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MHD flow in porous medium with small internal heat generation under variable electrical conductivity

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Abstract

MHD convective flow of a viscous incompressible fluid of variable electrical conductivity along a vertical isothermal porous plate in presence of external transverse magnetic field and small internal heat generation ($E. Pr \leq 1$) along the plate. While the heat generation decays exponentially, electrical conductivity of the fluid is considered to be the function of fluid temperature. Using similarity transformation techniques and considering fluid flow of low Prandtl Number, numerical solutions and results are obtained for different values of Darcy Number using Runge-Kutta and Shooting methods. The results are shown graphically for absent and present of internal heat generation within the fluid. Variation of Skin-friction and the Rate of heat transfer at the plate for various values of physical parameters are also shown. The results show significant effects of heat generation and external field on flow and heat transfer under consideration.

Keywords: Magneto hydrodynamic (MHD), porous-plate, isothermal, electrical-conductivity, skin-friction, heat-transfer

1. Introduction

The effect of heat generation or absorption in MHD flows can be effectively dealt by taking into account the variation of fluid properties along with temperature field, Herwig, *et al.* (1986) [14]. The effect of internal heat generation is especially pronounced for low Prandtl Number fluid ($\cong 0.001$ to 1.0) e.g. liquid metal like Mercury, Bismuth, KCl solution, NaCl solution etc. This is because of the fact that they have smaller Prandtl number but higher thermal conductivity which provides them ability to transport heat even if small temperature difference exists. The MHD flow with suitable electrically conducting fluid under magnetic field can control the rate of cooling while achieved desired results, Chakrabarti *et al.* (1979) [7]. Liquid metals having smaller Prandtl Number of the order 0.01 to 0.1 ; e.g. Bismuth= 0.01 , Mercury= 0.023 at 20° temperature etc. are generally used as coolants because of higher thermal conductivity. Many authors have studied problems of natural convection flow along vertical isothermal plate with such fluids of low Prandtl number. Flow of such kind of fluids at stagnation point have been discussed by Pai *et al.* (1956) [16]. Kay reported that thermal conductivity of liquids with low Prandtl number varies linearly with temperature in range of 0°F to 400°F . Arunachalam and Rajappa (1978) [3], considered forced convection in liquid metals with variable thermal conductivity and capacity in potential flow and derived explicit closed form of analytical solution. Chen (1998) [6], considered laminar mixed convection flow adjacent to vertical, continuously stretching sheet. Molla *et al.* (2004) [15], studied the natural convection flow along a horizontal cylinder in the presence of heat generation. Recently, Gorla *et al.* (2013) [11], Alam M. S. (2011) [4], Chain, T. C. (1998) [5], Hazen A. Allia (2002) [12], and many others have studied MHD flow with heat generation problem with various geometries. Boracic *et al.* (2010) [4], has studied natural convection MHD flow with variable electrical conductivity and heat generation along an isothermal plate. Crepeau, and Clarksean (1997) [10], had studied Natural Convection with internal heat generation. Chamkha and Khalad. (2001) [9] had given solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with internal heat generation or absorption. More recently, Sharma and Singh (2012) [18], have studied steady MHD natural convection flow with variable electrical conductivity and heat generation along an isothermal vertical plate.

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In this paper, MHD convective flow of a viscous incompressible fluid of variable electrical conductivity along a porous plate is discussed where fluid is sucked through vertical plate and maintained at constant suction velocity under the action of internal heat generation (small Eckert Number) and variable electrical conductivity in presence of transverse magnetic field. It is supposed that the heat generation decays exponentially and electrical conductivity of the fluid is considered to be the function of fluid temperature. Using similarity transformation techniques and considering fluid flow of low Prandtl Number, numerical solutions and results are obtained for different values of Darcy Number using Runge-Kutta and Shooting methods. Considering the energy dissipation due to porosity of the medium, the nature of flow parameters like fluid velocity, temperature, skin friction and heat transfer at the plate are analyzed graphically and thereby drawn to some conclusions.

2. Mathematical Formulation

Let us consider steady laminar natural convection flow of a viscous incompressible fluid over a vertical non-conducting plate in a medium. It is considered that the plate is porous and at constant temperature that generates an internal volumetric heat within the plate that affect fluid flow while the fluid electrical conductivity varies inversely with temperature. The x-axis is taken along the plate and y-axis is normal to the plate. A uniform magnetic field of intensity B_0 is applied normal to the plate. It is assumed that the electrical field due to polarization of charges and Hall Effect are negligibly small. Incorporating the Boussinesq's approximation within the boundary layer, the governing equations of continuity, momentum and energy, (Schlichting (1968) [17], respectively are given as follows.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T_w - T_\infty) \frac{\sigma_1 B_0^2}{\rho} u - \frac{\nu}{k_1} u = 0 \quad (2)$$

$$\rho C_p (u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}) = K \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{K_1} u^2 + Q \quad (3)$$

$$\sigma_1 = \frac{\sigma}{1 + \epsilon \theta} \quad (4)$$

The boundary conditions are

$$y = 0 : u = 0, v = 0, T = T_w$$

$$y \rightarrow \infty : u \rightarrow 0, T \rightarrow T_\infty \quad (5)$$

3. Method of Solution

Introducing the stream function $\Psi(x, y)$ such that

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = - \frac{\partial \Psi}{\partial x} \quad (6)$$

Where,

$$\Psi(x, y) = 4\nu f(\eta) \left(\frac{Gr}{4}\right)^{\frac{1}{4}} \text{ and } \eta = \frac{y}{x} \left(\frac{Gr}{4}\right)^{\frac{1}{4}} \quad (7)$$

Following Crepeau and Clarksean (1997), the volumetric rate of heat generation is

$$Q = S \left\{ k \left(\frac{T_w - T_\infty}{x^2} \right) \left(\frac{Gr}{4} \right)^{1/2} e^{-n} \right\} \quad (8)$$

Since equation (1) is identically satisfied equation (6), using equations (6), (7), and (8), equations (2) and (3), along with the equation (4), the resulted coupled non-linear ordinary differential equations, given are as follows

$$f'''' - 2f'^2 + 3ff'' + \theta - \left[\frac{M}{1 + \epsilon \theta} + \frac{1}{Da} \left(\frac{4}{Gr} \right)^{\frac{1}{2}} \right] f' = 0 \quad (9)$$

$$\theta'' + \theta' + 3Pr \theta' f - \left[16 \frac{E Pr}{Da} \left(\frac{Gr}{4} \right)^{\frac{1}{2}} (f')^2 \right] + Se^{-n} = 0 \quad (10)$$

