac{d^2 (u_0 + pu_1 + p^2 u_2 + \dots)}{dy^2} + M \right] + p \left[\frac{d^2 (u_0 + pu_1 + p^2 u_2 + \dots)}{dy^2} + 6\gamma \left(\frac{d^2 (u_0 + pu_1 + p^2 u_2 + \dots)}{dy^2} \right) \left(\frac{d(u_0 + pu_1 + p^2 u_2 + \dots)}{dy} \right)^2 - \beta^2 (u_0 + pu_1 + p^2 u_2 + \dots) + M \right] = 0 \quad (\text{A.7})$$

$$(1-p) \left[\frac{d^2 (\theta_0 + p\theta_1 + p^2 \theta_2 + \dots)}{dy^2} + \lambda \right] + p \left[\frac{d^2 (\theta_0 + p\theta_1 + p^2 \theta_2 + \dots)}{dy^2} + \lambda \left[\left((\theta_0 + p\theta_1 + p^2 \theta_2 + \dots) - \varepsilon (\theta_0 + p\theta_1 + p^2 \theta_2 + \dots)^2 \right) + \alpha \left(\frac{du}{dy} \right)^2 \left(1 + 2\gamma \left(\frac{du}{dy} \right)^2 \right) + \alpha \beta^2 u^2 \right] \right] = 0 \quad (\text{A.8})$$

Comparing the coefficients of like powers of p in the eqns. (A.7) and (A.8)

$$p^0 : \frac{d^2 u_0}{dy^2} + M = 0 \quad (\text{A.9})$$

$$p^0 : \frac{d^2 \theta_0}{dy^2} + \lambda = 0 \quad (\text{A.10})$$

$$p^1 : \frac{d^2 u_1}{dy^2} + 6\gamma \frac{d^2 u_0}{dy^2} \left(\frac{du_0}{dy} \right)^2 - \beta^2 u_0 = 0 \quad (\text{A.11})$$

$$p^1 : \frac{d^2 \theta_1}{dy^2} + \lambda \left[(\theta_0 - \varepsilon \theta_0^2) + \alpha \left(\frac{du_0}{dy} \right)^2 \left(1 + 2\gamma \left(\frac{du_0}{dy} \right)^2 \right) + \alpha \beta^2 u_0^2 \right] = 0 \quad (\text{A.12})$$

The initial approximations are as follows:

$$u_i(-1) = 0 = u_i(1) \quad (\text{A.13})$$

$$\theta_i(-1) = 0 = \theta_i(1) \quad i = 0, 1, 2, \dots \quad (\text{A.14})$$

Solving the eqns. (A.9) - (A.12) and with the help of the eqn. (A.13) and (A.14), we can obtain the following results.

$$u_0 = \frac{M}{2}(1 - y^2) \quad (\text{A.15})$$

$$u_1 = \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right) \frac{y^4}{12} + \frac{\beta^2 M}{4}(y^2 - 1) - \frac{1}{12} \left(6\gamma M^3 - \frac{\beta^2 M}{2} \right) \quad (\text{A.16})$$

$$\theta_0 = \frac{\lambda}{2}(1 - y^2) \quad (\text{A.17})$$

$$\theta_1 = -\lambda \left[A \frac{y^2}{2} + B y^4 + C y^6 + D y^8 + E y^{10} + F y^{12} + G y^{14} \right] + H \quad (\text{A.18})$$

Where the constants A, B, C, D, E, F, G & H are defined in the text eqns. (13) – (20) respectively
According to the HPM, we conclude that

$$u = \lim_{p \rightarrow 1} u(y) = u_0 + u_1 \quad (\text{A.19})$$

$$\theta = \lim_{p \rightarrow 1} \theta(y) = \theta_0 + \theta_1 \quad (\text{A.20})$$

Upon substituting the eqns. (A.15) & (A.16) into the eqn. (A.19), and (A.17) & (A.18) into the eqn. (A.20), we obtain the solutions in the text eqns.(7) and (8) respectively.

Appendix- B: Nomenclature

Symbol	Meaning
u'	Fluid velocity
p'	Fluid pressure
β_3	Material coefficient
k	Thermal conductivity
μ	Dynamic viscosity
γ	Dimensionless third grade material parameter
T	Fluid temperature
U	Characteristic velocity
T_a	Wall temperature
Q	Heat of reaction
A	Rate constant
R	Universal gas constant
E	Activation energy
C_0	Initial concentration of the reactant species
a	Channel width
y	Distance measured in the normal direction
α	Viscous heating parameter
θ	Dimensionless temperature
u	Dimensionless velocity
M	Dimensionless pressure gradient
β^2	Porous permeability parameter
Da	Darey parameter
K	Porous permeability
λ	Frank –Kamenetskii parameter