Where,

$$\nu = \frac{\mu}{\rho}; Gr = \left(\frac{g \beta (T_w - T_\infty) x^3}{\nu^2} \right); M = \frac{\sigma B_0^2 x^2}{\mu} \left(\frac{Gr}{4} \right)^{-1/2}; Pr = \frac{\mu C_p}{K}; Q = K \left(\frac{T_w - T_\infty}{x^2} \right) \left(\frac{Gr}{4} \right)^{\frac{1}{2}} e^{-n}; \theta = \frac{T - T_\infty}{T_w - T_\infty}; Da = \frac{K_1}{x^2}$$

The boundary conditions are reduced to

$$f(0) = 0, f'(0) = 0, f'(\infty) = 0, \theta(0) = 1 \text{ and } \theta(\infty) = 0 \quad (11)$$

The governing boundary layer equations (9) and (10) with boundary conditions (11) are solved using Runge-Kutta fourth order technique along with double shooting technique.

3.1 Skin Friction Coefficient

$$\tau = \frac{\tau}{\frac{1}{2}\rho u_0^2} = 2 \left(\frac{Gr}{4}\right)^{\frac{1}{4}} f''(0) \quad (12)$$

where,

$$(\tau)_{y=0} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)_{y=0}, \text{ shear stress at the plate and } u_0 = \sqrt{\{g\beta x(T_w - T_\infty)\}}, \text{ convective fluid velocity near the plate.}$$

3.2 Rate of Heat Transfer

The rate of heat transfer in terms of the Nusselt number at the plate is given by

$$Nu = \frac{qx}{K(T_w - T_\infty)} = -\left(\frac{Gr}{4}\right)^{\frac{1}{4}} \theta'(0)$$

$$\text{where, } q = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

Nomenclature

x, y : Cartesian coordinates	u, v: velocity components along x- and y- directions ;	
ρ : density of fluid;	g: Acceleration due to gravity of the Earth.	
Ψ : stream function;	μ : coefficient of viscosity	ν : kinematic viscosity
T: fluid temperature;	T_∞ : Temperature of fluid far away from plate;	
η : similarity variable	f: dimensionless stream function;	
τ : skin-friction coefficient;	Q: volumetric rate of heat generation	
θ : dimensionless temperature	β : coefficient of thermal expansion;	
Pr: Prandtl Number	k: coefficient of thermal conductivity	
Nu: Nusselt number	C_p : specific heat at constant pressure :	
B_o : magnetic field intensity	M: Hartmann number;	σ : electrical conductivity
E: Eckert Number	ε : electrical conductivity parameter	
Da: Darcy number	σ_1 : variable electrical conductivity	

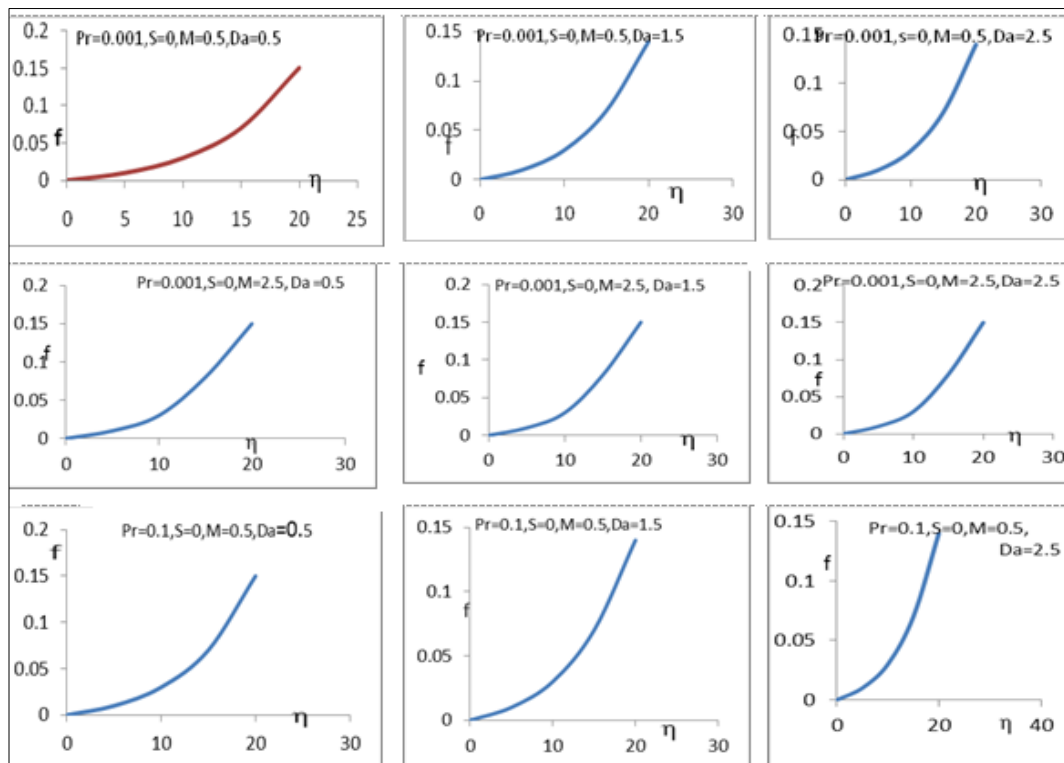
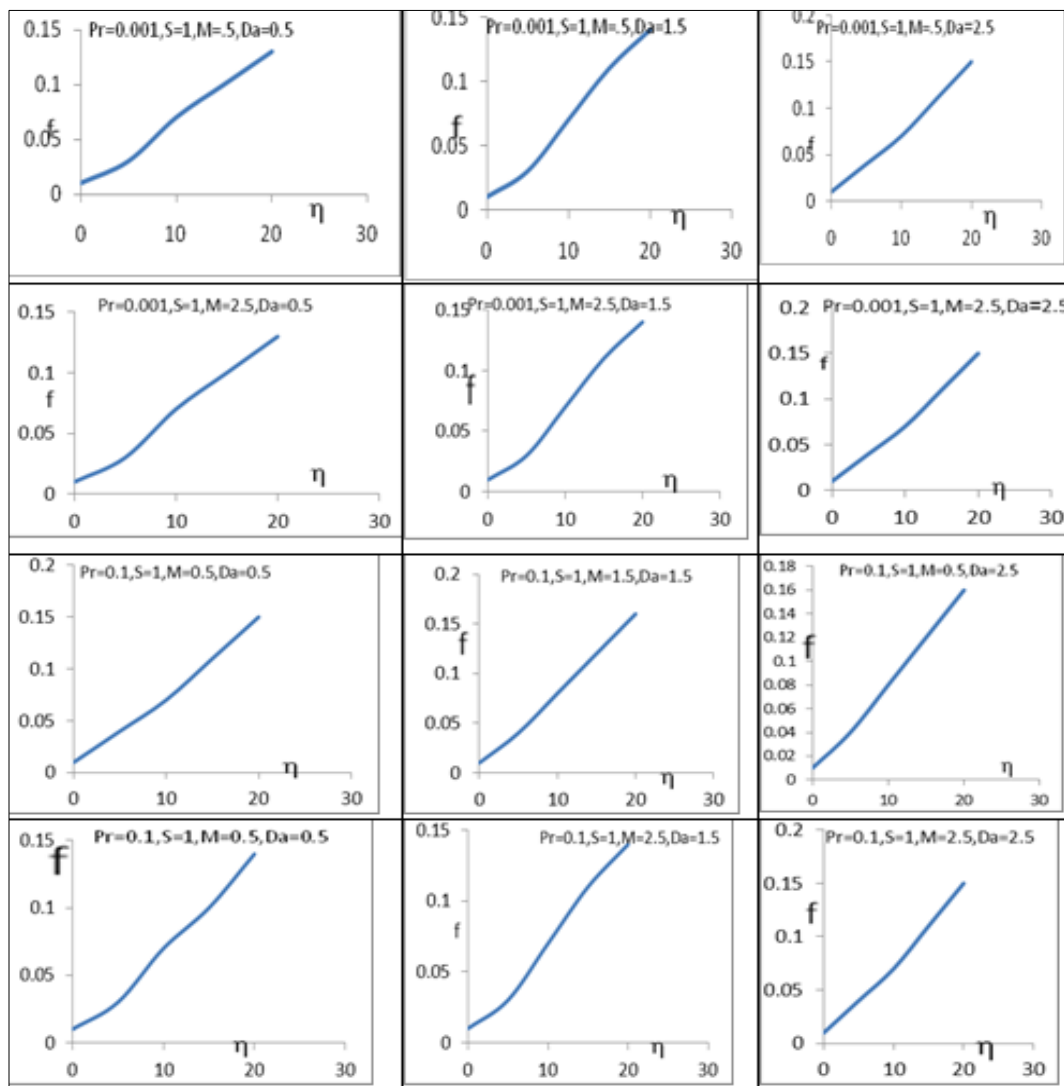
4. Solutions of Equations

Solution for the equations (9 & 10) subject to the boundary condition (11) are obtained using Shooting iteration technique along with fourth order Runge-KUTTA method for different values of physical parameters. In calculating numerical results for physical quantities of our interest f , T , τ & Nu we have considered $S = 0.0$ which means absence of internal heat generation while $S > 0$ means presence of it, $Gr = 5.00$ because it relates to many important magnetohydrodynamic phenomena such as problems of cooling in nuclear reactors etc. ; $Pr = 0.001$ to 0.10 since it is related to the popular liquids metals ; $E = 1.0$ which means low heat generation, $n = 1.0$ (chosen arbitrarily). The physical parameters whose effects on flow motion are the objectives of this study, varied as $Da = 1.0$ to 2.5 ; $\varepsilon = 0.01$ (KCl solution $\varepsilon = 1.05$ at 15°C), to 20.0 (NaCl solution $\varepsilon = 20.14$ at 15°C); $M = 0.2$ to 1.5 ; $S = 0.0$ to 2.5 . The various values of non-dimensional parameters fluid-velocity (f), fluid-temperature (T), Skin-friction at the plate (τ) $_{\eta=0}$, and the rate of heat transfer at the plate (Nu) $_{\eta=0}$, has been obtained from the numerical solutions. Values are plotted for above mentioned values of Da , ε , M and S ; the results are shown in the figures (1-10).

4.1 Numerical Solutions

The system of non-linear ordinary differential equations (9 - 10) together with the boundary conditions (11) are solved numerically using Nachtsheim-Swigert shooting iteration technique (guessing the missing values) along with fourth order Runge-Kutta initial value solver. Chakraborty *et al.* (2001) ^[8], Hazarika *et. al.* (2002) ^[13], Alam *et al* (2011) ^[2] have also used same technique to solve their problems.

5. Results

Fig 1: (i – ix) Velocity Profile for $S=0.0$ Fig 2: (i – xii) Velocity Profile at $S = 1.0$

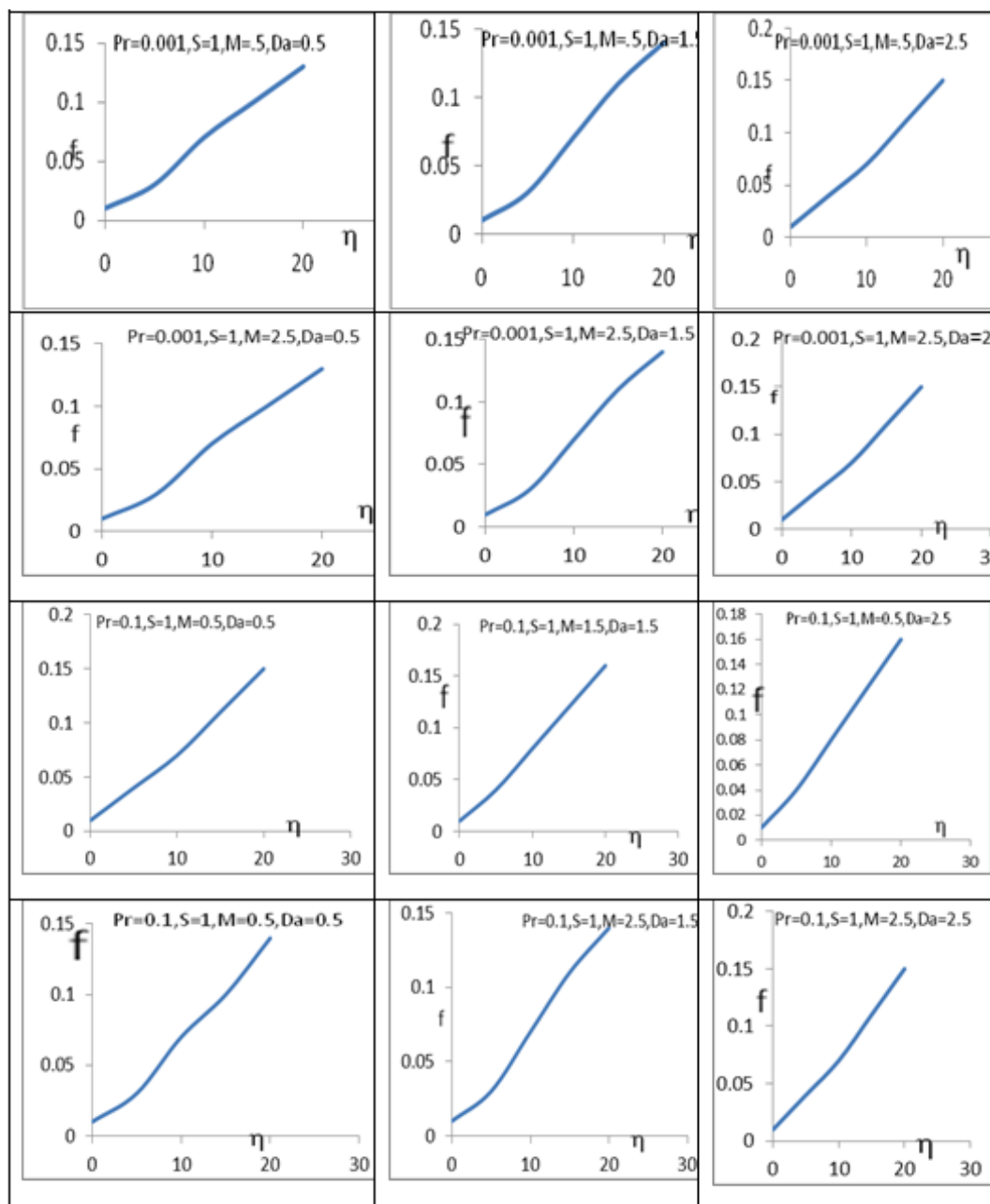
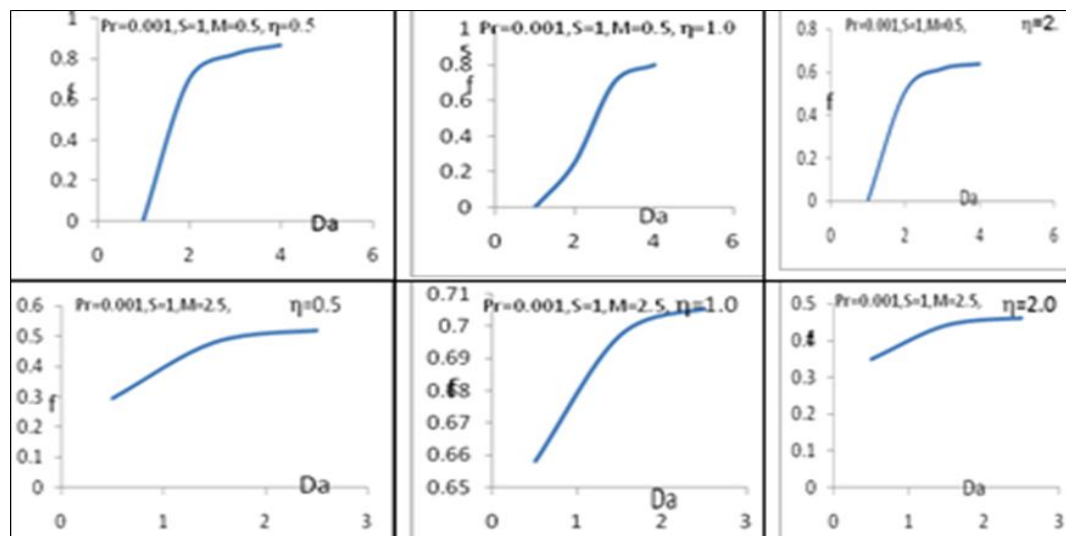


Fig 3: (i – xv) Velocity variation with Da , for $S = 0.0$



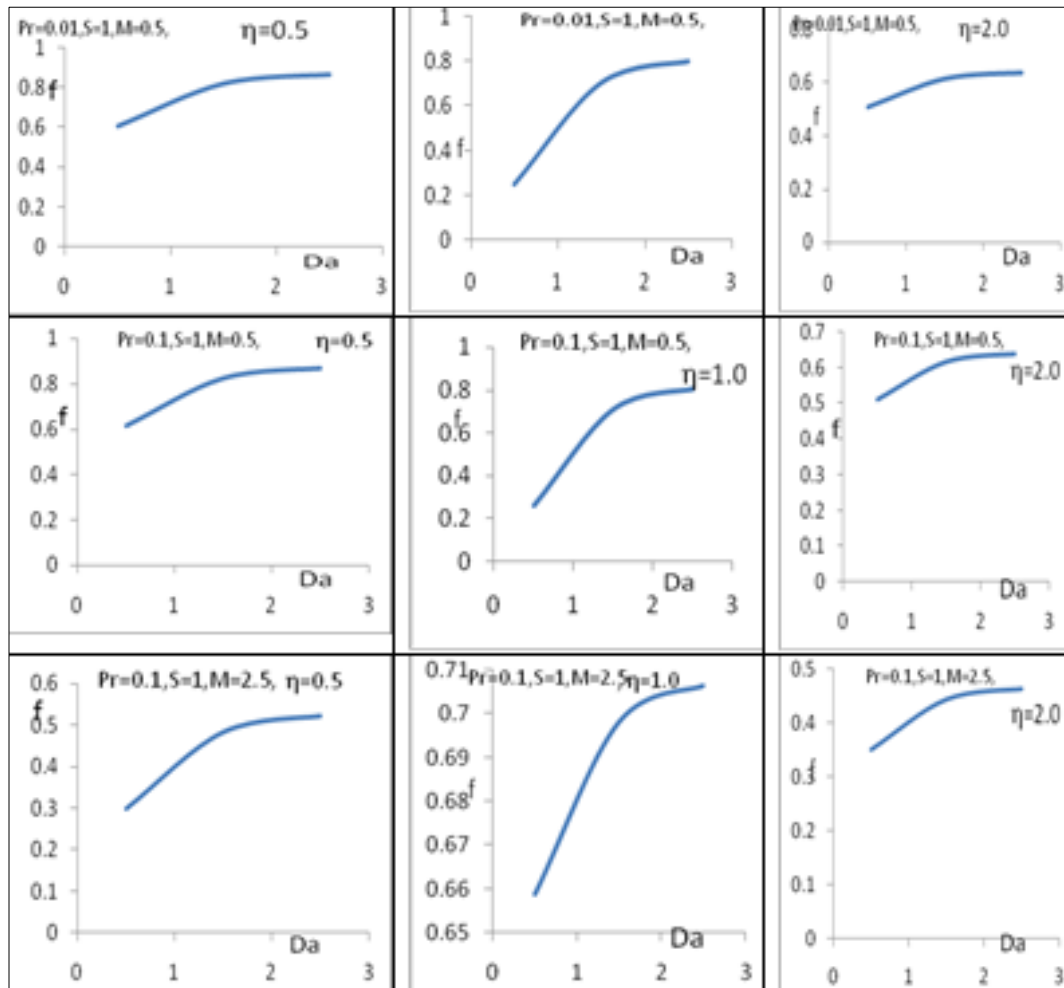


Fig 4: (i – xviii), Velocity variation with Da, at $S = 1.0$

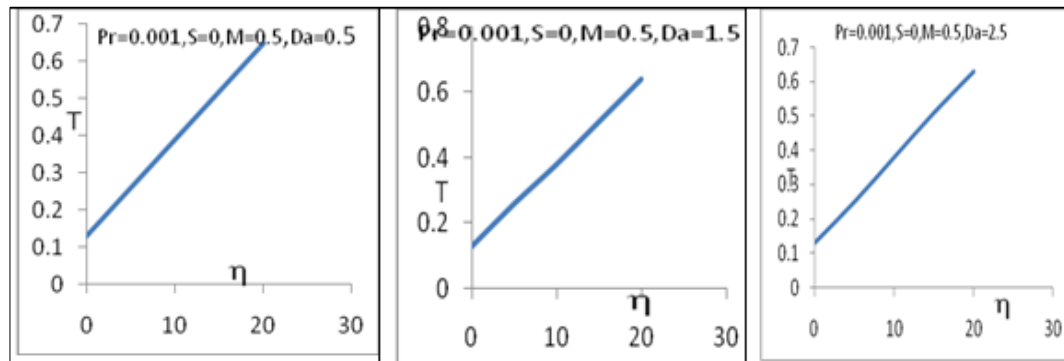


Fig 5: (i – xii), Temperature Profile at $S = 0.0$

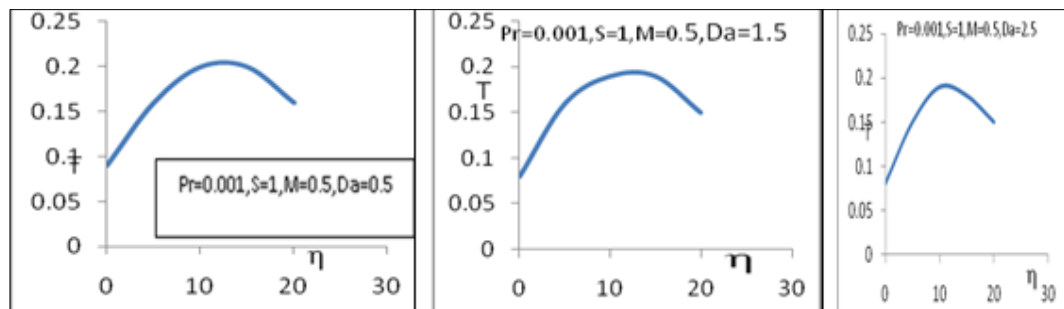


Fig. 6: (I – xii), Temperature Profile at $S = 1.0$

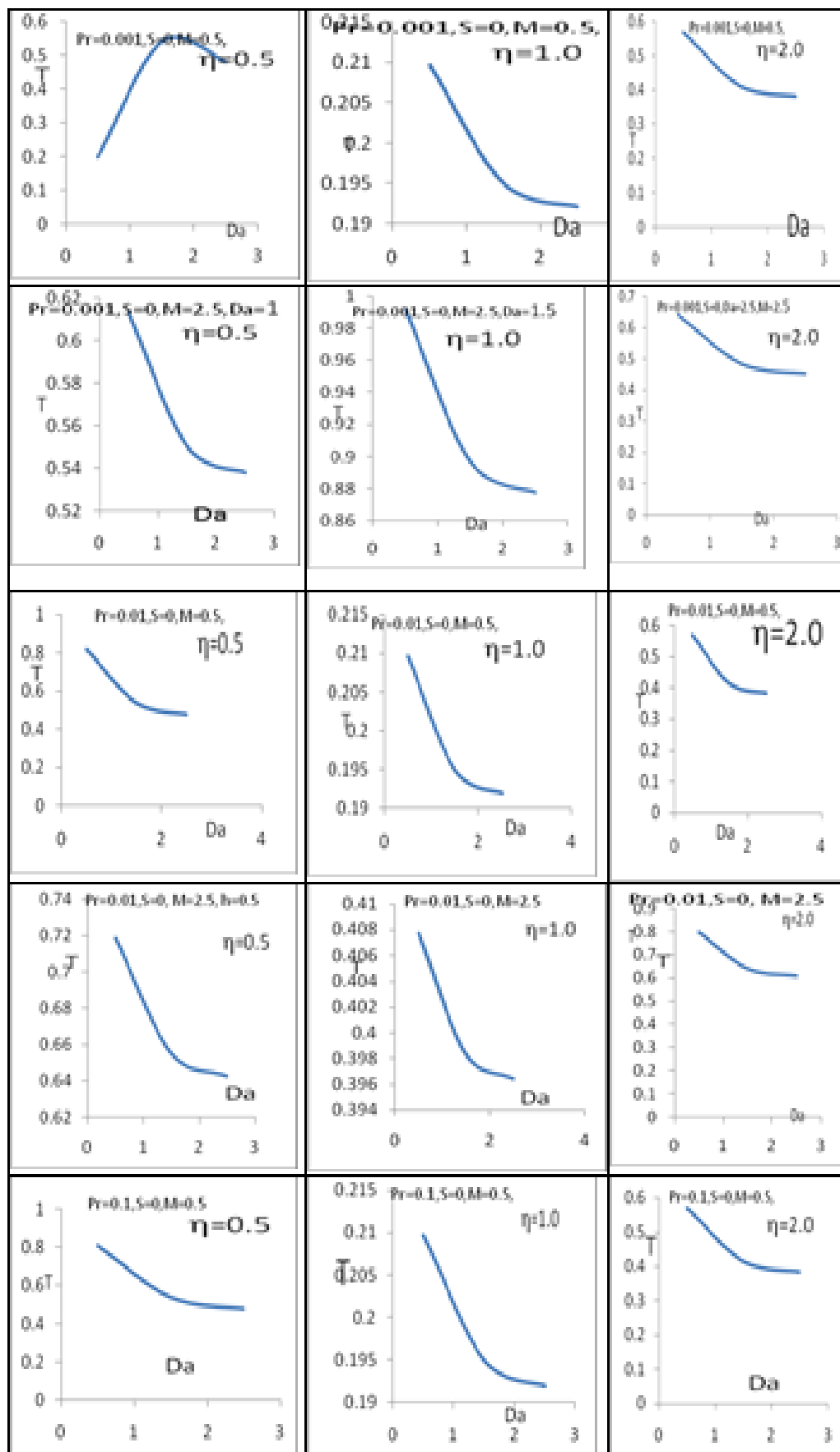


Fig 7: (i – xv) Temperature with Da ; $S = 0.0$

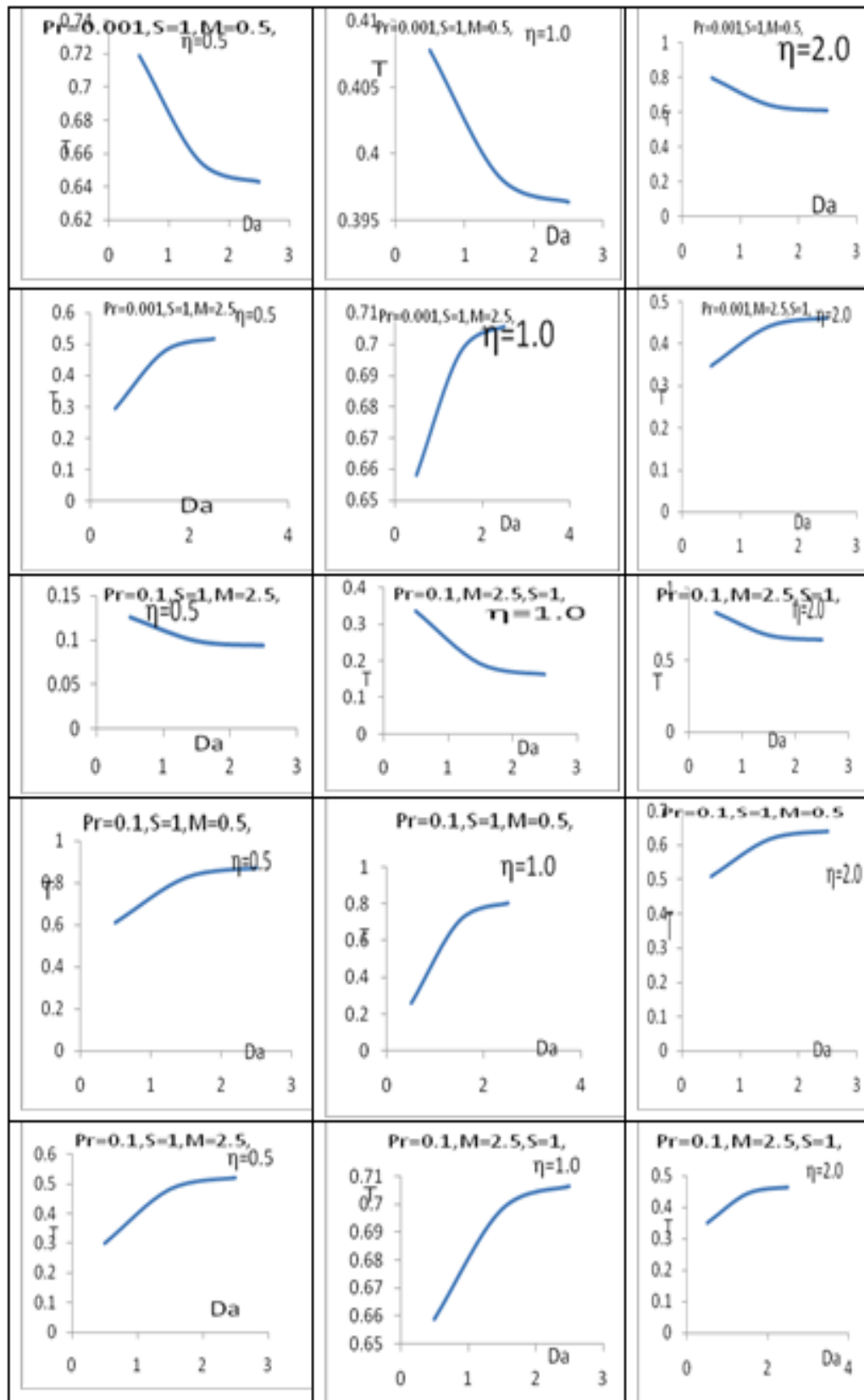


Fig 8: (i – xv) Temperatuare with Da ; $S = 1.0$

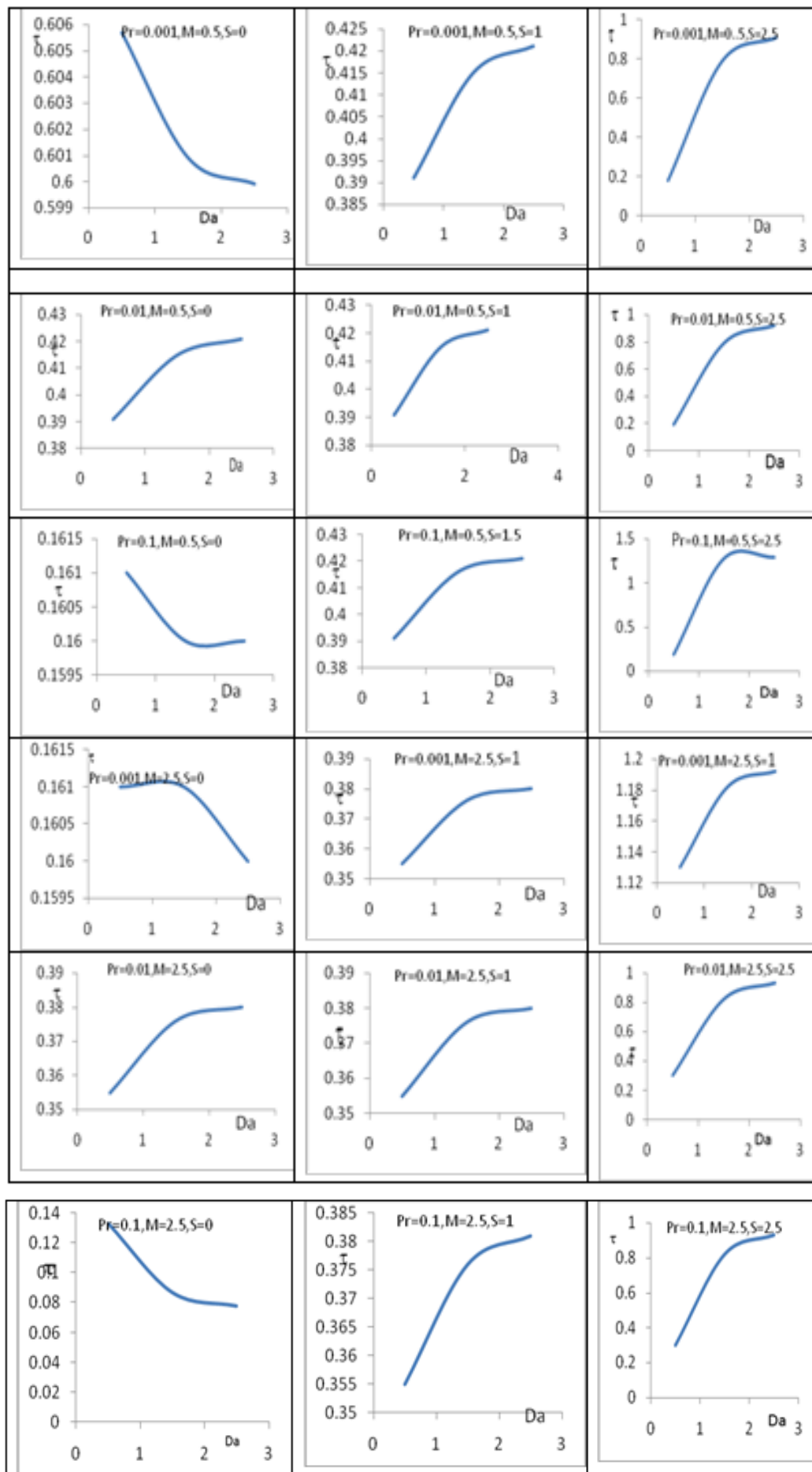


Fig 9: (i-xxi), Skin Friction Profile

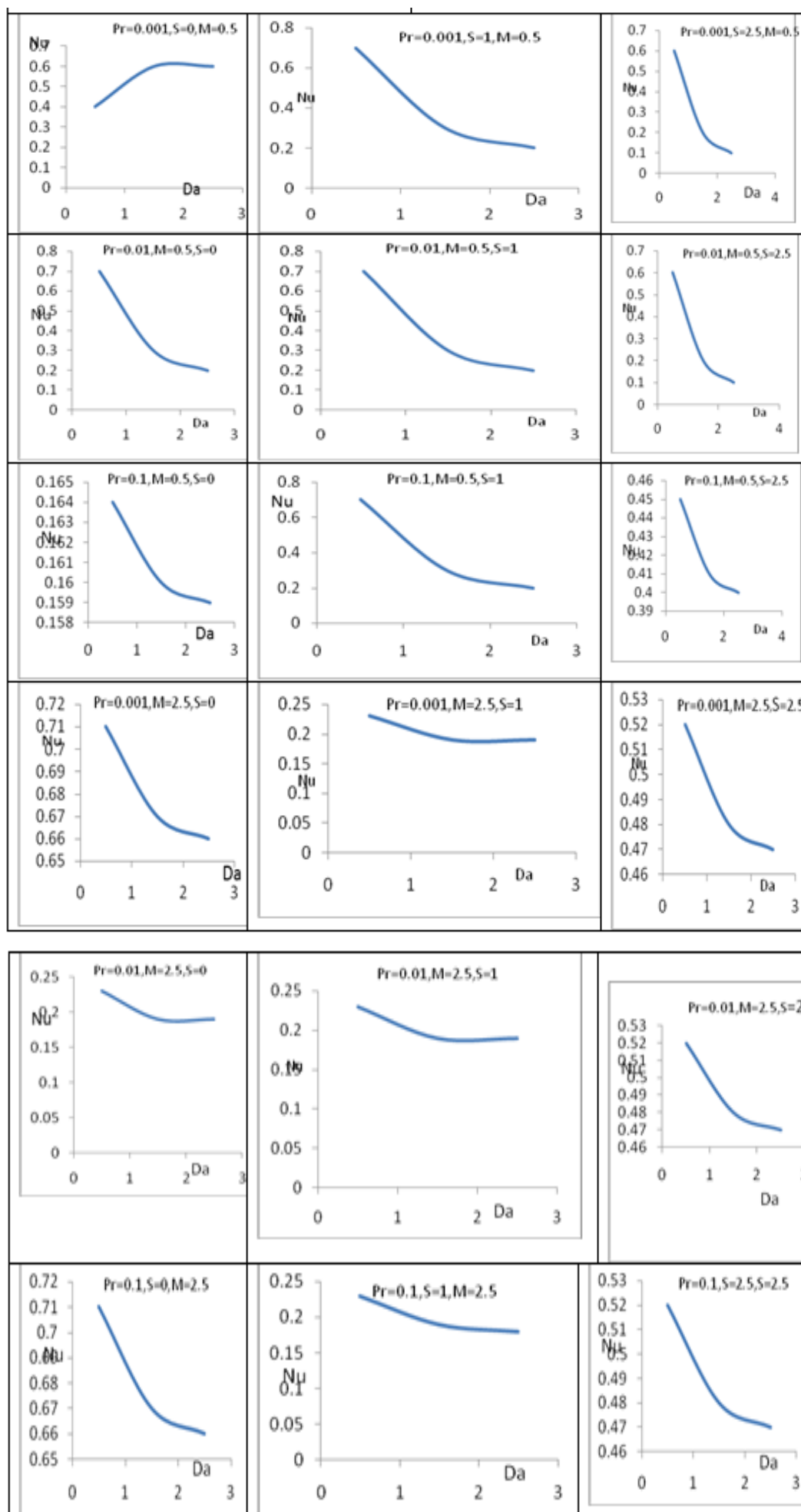


Fig. 10. Heat transfer Profile

Discussions

- Fig.1 (i – iii), at $S=0.0$, f increases exponentially with η for Da ($= 0.5$ to 2.5), M ($= 0.5$ to 2.5), Pr ($= 0.001$ to 0.1). With the increase of Da ($= 0.5$ to 2.5), keeping M & Pr constant, f slowly decreases fig1. (i-iii). With increase of M ($= 0.5$ to 2.5), keeping Da & Pr constant, f slowly decreases fig1. (I & iv, ii & v, iii & vi). For higher value of Pr , f increases slowly when Da & M are constant fig1. (i & vii, ii & viii, iii & ix)
- Fig. 2(i – xii), at $S=1.0$, f increases linearly with η away from the plate for Da ($=0.5$ to 2.5), M ($= 0.5$ to 2.5) and Pr ($=0.001$ to 0.1). With the increase of Da , keeping M & Pr constant, f slightly increases fig2.(i-iii, iv –vi, vii-ix). With increase of M ($= 0.5$ to 2.5), keeping Da & Pr constant, f slowly decreases. For higher value of Pr , f increases slowly when Da & M are constant fig2.(I & vii, ii & viii, iii & ix).
- Fig. 3 (i – xv), at $S = 0.0$, f decreases exponentially with the rise of Da . Magnitude of f rises with the rise of Pr but decreases with the rise of M . The variation of f with Da is found much for $\eta \cong 1.0$
- Fig. 4(i – xviii), at $S = 1.0$, away from the plate ($\eta \geq 0$), f increases with Da , for all values of M & Pr . For higher values of M , f rises exponentially with Da , fig(i & iv; iii & vi) ; similarly when Pr is increased fig(i & x; iii & xiii). The magnitude of f decreases at higher value of M , fig.(i & iv; iii & vi). f decreases further when both M & Pr are higher fig (xiii - xvi).
- Fig. 5 (i – iii), Temperature Profile at $S = 0.0$, T increases sharply away from the plate ($\eta \geq 0$), for Da (0.5 to 2.5), M ($= 0.5$ to 2.5) and Pr ($=0.001$ to 0.1). T falls slowly with the increase of Da , keeping M and Pr constant.
- Fig. 6 (i – xii), Temperature Profile at $S = 1.0$, the nature of variation of T away from the plate is different from those for $S=0.0$. At $S=1.0$, T first increases then gradually decreases fig.6 (i-iii).
- Fig. 7(i – xv) at $S = 0.0$, T decreases slowly with the rise of Da , magnitude of T rises with the rise of Pr fig. 7(i & xiii, ii & xiv, iii & xv). Exception is that at very near to the plate and smaller values of M & Pr , T rises than decreases slowly fig.7(i). For higher value of M , magnitude of T increases fig7(I & iv, ii & v, iii & vi); similarly for Pr . The variation of T with Da is found much for $\eta \cong 1.0$.
- Fig. 8(i – xv) at $S = 1.0$, the nature of variation of T with Da for $Pr = 0.001$ is different for $Pr > 0.001$. For $Pr = 0.001$, T falls exponentially with the rise of Da fig.8 (i-iii), while for higher values of Pr (> 0.001), T rises exponentially. The nature of variation of T with Da is opposite for $M= 0.5$ (exponential decrease) to that $M=2.5$ (exponential increase)fig.8(x-iii) & fig.8 (vii-ix) respectively; oppositely for Pr , fig8.(iv-vi) & (vii-ix).
- Fig.9 (i-xxi), the nature of variation of Skin friction (τ) with the rise of Da is almost opposite for $S=0$ and when M & Pr have lower values fig.9(i,vii,xiii, ixx) to that for higher values of $S(= \geq 1)$, M & Pr . With the rise of Pr (for M & S constant), τ decreases (fig I, iv, vii) whereas, with the rise of M (for Pr & S constant) τ increases fig9.(ii & xi, v & xiv, viii & xx)
- From fig. 10(i-xvii), heat transfer Nu decreases exponentially with Da for all values of S , M & Pr except when all these values are very small (i.e. $S=0.0$, $Pr = 0.001$, $M=0.5$) fig10.(i). For higher value of M , Nu decreases with the increase of Da fig.10 (i & vii, iii & ix); this is almost opposite in case of Pr fig10. (i & vii, ii & viii, iii & ix).

7. Conclusions

- In absence of internal heat generation ($S=0.0$), fluid velocity increases exponentially away from the plate whereas in presence ($S=1.0$), it increases linearly. With the increase of Da , for $S=0.0$ fluid velocity slowly decreases whereas it increases linearly for $S=1.0$.
- In absence of internal heat generation $S = 0.0$, fluid temperature T increases sharply away from the plate ; whereas for $S=1.0$, T first increases then decreases gradually. The variation of T with Da significantly depends upon value of Pr in case of $S=1.0$ to that $S=0.0$
- The nature of variation of Skin friction (τ) with the rise of Da is almost opposite in presence of heat generation $S=0$ and lower values of M & Pr to that in presence of heat generation $S(= \geq 1)$, and for higher values M , Pr and $S(= \geq 1)$.
- The rate of Heat transfer Nu decreases exponentially with Da for all values of S , M & Pr except when all these values are very small (i.e. $S=0.0$, $Pr=0.001$, $M=0.5$).

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